b. Since $\sum_{k=1}^{50} 3a_k = 3\sum_{k=1}^{50} a_k$ by Property III, then

$$3\sum_{k=1}^{50} a_k = 600$$

$$\sum_{k=1}^{50} a_k = 200.$$

Now work margin exercise 3.

Margin Exercise Answers

1. a. $\sum_{k=1}^{4}\left(k^2-1\right)=0+3+8+15=26$ **b.** $\sum_{k=1}^{5}\frac{(-1)^k}{k}=-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}=-\frac{47}{60}$

2. a. $\sum_{k=5}^{9}2k$ or $\sum_{k=1}^{5}(2k+8)$ **b.** $\sum_{k=2}^{6}\frac{(-1)^k}{2k}$ **3. a.** $c=6$ **b.** $\sum_{k=1}^{20}a_k=68$

13.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A sequence has a/an _____ number of terms.

2. Finding the sum of a few terms of a sequence is called finding a/an _____ sum.

3. In the notation $\sum_{k=1}^{n} a_k$, the value k is called the _____ of summation.

4. The notation … is called a/an _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. In sigma notation, the lower limit of summation must always be 1.

6. The associative, commutative, and distributive properties hold for sums of real numbers.

Practice

For each sequence, write out the partial sums S_1, S_2, S_3, and S_4, then evaluate each partial sum.

1. $\{3k-1\}$

2. $\{2k+5\}$

3. $\left\{\frac{k}{k+1}\right\}$

4. $\left\{\frac{k+1}{k}\right\}$

5. $\left\{(-1)^{k-1}k^2\right\}$

6. $\left\{(-1)^k k^3\right\}$

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7. $\left\{\frac{1}{2^k}\right\}$

8. $\left\{\left(\frac{2}{3}\right)^k\right\}$

9. $\left\{2k-k^2\right\}$

10. $\left\{k^2-k\right\}$

Write the indicated sums as expanded sums of the terms and find the value of each sum.

11. $\sum_{k=1}^{5} 2k$

12. $\sum_{k=1}^{8} 3k$

13. $\sum_{k=2}^{6} (k+3)$

14. $\sum_{k=9}^{11} (2k+1)$

15. $\sum_{k=2}^{4} \frac{1}{k}$

16. $\sum_{k=1}^{3} \frac{1}{2k}$

17. $\sum_{k=1}^{3} 2^k$

18. $\sum_{k=10}^{15} (-1)^k$

19. $\sum_{k=4}^{8} k^2$

20. $\sum_{k=1}^{5} k^3$

21. $\sum_{k=3}^{6} (9-2k)$

22. $\sum_{k=2}^{7} (4k-1)$

23. $\sum_{k=2}^{5} (-1)^k \left(k^2+k\right)$

24. $\sum_{k=1}^{6} (-1)^k \left(k^2-2\right)$

25. $\sum_{k=1}^{5} \frac{k}{k+1}$

26. $\sum_{k=3}^{5} \frac{k+1}{k^2}$

Write each sum in sigma notation. There may be more than one correct answer.

27. $1+3+5+7+9$

28. $4+7+10+13+16$

29. $-1+1+(-1)+1+(-1)$

30. $-2+4-8+16-32$

31. $16+25+36+49$

32. $8+15+24+35+48$

33. $\frac{1}{8}-\frac{1}{27}+\frac{1}{64}-\frac{1}{125}+\frac{1}{216}$

34. $-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}$

35. $\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\cdots+\frac{15}{16}$

36. $\frac{6}{25}+\frac{7}{36}+\frac{8}{49}+\frac{9}{64}+\cdots+\frac{13}{144}$

Use the provided information to find the indicated sums.

37. $\sum_{k=1}^{14} a_k = 18$ and $\sum_{k=1}^{14} b_k = 21$. Find $\sum_{k=1}^{14} (a_k+b_k)$.

38. $\sum_{k=1}^{19} a_k = 23$ and $\sum_{k=1}^{19} b_k = 16$. Find $\sum_{k=1}^{19} (a_k-b_k)$.

39. $\sum_{k=1}^{15} a_k = 19$. Find $\sum_{k=1}^{15} 3a_k$.

40. $\sum_{k=1}^{11} a_k = 35$. Find $\sum_{k=1}^{11} 2a_k$.

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41. $\sum_{k=1}^{25} a_k = 63$ and $\sum_{k=1}^{11} a_k = 15$. Find $\sum_{k=12}^{25} a_k$.

42. $\sum_{k=1}^{16} a_k = 56$ and $\sum_{k=17}^{40} a_k = 42$. Find $\sum_{k=1}^{40} a_k$.

43. $\sum_{k=13}^{29} a_k = 84$ and $\sum_{k=1}^{29} a_k = 143$. Find $\sum_{k=1}^{12} 5a_k$.

44. $\sum_{k=1}^{27} a_k = 46$ and $\sum_{k=1}^{10} a_k = 122$. Find $\sum_{k=11}^{27} 2a_k$.

45. $\sum_{k=1}^{18} a_k = 41$ and $\sum_{k=1}^{18} b_k = 62$. Find $\sum_{k=1}^{18} (3a_k - 2b_k)$.

46. $\sum_{k=1}^{21} a_k = -68$ and $\sum_{k=1}^{21} b_k = 39$. Find $\sum_{k=1}^{21} (a_k + 2b_k)$.

47. $\sum_{k=1}^{20} b_k = 34$ and $\sum_{k=1}^{20} (2a_k + b_k) = 144$. Find $\sum_{k=1}^{20} a_k$.

48. $\sum_{k=1}^{16} a_k = 28$ and $\sum_{k=1}^{16} (b_k - 3a_k) = -12$. Find $\sum_{k=1}^{16} b_k$.

Writing & Thinking

49. Use the sum of two expressions in sigma notation to represent the following sum:
$-22 + 3 - 24 + 6 - 26 + 9 - 28 + 12 - 30 + 15$

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