5.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The standard form of a linear equation in three variables is _____.
2. Solutions to a linear equation in three variables are called _____ _____.
3. There can be 0, 1, or a/an _____ number of solutions to a system of three linear equations in three variables.
4. Graphs can be created in three dimensions by using a coordinate system involving three mutually perpendicular number lines—the ___-axis, ___-axis, and ___-axis.
5. The three coordinate axes separate space into eight regions called _____.
6. If three distinct planes intersect, they will do so in a straight line or in a single _____ represented by an ordered triple.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. To find the solution for a system of three linear equations in three variables, start by choosing two variables and eliminating one equation.
8. An ordered triple believed to be a solution to a system of three linear equations in three variables should be checked in all three original equations.
9. If two distinct planes intersect, that intersection forms a straight line.
10. Two distinct planes will always intersect.

Practice

Solve each system of equations. State which systems, if any, have no solution or an infinite number of solutions. See Examples 1 through 3.

1. $\begin{cases} x+y-z=0 \\ 3x+2y+z=4 \\ x-3y+4z=5 \end{cases}$

2. $\begin{cases} x-y+2z=3 \\ -6x+y+3z=7 \\ x+2y-5z=-4 \end{cases}$

3. $\begin{cases} 2x-y-z=1 \\ 2x-3y-4z=0 \\ x+y-z=4 \end{cases}$

4. $\begin{cases} y+z=6 \\ x+5y-4z=4 \\ x-3y+5z=7 \end{cases}$

5. $\begin{cases} x+y-2z=4 \\ 2x+y=1 \\ 5x+3y-2z=6 \end{cases}$

6. $\begin{cases} x-y+5z=-6 \\ x+2z=0 \\ 6x+y+3z=0 \end{cases}$

7. $\begin{cases} y+z=2 \\ x+z=5 \\ x+y=5 \end{cases}$

8. $\begin{cases} 2y+z=-4 \\ 3x+4z=11 \\ x+y=-2 \end{cases}$

9. $\begin{cases} x-y+2z=-3 \\ 2x+y-z=5 \\ 3x-2y+2z=-3 \end{cases}$

10. $\begin{cases} x-y-2z=3 \\ x+2y+z=1 \\ 3y+3z=-2 \end{cases}$

11. $\begin{cases} 2x-y+5z=-2 \\ x+3y-z=6 \\ 4x+y+3z=-2 \end{cases}$

12. $\begin{cases} 2x-y+5z=5 \\ x-2y+3z=0 \\ x+y+4z=7 \end{cases}$

13. $\begin{cases} 3x+y+4z=-6 \\ 2x+3y-z=2 \\ 5x+4y+3z=2 \end{cases}$

14. $\begin{cases} 2x+y-z=-3 \\ -x+2y+z=5 \\ 2x+3y-2z=-3 \end{cases}$

15. $\begin{cases} x-2y+z=7 \\ x-y-4z=-4 \\ x+4y-2z=-5 \end{cases}$

16. $\begin{cases} 2x-2y+3z=4 \\ x-3y+2z=2 \\ x+y+z=1 \end{cases}$

17. $\begin{cases} 2x+3y+z=4 \\ 3x-5y+2z=-5 \\ 4x-6y+3z=-7 \end{cases}$

18. $\begin{cases} x+y+z=3 \\ 2x-y-2z=-3 \\ 3x+2y+z=4 \end{cases}$

19. $\begin{cases} 2x-3y+z=-1 \\ 6x-9y-4z=4 \\ 4x+6y-z=5 \end{cases}$

20. $\begin{cases} x+6y+z=6 \\ 2x+3y-2z=8 \\ 2x+4z=3 \end{cases}$

Applications

Solve.

21. A florist is creating the bridesmaids' bouquets for a wedding. Each bouquet will cost $92 and have a mixture of 16 flowers consisting of tiger lilies, cream roses, and white daisies. The lilies cost $10 each, the roses cost $6 each, and the daisies cost $4 each. If each bouquet will have as many daisies as it has roses and lilies combined, how many of each type of flower will be in the bouquet?

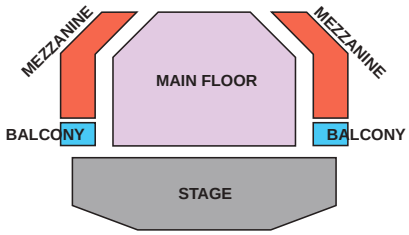
a. Write a system of linear equations in three variables to describe the situation. Remember that you will have three equations. Use the variable x to represent the number of lilies, the variable y to represent the number of roses, and the variable z to represent the number of daisies.

b. Solve the system of equations from part a.

c. Use the solution from part b. to answer the question from the problem statement. Write a complete sentence.

22. A theater has seats for a theatrical production located on the main floor, the balcony, and the mezzanine. For an upcoming musical, main floor tickets cost $60 each, balcony tickets cost $45 each, and mezzanine tickets cost $30 each. On opening night, the ticket sales totaled $27,600. The box office sold 20 more tickets for the main floor than they did for the balcony and mezzanine combined. The number of tickets sold for the mezzanine was 40 more than twice the number of tickets sold for the balcony. How many of each type of ticket did the box office sell?

a. Write a system of linear equations in three variables to describe the situation. Remember that you will have three equations. Use the variable x to represent the number of main floor tickets sold, the variable y to represent the number of balcony tickets sold, and the variable z to represent the number of mezzanine tickets sold.



b. Solve the system of equations from part a.

c. Use the solution from part b. to answer the question from the problem statement. Write a complete sentence.

23. The sum of three integers is 67. The sum of the first and second integers is 13 more than the third integer. The third integer is 7 less than the first. Find the three integers.

24. The sum of three integers is 189. The first integer is 28 less than the second. The second integer is 21 less than the sum of the first and third integers. Find the three integers.

25. A wallet contains $218 in $10, $5, and $1 bills. There are forty-six bills in all and four more fives than tens. How many bills of each kind are there?

26. Ava has 23 coins in her purse, including nickels, dimes, and quarters. She has two more dimes than quarters, and the total value of the coins is $2.50. How many of each kind of coin does she have?

27. The perimeter of a triangle is 73 cm. The longest side is 13 cm less than the sum of the other two sides. The shortest side is 11 cm less than the longest side. Find the lengths of the three sides.

28. The sum of the measures of the three angles of a triangle is 180°. In one particular triangle, the largest angle is 10° more than three times the smallest angle, and the third angle is one-half the largest angle. What are the measures of the three angles?

29. At Steve's Fruit Stand, 4 pounds of bananas, 2 pounds of apples, and 3 pounds of grapes cost $16.40. Five pounds of bananas, 4 pounds of apples, and 2 pounds of grapes cost $16.60. Two pounds of bananas, 3 pounds of apples, and 1 pound of grapes cost $9.60. Find the price per pound of each kind of fruit.

30. The Tates are having a house built. The cost is split into three parts: the house, the lot, and the improvements. The cost of building the house is $16,000 more than three times the cost of the lot. The cost of the improvements (the landscaping, sidewalks, and upgrades) is one-third the cost of the lot. If the total cost is $159,000, what is the cost of each part of the construction?

31. Kai inherited $100,000 from his aunt and decided to invest in three different accounts: savings, bonds, and stocks. The amount in his bond account was $10,000 more than three times the amount in his stock account. At the end of the first year, the savings account returned 5%, the bond 8%, and the stocks 10% for total interest of $7400. How much did he invest in each account?

32. Summer has saved a total of $30,000 and wants to invest in three different stocks: Apple, Netflix, and Amazon.com, Inc. She wants the Apple amount to be $1000 less than twice the Netflix amount and the Amazon.com, Inc. amount to be $2000 more than the total in the other two stocks. How much should she invest in each stock?

33. A chemist wants to mix 9 liters of a 25% acid solution. Because of limited amounts on hand, the mixture needs to be created from three different solutions, one with 10% acid, another with 30% acid, and a third with 40% acid. The amount of the 10% solution must be twice the amount of the 40% solution, and the amount of the 30% solution must equal the total amount of the other two solutions. How much of each solution must be used?

34. An appliance company makes three versions of their popular stand mixer. The standard model has production costs of $100, the deluxe model of $175, and the premium model of $250. On any given day the factory line makes 140 mixers and spends $21,500 on production costs. If the number of standard mixers made equals the sum of the premium and deluxe models made, how many of each kind of mixer are made each day?

Writing & Thinking

35. Is it possible for three linear equations in three unknowns to have exactly two solutions? Explain your reasoning in detail.

36. In geometry, three non-collinear points determine a plane. (That is, if three points are not on a line, then there is a unique plane that contains all three points.). Find the values of A, B, and C (and therefore the equation of the plane) given $Ax + By + Cz = 3$ and the three points on the plane $(0, 3, 2), (0, 0, 1)$ and $(-3, 0, 3)$. Sketch the plane in three dimensions as best you can by locating the three given points.

37. As stated in Exercise 36, three non-collinear points determine a plane. Find the values of A, B, and C (and therefore the equation of the plane) given $Ax + By + Cz = 10$ and the three points on the plane $(2, 0, -2), (3, -1, 0)$ and $(-1, 5, -4)$. Sketch the plane in three dimensions as best you can by locating the three given points.