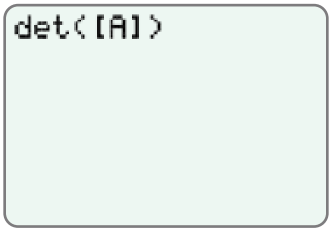
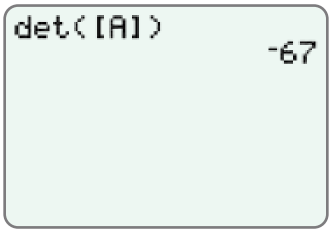
Step 3: Press [2nd] MATRIX again and on the NAMES menu, choose [A] 3x3.
Press [ENTER] and press [)].



Step 4: Press [ENTER] and the display will appear with the solution, $\det(A) = -67$.



Now work margin exercise 5.

Margin Exercise Answers

1. a. -18; **b.** -8; **c.** 0 **2. a.** $\begin{vmatrix} 4 & 8 \\ 7 & -6 \end{vmatrix}$; **b.** $\begin{vmatrix} 4 & 0 \\ -1 & 3 \end{vmatrix}$; **c.** $\begin{vmatrix} 4 & 8 \\ -1 & 2 \end{vmatrix}$ **3. a.** 26 **b.** 175 **4.** -7; **5.** 229

5.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. In a/an _____ matrix, the number of rows is equal to the number of columns.

2. A determinant is indicated by closing the array of real numbers between two _____ _____.

3. Every determinant with real-number entries simplifies to a/an _____ value.

4. One method of evaluating a 3 × 3 determinant is called _____ by minors.

5. When evaluating a 3 × 3 determinant, each minor is multiplied by its corresponding entry and the value ___ or ___.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. A 2 × 2 determinant has three rows and three columns.

7. To find the value of a 3 × 3 determinant, you need to find the value of several 2 × 2 determinants.

8. A minor of a_{13} of a determinant is found by deleting the third row and the first column.

Practice

Find the determinant of each 2 × 2 matrix. See Example 1.

1. $A = \begin{bmatrix} 2 & 7 \\ 4 & 3 \end{bmatrix}$

2. $A = \begin{bmatrix} 7 & 3 \\ 8 & 5 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

4. $A = \begin{bmatrix} 7 & 2 \\ 3 & -6 \end{bmatrix}$

5. $A = \begin{bmatrix} 6 & 3 \\ -11 & -5 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$

7. $A = \begin{bmatrix} 9 & 4 \\ 4 & 7 \end{bmatrix}$

8. $A = \begin{bmatrix} 3 & -4 \\ 8 & -6 \end{bmatrix}$

Given the following determinant, find the minor of each chosen entry. See Example 2.

$$\det(A) = \begin{vmatrix} 3 & -5 & 9 \\ 2 & 0 & -7 \\ 4 & 1 & 1 \end{vmatrix}$$

9. a_{31}

10. a_{12}

11. a_{32}

12. a_{23}

Evaluate the value of each 3 × 3 determinant. See Example 3.

13. $\begin{vmatrix} 0 & -1 & 2 \\ 3 & 5 & -7 \\ -3 & 4 & 1 \end{vmatrix}$

14. $\begin{vmatrix} 1 & 0 & -1 \\ -2 & 3 & 5 \\ 6 & -3 & 4 \end{vmatrix}$

15. $\begin{vmatrix} 1 & -1 & 2 \\ -2 & 5 & -7 \\ 6 & 4 & 1 \end{vmatrix}$

16. $\begin{vmatrix} 2 & -1 & -3 \\ 5 & 9 & 4 \\ 7 & 6 & -2 \end{vmatrix}$

17. $\begin{vmatrix} 2 & 1 & 3 \\ 3 & 4 & 5 \\ 1 & 7 & 2 \end{vmatrix}$

18. $\begin{vmatrix} -3 & 2 & 1 \\ 1 & -4 & -1 \\ 2 & 5 & 3 \end{vmatrix}$

19. $\begin{vmatrix} 2 & 1 & -1 \\ 4 & 3 & 2 \\ 1 & 5 & 5 \end{vmatrix}$

20. $\begin{vmatrix} 6 & 7 & 1 \\ 0 & 3 & 3 \\ 4 & 1 & -5 \end{vmatrix}$

21. $\begin{vmatrix} 3 & -1 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 2 \end{vmatrix}$

22. $\begin{vmatrix} 2 & 3 & 2 \\ 1 & -1 & 5 \\ 0 & 5 & 1 \end{vmatrix}$

Solve for the variable. See Example 4.

23. $\begin{vmatrix} 1 & 3 & 4 \\ 2 & x & 3 \\ 1 & 3 & 5 \end{vmatrix} = 1$

24. $\begin{vmatrix} -2 & -1 & 1 \\ x & 1 & -1 \\ 4 & 3 & -2 \end{vmatrix} = 7$

25. $\begin{vmatrix} -2 & 0 & x \\ 0 & 4 & -8 \\ 6 & 1 & 3 \end{vmatrix} = -16$

26. $\begin{vmatrix} 5 & 3 & -2 \\ -1 & 0 & x \\ 2 & 1 & -1 \end{vmatrix} = 3$

27. $\begin{vmatrix} -4 & 1 & 3 \\ 2 & x & x \\ 0 & 5 & -3 \end{vmatrix} = -28$

28. $\begin{vmatrix} 1 & x & x \\ 2 & -2 & 1 \\ -1 & 3 & 2 \end{vmatrix} = 0$

29. $\begin{vmatrix} x & x & 1 \\ 1 & 5 & 0 \\ 0 & 1 & -2 \end{vmatrix} = -15$

30. $\begin{vmatrix} 3 & 1 & -2 \\ 1 & x & 4 \\ 2 & x & 0 \end{vmatrix} = 38$

The equation $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ is an equation of a line passing through two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Find an equation in standard form for the line determined by the following pairs of points.

31. $(3,2), (-1,4)$

32. $(-2,1), (5,3)$

33. $(4,-4), (0,6)$

34. $(1,3), (-2,-1)$

The area of a triangle having the vertices $P_1(x_1, y_1), P_2(x_2, y_2)$, and $P_3(x_3, y_3)$ is given by the absolute value of the expression $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$. Draw the triangle with the given points as vertices and then find the area of the triangle.

35. $(3,1), (1,-1), (5,2)$

36. $(4,0), (5,-2), (7,1)$

37. $(-1,3), (-4,-1), (3,-2)$

38. $(1,5), (-1,-2), (3,0)$

Use a graphing calculator to find the value of each determinant.

39. $\begin{vmatrix} 3 & -4 & 6 \\ 2 & 4 & -1 \\ 7 & 9 & -1 \end{vmatrix}$

40. $\begin{vmatrix} 2.1 & 3.5 & -3.4 \\ 2.6 & 5.0 & 1.2 \\ -1.0 & 3.4 & 6.3 \end{vmatrix}$

41. $\begin{vmatrix} 1.6 & \frac{1}{2} & -5.9 \\ 0.7 & \frac{3}{4} & 1.7 \\ 5.0 & 8.2 & -4.1 \end{vmatrix}$

42. $\begin{vmatrix} -10 & 15 & 25 \\ 0 & -7 & 5 \\ 16 & -12 & 8 \end{vmatrix}$

Writing & Thinking

43. Explain in your own words the position the three points $P_1(x_1, y_1), P_2(x_2, y_2)$, and $P_3(x_3, y_3)$ if the expression $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ has a value of 0. (**Hint:** Refer to the discussion before Exercise 35.)

44. Suppose that in a 2×2 determinant two rows are identical. What will be the value of this determinant? Give two specific examples and a general example to back up your conclusion.

45. Suppose that in a 3×3 determinant one row is all 0s. What will be the value of this determinant? Give two specific examples and a general example to back up your conclusion.

46. In each part, give two specific examples and a general example to back up your conclusion.

a. Suppose that in a 2×2 determinant two rows (or columns) are switched. How will the value of this new determinant relate to the value of the original determinant?

b. Suppose that in a 3×3 determinant two rows (or columns) are switched. How will the value of this new determinant relate to the value of the original determinant?