$$D_y = \begin{vmatrix} 1 & 7 & 3 \\ 2 & -4 & -3 \\ 5 & -5 & 0 \end{vmatrix} = 1\begin{vmatrix} -4 & -3 \\ -5 & 0 \end{vmatrix} - 7\begin{vmatrix} 2 & -3 \\ 5 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & -4 \\ 5 & -5 \end{vmatrix}$$
$$= 1(-15) - 7(15) + 3(10)$$
$$= -90$$

$$D_z = \begin{vmatrix} 1 & 1 & 7 \\ 2 & -1 & -4 \\ 5 & -2 & -5 \end{vmatrix} = 1\begin{vmatrix} -1 & -4 \\ -2 & -5 \end{vmatrix} - 1\begin{vmatrix} 2 & -4 \\ 5 & -5 \end{vmatrix} + 7\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$
$$= 1(-3) - 1(10) + 7(1)$$
$$= -6$$

Applying Cramer's rule, we can solve for x, y, and z.

$$x = \frac{D_x}{D} = \frac{-18}{-18} = 1, \quad y = \frac{D_y}{D} = \frac{-90}{-18} = 5, \quad \text{and} \quad z = \frac{D_z}{D} = \frac{-6}{-18} = \frac{1}{3}$$

Therefore, the solution to the system is $\left(1, 5, \frac{1}{3}\right)$.

Now work margin exercise 4.

The determinants shown in Examples 3 and 4 are expanded by the first row. However, you should remember that any row or column can be used in the expansion as long as the corresponding adjustments in the + and − signs are used with the minors. This may be particularly useful when a row or column has one or more 0s because multiplication by 0 will always give 0, and this will reduce the time needed for the expansion.

Margin Exercise Answers

1. $\left(-\frac{2}{5}, \frac{11}{5}\right)$ **2.** $(2, -3)$ **3.** $D = 0$, so Cramer's Rule cannot be used. **4.** $(5, 2, -1)$

5.9 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Cramer's Rule is a method of solving systems of linear equations using _____.
2. Cramer's Rule can be used if D ___ 0.
3. Cramer's Rule can be used when there is a/an _____ solution.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

4. The numerators of both the x- and y-values in the solution are equal to the value of the determinant of the coefficient matrix.
5. When using Cramer's Rule to solve a system of three linear equations, you can expand the 3 × 3 determinant across any row or down any column.

Practice

Use Cramer's Rule to solve the system of linear equations, if possible. If the determinant of the coefficient is zero, solve the system using addition or substitution to determine whether the system has no solution or infinitely many solutions. See examples 1 through 4.

1. $\begin{cases} 2x-5y=-7 \\ 3x-2y=6 \end{cases}$

2. $\begin{cases} 3x+5y=17 \\ x+3y=15 \end{cases}$

3. $\begin{cases} 6x-4y=5 \\ 3x+8y=0 \end{cases}$

4. $\begin{cases} 3x+4y=24 \\ 2x+y=11 \end{cases}$

5. $\begin{cases} 3x+y=1 \\ -9x-3y=2 \end{cases}$

6. $\begin{cases} 4x+8y=12 \\ 3x+6y=9 \end{cases}$

7. $\begin{cases} 12x+4y=3 \\ -10x+3y=7 \end{cases}$

8. $\begin{cases} 4x-9y=2 \\ 8x-15y=3 \end{cases}$

9. $\begin{cases} 2x+3y=4 \\ 3x-4y=5 \end{cases}$

10. $\begin{cases} 5x+2y=7 \\ 2x-3y=4 \end{cases}$

11. $\begin{cases} 7x+3y=9 \\ 4x+8y=11 \end{cases}$

12. $\begin{cases} 5x-9y=3 \\ 11x+6y=12 \end{cases}$

13. $\begin{cases} 6x-13y=21 \\ 5x-12y=18 \end{cases}$

14. $\begin{cases} 10x+7y=15 \\ 13x-4y=11 \end{cases}$

15. $\begin{cases} 8x-9y=-14 \\ 15x+6y=7 \end{cases}$

16. $\begin{cases} 17x-5y=21 \\ 4x+3y=6 \end{cases}$

17. $\begin{cases} 0.8x+0.3y=4 \\ 0.9x-1.2y=5 \end{cases}$

18. $\begin{cases} 0.4x+0.7y=3 \\ 0.5x+y=6 \end{cases}$

19. $\begin{cases} 1.6x-4.5y=1.5 \\ 0.4x+1.2y=3.1 \end{cases}$

20. $\begin{cases} 2.3x+1.8y=4.6 \\ 0.8x-1.4y=3.2 \end{cases}$

21. $\begin{cases} x-2y-z=-7 \\ 2x+y+z=0 \\ 3x-5y+8z=13 \end{cases}$

22. $\begin{cases} 2x+3y+z=0 \\ 5x+y-2z=9 \\ 10x-5y+3z=4 \end{cases}$

23. $\begin{cases} 5x-4y+z=17 \\ x+y+z=4 \\ -10x+8y-2z=11 \end{cases}$

24. $\begin{cases} 9x+10y=2 \\ 2x+6z=4 \\ -3y+3z=1 \end{cases}$

25. $\begin{cases} 2x-3y-z=-4 \\ -x+2y+z=6 \\ x-y+2z=14 \end{cases}$

26. $\begin{cases} 2x-3y-z=4 \\ x-2y-z=1 \\ x-y+2z=9 \end{cases}$

27. $\begin{cases} 3x+2y+z=5 \\ 2x+y-2z=4 \\ 5x+3y-z=9 \end{cases}$

28. $\begin{cases} 8x+3y+2z=15 \\ 3x+5y+z=-4 \\ 2x+3y=-7 \end{cases}$

29. $\begin{cases} 2x-y+3z=1 \\ 5x+2y-z=2 \\ x-2y+5z=2 \end{cases}$

30. $\begin{cases} 2x+3y+2z=-5 \\ 2x-2y+z=-1 \\ 5x+y+z=1 \end{cases}$

Applications

Set up a system of linear equations, then solve the system using Cramer's Rule. (**Hint:** Remember to write the equations in standard form before using Cramer's Rule.)

31. The three sides of a triangle are related as follows: the perimeter is 43 feet, the second side is 5 feet more than twice the first side, and the third side is 3 feet less than the sum of the other two sides. Find the lengths of the three sides of the triangle.

32. Joel loves candy bars and ice cream. Each candy bar contains 5 grams of fat and 280 calories, and each serving of ice cream contains 10 grams of fat and 150 calories. How many candy bars and how many servings of ice cream did he eat the week that he consumed 85 grams of fat and 2300 calories from these two foods?

33. A financial advisor has $6 million to invest for her clients. She chooses, for one month, to invest in mutual funds and technology stocks. If the mutual funds earned 2% and the stocks earned 4% for a total of $170,000 in earnings for the month, how much money did she invest in each type of investment?

34. A farmer plants corn, wheat, and soybeans and rotates the planting each year on his 500-acre farm. In one particular year, the profits were: $120 per acre for corn, $100 per acre for wheat, and $80 per acre for soybeans. He planted twice as many acres with corn as with soybeans. How many acres did he plant with each crop that year, if he made a total profit of $51,800?

Writing & Thinking

35. Describe the benefits of using Cramer's Rule over solving a system of linear equations with the addition or substitution methods.