b. For the first job offer, use $n = 10$, $a_1 = \$35{,}000$, and $a_n = \$53{,}000$. For the second job offer, use $n = 10$, $a_1 = \$40{,}000$, and $a_n = \$50{,}800$.

$$\text{First job: } S_{10} = \frac{10}{2}(\$35{,}000 + \$53{,}000) = \$440{,}000$$

$$\text{Second job: } S_{10} = \frac{10}{2}(\$40{,}000 + \$50{,}800) = \$454{,}000$$

Over the first 10 years, the second job would pay more in total salary.

Now work margin exercise 10.

Margin Exercise Answers

1. Arithmetic **2.** Not arithmetic **3.** $a_{12} = 68$ **4.** $a_{25} = 76$ **5.** $a_1 = -8$ and $d = 6$ **6.** $k = 33$ **7.** 1215 **8.** 6806 **9.** –2662 **10. a.** First company: \$283; Second company: \$290; The second company pays more for the 20th contract. **b.** First company: \$4330; Second company: \$3900; The first company pays more if you take 20 contracts.

13.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Any two consecutive terms in an arithmetic sequence have the _____ difference.

2. For an arithmetic sequence, the letter d denotes the _____.

3. The formula for the general term of an arithmetic sequence is _____.

4. The n^{th} partial sum S_n is the sum of the first ___ terms of an arithmetic sequence.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. Another name for an arithmetic sequence is *arithmetic progression*.

6. To determine whether a sequence is arithmetic, find the ratio of consecutive terms and determine if that ratio is constant.

Practice

Determine whether each sequence is arithmetic. If the sequence is arithmetic, find the common difference and the general n^{th} term.

1. 2, 5, 8, 11, ...

2. –3, 1, 5, 9, ...

3. 7, 5, 3, 1, ...

4. 5, 6, 7, 8, ...

5. 1, 2, 3, 5, 8, ...

6. 2, 4, 8, 16, ...

7. 6, 2, –2, –6, ...

8. 4, –1, –6, –11, ...

9. $0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

10. $2, \frac{7}{3}, \frac{8}{3}, 3, \ldots$

Write the first five terms of each sequence, then determine whether the sequence is arithmetic.

11. $\{2n-1\}$

12. $\{4-n\}$

13. $\left\{\frac{1}{n+1}\right\}$

14. $\left\{\frac{1}{2n}\right\}$

15. $\left\{n+\frac{n}{2}\right\}$

16. $\{5-6n\}$

17. $\left\{7-\frac{n}{3}\right\}$

18. $\left\{\frac{2}{3}n-\frac{7}{3}\right\}$

19. $\left\{(-1)^n(3n-2)\right\}$

20. $\left\{(-1)^{n+1}(2n+1)\right\}$

Use the given information to find the general form $\{a_n\}$ of each arithmetic sequence.

21. $a_1=1,\ d=\frac{2}{3}$

22. $a_1=9,\ d=-\frac{1}{3}$

23. $a_1=7,\ d=-2$

24. $a_1=-3,\ d=\frac{4}{5}$

25. $a_1=10,\ a_3=13$

26. $a_1=6,\ a_5=4$

27. $a_{10}=13,\ a_{12}=3$

28. $a_5=7,\ a_9=19$

29. $a_{13}=60,\ a_{23}=75$

30. $a_{11}=54,\ a_{29}=180$

Assume each sequence is arithmetic. Find the indicated value.

31. $a_1=8,\ a_{11}=168$. Find a_{15}.

32. $a_1=17,\ a_9=-55$. Find a_{20}.

33. $a_6=8,\ a_4=2$. Find a_{18}.

34. $a_{16}=12,\ a_7=30$. Find a_9.

35. $a_{13}=34,\ d=2,\ a_n=22$. Find n.

36. $a_4=20,\ d=3,\ a_n=44$. Find n.

37. $a_{10}=41,\ d=4,\ a_n=77$. Find n.

38. $a_3=15,\ d=-\frac{3}{2},\ a_n=6$. Find n.

Use the formula for partial sums of arithmetic sequences to calculate the partial sums.

39. $-2+0+2+4+\cdots+24$

40. $3+6+9+\cdots+33$

41. $1+6+11+16+\cdots+46$

42. $5+9+13+17+\cdots+49$

43. $\sum_{k=1}^{9}(3k-1)$

44. $\sum_{k=1}^{12}(4-5k)$

45. $\sum_{k=1}^{11}(4k-3)$

46. $\sum_{k=1}^{10}(2k+7)$

47. $\sum_{k=1}^{13}\left(\frac{2k}{3}-1\right)$

48. $\sum_{k=1}^{28}(8k-5)$

49. $\sum_{k=1}^{9}\left(k+\frac{k}{3}\right)$

50. $\sum_{k=1}^{16}\left(9-\frac{k}{3}\right)$

Use the properties of sigma notation to find the indicated sums.

51. If $\sum_{k=1}^{33} a_k = -12$, find $\sum_{k=1}^{33} (5a_k + 7)$.

52. If $\sum_{k=1}^{15} a_k = 60$, find $\sum_{k=1}^{15} (-2a_k - 5)$.

53. If $\sum_{k=1}^{100} (-3a_k + 4) = 700$, find $\sum_{k=1}^{100} a_k$.

54. If $\sum_{k=1}^{50} (2b_k - 5) = 32$, find $\sum_{k=1}^{50} b_k$.

Applications

Solve.

55. On a certain project, a construction company was penalized for taking more than the contractual time to finish the project. For each day past the contractual finish date, the company forfeited \$750 the first day, \$900 the second day, \$1050 the third day, and so on. How many additional days were needed to complete the project if the total penalty was \$12,150?

56. A piece of property is currently valued at \$480,000. The property is estimated to appreciate in value as follows: \$14,000 the first year, \$14,500 the second year, \$15,000 the third year, and so on. Based on this estimate, what will be the value of the property after 10 years?

57. The rungs of a ladder on a playground decrease uniformly in width, from bottom to top. The bottom rung is 84 cm long and the top rung is 46 cm long. What is the total length of the wood needed to make the rungs if there are 25 rungs?

58. A pile of blocks has 19 blocks in the first layer, 17 in the second layer, 15 in the third layer, and so on, with only 1 block in the top layer. How many blocks are in the pile?

59. Theater seats are arranged in arcs so that there are 6 additional seats in each semicircular row moving away from the stage. The first row (the one closest to the stage) has 20 seats, and the theater has 20 rows of seats. How many seats are in the last row of the theater? How many seats are in the theater?

60. Samantha accumulated \$50,000 in student loans during her four years in college. She has agreed to pay \$1000 towards her loans during the first year of repayment and increase the payment by \$500 each year thereafter. How much will she have paid over 12 years of repayment?

Writing & Thinking

61. Explain why an alternating sequence (one in which the terms alternate between positive and negative values) cannot be an arithmetic sequence.