Comparing the denominators, we see that $2^n < 2^{n+1}$. This means that

$$\frac{1}{2^n} > \frac{1}{2^{n+1}} \text{ and } a_n > a_{n+1}.$$

Therefore, the sequence $\{a_n\} = \left\{\frac{1}{2^n}\right\}$ is decreasing.

b. The sequence $\{b_n\} = \{2 + (-1)^n\}$ has a term of $(-1)^n$, which means it is likely an alternating sequence. Consider the first four terms of the sequence $\{b_n\}$.

$$b_1 = 2 + (-1)^1 = 2 - 1 = 1$$

$$b_2 = 2 + (-1)^2 = 2 + 1 = 3$$

$$b_3 = 2 + (-1)^3 = 2 - 1 = 1$$

$$b_4 = 2 + (-1)^4 = 2 + 1 = 3$$

From this pattern, we see that the sequence is 1, 3, 1, 3, …, which is neither increasing nor decreasing.

c. In formula form, we have $c_n = n + 3$ and $c_{n+1} = (n+1) + 3 = n + 4$. Because $n + 3 < n + 4$, we have $c_n < c_{n+1}$, which means the sequence is increasing.

Now work margin exercise 5.

Margin Exercise Answers

1. a. $c_1 = 0, c_2 = \frac{1}{2}, c_3 = \frac{2}{3}, c_{50} = \frac{49}{50}$ **b.** $d_1 = 3, d_2 = 8, d_3 = 13, d_{50} = 248$

2. a. $a_n = \frac{1}{n}$ **b.** $a_n = 2^{n-1}$ **3.** \$960, \$768, \$614.40 **4.** –3, 5, –7, 9, –11

5. a. neither **b.** decreasing **c.** increasing

13.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A sequence is a list of ________ that occur in a certain order.
2. The general term a_n is called the ______ term of the sequence.
3. A/An _______ sequence is formed by each successive term in a sequence being found by referring to a previous term.
4. An alternating sequence has terms that alternate in ______.
5. If a sequence is _______, each successive term becomes smaller.
6. If a sequence is _______, each successive term becomes larger.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Alternating sequences generally involve expressions with a factor of $(-1)^n$ or $(-1)^{n+1}$.

8. In a sequence, a_2 is the second term of the sequence.

9. The terms in an alternating sequence are always negative.

Practice

Write the first five terms of each sequence. See Example 1.

1. $\{3n-1\}$
2. $\{4n+1\}$
3. $\left\{1+\frac{1}{n}\right\}$
4. $\left\{\frac{n+3}{n+1}\right\}$
5. $\{n^2+n\}$
6. $\{n-n^2\}$
7. $\{2^n\}$
8. $\{2^n-n^2\}$
9. $\left\{\left(\frac{1}{2}\right)^n\right\}$
10. $\left\{\left(-\frac{1}{2}\right)^{n+1}\right\}$
11. $\left\{(-1)^n(n^2+1)\right\}$
12. $\left\{(-1)^{n-1}(3^n)\right\}$
13. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$
14. $\left\{(-1)^n\left(\frac{1}{2n+3}\right)\right\}$
15. $\left\{\frac{2}{n(n+1)}\right\}$
16. $\left\{\frac{2n}{n+1}\right\}$
17. $\left\{\frac{n(n-1)}{2}\right\}$
18. $\left\{\frac{1+(-1)^n}{2}\right\}$

Find the formula for the general term of each sequence. See Example 2.

19. 2, 5, 8, 11, 14, …
20. 5, 9, 13, 17, 21, …
21. 1, 4, 9, 16, 25, …
22. 2, 5, 10, 17, 26, …
23. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$
24. $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \ldots$
25. 6, 12, 18, 24, 30, …
26. 5, 10, 20, 40, 80, …
27. 1, –3, 5, –7, 9, …
28. –3, 7, –11, 15, –19, …

Determine whether each sequence is decreasing, increasing, or neither. See Example 5.

29. $\{n+4\}$
30. $\{1-2n\}$
31. $\{n^0\}$
32. $\left\{\frac{1}{n+3}\right\}$

33. $\left\{\frac{1}{3^n}\right\}$

34. $\left\{\frac{2n+1}{n}\right\}$

35. $\left\{(-1)^{n+1}\left(\frac{2}{n+1}\right)\right\}$

36. $\left\{\frac{n}{n+1}\right\}$

Applications

Write the terms of the finite sequence described, then answer the stated question.

37. A certain automobile costs \$40,000 new and depreciates at a rate of $\frac{3}{10}$ of its current value each year. What will be its value after 3 years?

38. A culture of bacteria triples every day. If there were 100 bacteria in the original culture, how many would be present after 4 days?

39. A ball is dropped from a height of 10 meters. Each time it bounces, it rises to $\frac{2}{5}$ of its previous height. How high will it bounce on its fourth bounce?

40. A local university is experiencing a declining enrollment of 6% per year. If the present enrollment is 20,000, what is the projected enrollment after 5 years?

41. A certain medication has a half-life of 1 day, which means that after 1 day, half of the dosage remains in a person's system. The general formula for the remaining amount of medication in a person's system in this situation is $h_n = A\left(\frac{1}{2}\right)^n$, where A is the original amount of medication administered and n is the number of days that have passed. How much medication remains in a person's system after 3 days if the original dosage was 100 mg?

42. The population of a certain city is expected to grow by 3.5% of its current value each year. If the current population is 15,500 people, how many people are expected to reside in the city after 4 years? (Round each of your answers up to the nearest person.)

Writing & Thinking

43. Show that the sequence of digits from the irrational number $\pi = 3.1415926535...$ is neither increasing nor decreasing. (**Note:** There is no formula for a_n.)

44. Show that the sequence $\left\{\frac{(-1)^{n+1}}{3n}\right\}$ is neither increasing nor decreasing. Is this an alternating sequence?

45. The famous Fibonacci sequence is an example of a recurrence sequence. That is, each term depends on previous terms.

a. Write the first eight terms of the Fibonacci sequence defined as $F_{n+2} = F_{n+1} + F_n$, where $F_1 = 1$ and $F_2 = 1$.

b. Form the sequence of the differences of successive terms. What do you notice?