12.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Ellipses are conic sections that are _______ in shape.

2. An ellipse is the set of all points in a plane for which the _______ of the distances from two fixed points is _______.

3. Each fixed point of the ellipse is called a/an _______.

4. A hyperbola is a set of all points in a plane such that the absolute value of the _______ of the distances from two fixed points is _______.

5. When $a^2 > b^2$, the segment of length $2b$ joining the y-intercepts is called the _______ axis.

6. When $b^2 > a^2$, the segment of length $2b$ joining the y-intercepts is called the _______ axis.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. An ellipse's center is the point midway between the two foci.

8. The foci of an ellipse lie on the ellipse.

9. The midway point between the foci of a hyperbola is called the origin.

Practice

Write each of the equations in standard form. Then sketch the graph. For hyperbolas, graph the asymptotes as well. See Examples 1 through 4.

1. $x^2+9y^2=36$

2. $x^2+4y^2=16$

3. $4x^2+25y^2=100$

4. $4x^2+9y^2=36$

5. $16x^2+y^2=16$

6. $36x^2+9y^2=36$

7. $x^2-y^2=1$

8. $x^2-y^2=4$

9. $9x^2-y^2=9$

10. $4x^2-y^2=4$

11. $4x^2-9y^2=36$

12. $9x^2-16y^2=144$

13. $2x^2+y^2=8$

14. $3x^2+y^2=12$

15. $x^2+5y^2=20$

16. $x^2+7y^2=28$

17. $y^2-x^2=9$

18. $y^2-x^2=16$

19. $y^2-2x^2=8$

20. $y^2-3x^2=12$

21. $y^2-2x^2=18$

22. $y^2-5x^2=20$

23. $3x^2+2y^2=18$

24. $4x^2+3y^2=12$

25. $4x^2+5y^2=20$

26. $3x^2+8y^2=48$

27. $9y^2-8x^2=72$

28. $4x^2-7y^2=28$

29. $3y^2-4x^2=36$

30. $3x^2-5y^2=75$

Match the equations with the given graphs. See Examples 5 and 6.

31. $\frac{(x-1)^2}{4}+\frac{(y-3)^2}{25}=1$

32. $\frac{(x+1)^2}{4}+\frac{(y+3)^2}{25}=1$

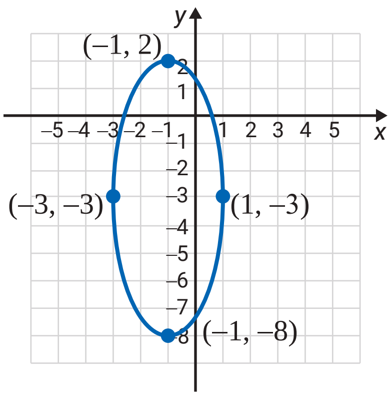
33. $\frac{(x-1)^2}{25}+\frac{(y-3)^2}{4}=1$

34. $\frac{(x+1)^2}{25}+\frac{(y+3)^2}{4}=1$

35. $\frac{(x+1)^2}{25}-\frac{(y+3)^2}{4}=1$

36. $\frac{(y+3)^2}{4}-\frac{(x+1)^2}{25}=1$

a.



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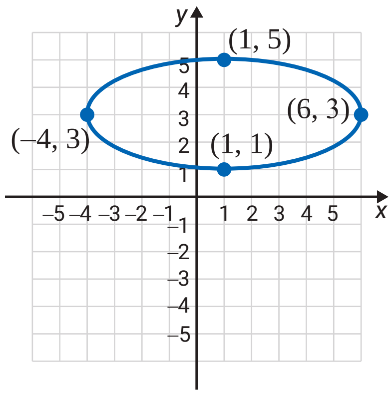

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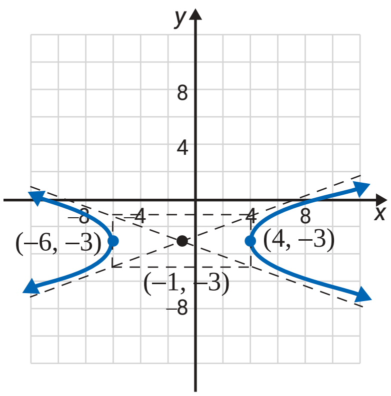

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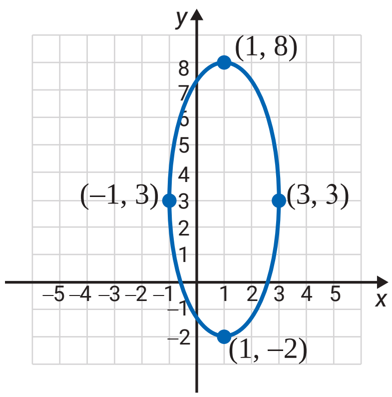

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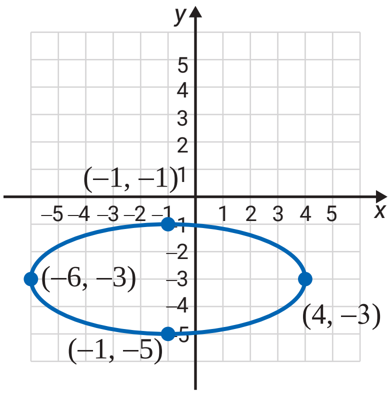

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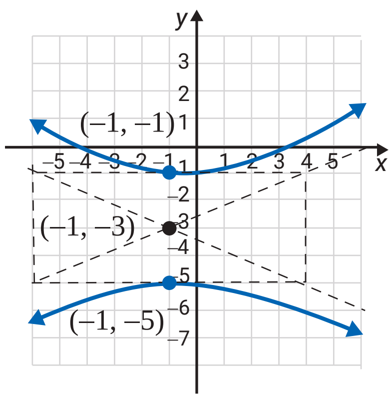


Use your knowledge of translations to graph each of the following equations. These graphs are ellipses and hyperbolas with centers at points other than the origin. See Examples 5 and 6.

37. $\frac{(x-2)^2}{25}+\frac{(y-1)^2}{9}=1$

38. $\frac{(x+1)^2}{16}+\frac{(y-4)^2}{1}=1$

39. $\frac{(x+5)^2}{1}-\frac{(y+2)^2}{16}=1$

40. $\frac{(x-4)^2}{9}-\frac{(y-3)^2}{36}=1$

41. $\frac{(x+1)^2}{49}+\frac{(y-6)^2}{100}=1$

42. $\frac{(x+2)^2}{4}+\frac{(y+3)^2}{36}=1$

43. $\frac{(y-2)^2}{9}-\frac{(x+2)^2}{4}=1$

44. $\frac{(y+5)^2}{25}-\frac{(x-1)^2}{64}=1$

Writing & Thinking

45. The definition of an ellipse is given in the text as follows.

An ellipse is the set of all points in a plane for which the sum of the distances from two fixed points is constant.

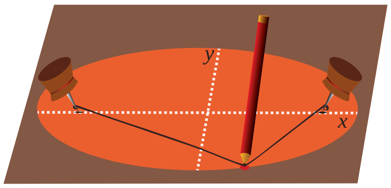
a. Draw an ellipse by proceeding as follows.

Step 1: Place two thumb tacks in a piece of cardboard.

Step 2: Select a piece of string slightly longer than the distance between the two tacks.

Step 3: Tie the string to each thumb tack and stretch the string taut using a pencil.

Step 4: Use the pencil to trace the path of an ellipse on the cardboard by keeping the string taut. (The length of the string represents the fixed distance from points on the ellipse to the two foci.)



b. Show that the equation of an ellipse with foci at $(-c, 0)$ and $(c, 0)$, center at the origin, and $2a$ as the constant sum of the lengths to the foci can be written in the form $\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$.

c. In the equation in part b., substitute $b^2=a^2-c^2$ to get the standard form for the equation of an ellipse. Show that the points $(0, -b)$ and $(0, b)$ are the y-intercepts and a is the distance from each y-intercept to a focus.