

## 13.5 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- The factorial  $n!$  is defined to be the product of all positive integers from \_\_\_ through \_\_\_.
- The notation  $\binom{n}{r}$  is called a \_\_\_\_\_ coefficient.
- \_\_\_\_\_ triangle is formed using the coefficients in the binomial expansions of  $(a+b)^n$ , where  $n$  is a positive integer.
- The \_\_\_\_\_ Theorem written in sigma notation is  $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$ .
- Another term for writing out the simplified product of  $(a+b)^n$  is \_\_\_\_\_.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- The factorial  $0!$  is defined to be 0.
- The values of  $\binom{6}{2}$  and  $\binom{6}{4}$  are equal.
- The sum of the exponents of  $a$  and  $b$  in each term of the expansion of  $(a+b)^n$  is equal to  $n$ .

### Practice

Simplify. See Examples 1 and 2.

- |                       |                              |                     |
|-----------------------|------------------------------|---------------------|
| 1. $\frac{8!}{6!}$    | 7. $\frac{n!}{n}$            | 12. $\binom{5}{4}$  |
| 2. $\frac{11!}{7!}$   | 8. $\frac{n!}{(n-3)!}$       | 13. $\binom{7}{3}$  |
| 3. $\frac{3!8!}{10!}$ | 9. $\frac{(k+3)!}{k!}$       | 14. $\binom{8}{5}$  |
| 4. $\frac{5!7!}{8!}$  | 10. $\frac{n(n+1)!}{(n+2)!}$ | 15. $\binom{10}{0}$ |
| 5. $\frac{5!4!}{6!}$  | 11. $\binom{6}{3}$           | 16. $\binom{6}{2}$  |
| 6. $\frac{7!4!}{10!}$ |                              |                     |

Determine the first four terms of the expansion of each binomial expression.

- |                    |                     |
|--------------------|---------------------|
| 17. $(x + y)^7$    | 23. $(x + 2y)^6$    |
| 18. $(x + y)^{11}$ | 24. $(x + 3y)^5$    |
| 19. $(x + 1)^9$    | 25. $(3x - y)^7$    |
| 20. $(x + 1)^{12}$ | 26. $(2x - y)^{10}$ |
| 21. $(x + 3)^5$    | 27. $(x^2 - 4y)^9$  |
| 22. $(x - 2)^6$    | 28. $(x^2 - 2y)^7$  |

Expand each expression using the Binomial Theorem. See Example 3.

- |                  |                    |
|------------------|--------------------|
| 29. $(x + y)^6$  | 35. $(x + 2y)^4$   |
| 30. $(x + y)^8$  | 36. $(x + 3y)^5$   |
| 31. $(x - 1)^7$  | 37. $(3x - 2y)^4$  |
| 32. $(x - 1)^9$  | 38. $(5x + 2y)^3$  |
| 33. $(3x + y)^5$ | 39. $(x^2 + 2y)^4$ |
| 34. $(2x + y)^6$ | 40. $(3x^2 - y)^5$ |

Find the specified term of each expression. See Example 4.

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|-----------------------------------|---|
| 41. $(x - 2y)^{10}$ ; fifth term  | 44. $(4x - 1)^9$ ; seventh term         |
| 42. $(x + 3y)^{12}$ ; third term  | 45. $(5x^2 - y^2)^{12}$ ; tenth term    |
| 43. $(2x + 3)^{11}$ ; fourth term | 46. $(2x^2 + y^2)^{15}$ ; eleventh term |

Approximate the value of each expression to the nearest thousandth. See Example 5.

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|-------------------|----------------|
| 47. $(1.01)^6$    | 51. $(2.3)^5$  |
| 48. $(0.96)^8$    | 52. $(2.8)^6$  |
| 49. $(0.97)^7$    | 53. $(0.98)^8$ |
| 50. $(1.02)^{10}$ | 54. $(1.03)^9$ |

## Writing & Thinking

55. Use the Binomial Theorem to factor  $x^4 + 8x^3 + 24x^2 + 32x + 16$ .