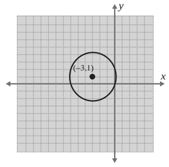
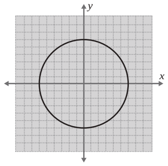
Margin Exercise Answers

1. Distance: 5 **2.** Triangle DEF is a right triangle since $\left(\sqrt{45}\right)^2+\left(\sqrt{80}\right)^2=\left(\sqrt{125}\right)^2$.
3. Midpoint: $\left(\frac{5}{2},-3\right)$ **4.** $x^2+y^2=5$. Both $\left(\sqrt{2},\sqrt{3}\right)$ and $(1,2)$ are on the circle.
5. $(x-3)^2+(y+2)^2=16$, $(7,-2)$ is on the circle. **6.** The equation can be written in the form $(x+3)^2+(y-1)^2=10$. The center is at $(-3,1)$ and the radius is $\sqrt{10}$.



7.



12.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in a plane can be determined with the formula $d=$ ____________.
2. When calculating the distance d, be sure to add the ______ before taking the square root.
3. The midpoint is found by ______ the corresponding coordinates of the endpoints.
4. A circle is a set of points on a plane that are a fixed ______ from a fixed ______.
5. The diameter is ______ the length of the radius.
6. The equation of a circle with radius r and center at (h, k) is $r^2=$ ______.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The Pythagorean Theorem is used to derive the distance formula.
8. The distance from the center of a circle to any point on the circle is called the diameter of the circle.
9. To find the equation of a circle with center at $(2, 5)$ and radius 6, the distance formula can be used.
10. The hypotenuse of a right triangle is the longest side of the triangle.

Practice

Find the distance between the two given points and the coordinates of the midpoint of the line segment joining the two points. See Examples 1 and 3.

1. $(2, 4), (6, 7)$
2. $(1, 0), (6, 12)$
3. $(-3, 2), (9, 7)$
4. $(-6, 3), (-2, 0)$

5. $(1, 7), (3, 2)$

6. $(-2, 1), (3, -4)$

7. $(4, -3), (7, -3)$

8. $(-2, 6), (5, 6)$

9. $(5, -2), (7, -5)$

10. $(6, 4), (8, -5)$

11. $(-7, 3), (1, -12)$

12. $(3, 8), (-2, -4)$

Find equations for each of the circles. See Examples 4 and 5.

13. Center $(0, 0)$; $r = 4$

14. Center $(0, 0)$; $r = 6$

15. Center $(0, 0)$; $r = \sqrt{3}$

16. Center $(0, 0)$; $r = \sqrt{7}$

17. Center $(0, 0)$; $r = \sqrt{11}$

18. Center $(0, 0)$; $r = \sqrt{13}$

19. Center $(0, 0)$; $r = \frac{2}{3}$

20. Center $(0, 0)$; $r = \frac{7}{4}$

21. Center $(0, 2)$; $r = 2$

22. Center $(0, 5)$; $r = 5$

23. Center $(4, 0)$; $r = 1$

24. Center $(-3, 0)$; $r = 4$

25. Center $(-2, 0)$; $r = \sqrt{8}$

26. Center $(5, 0)$; $r = \sqrt{2}$

27. Center $(3, 1)$; $r = 6$

28. Center $(-1, 2)$; $r = 5$

29. Center $(3, 5)$; $r = \sqrt{12}$

30. Center $(4, -2)$; $r = \sqrt{14}$

31. Center $(7, 4)$; $r = \sqrt{10}$

32. Center $(-3, 2)$; $r = \sqrt{7}$

Write each of the equations in standard form. Find the center and radius of the circle and then sketch the graph. See Example 6.

33. $x^2 + y^2 = 9$

34. $x^2 + y^2 = 16$

35. $x^2 = 49 - y^2$

36. $y^2 = 25 - x^2$

37. $x^2 + y^2 = 18$

38. $x^2 + y^2 = 12$

39. $x^2 + y^2 + 2x = 8$

40. $x^2 + y^2 + 6x = 0$

41. $x^2 + y^2 - 4y = 0$

42. $x^2 + y^2 - 4x = 12$

43. $x^2 + y^2 + 2x + 4y = 11$

44. $x^2 + y^2 + 4x + 4y = 8$

45. $x^2 + y^2 - 4x + 10y + 20 = 0$

46. $x^2 + y^2 - 6x - 8y + 9 = 0$

47. $x^2 + y^2 - 4x - 6y + 5 = 0$

48. $x^2 + y^2 + 10x - 2y + 14 = 0$

Use the Pythagorean Theorem to decide if the triangle determined by the given points is a right triangle. See Example 2.

49. $A(1, -2), B(7, 1), C(5, 5)$

50. $A(-5, -1), B(2, 1), C(-1, 6)$

Show that the triangle determined by the given points is an isosceles triangle (has two equal sides).

51. $A(1,1),\ B(5,9),\ C(9,5)$

52. $A(1,-4),\ B(3,2),\ C(9,4)$

Show that the triangle determined by the given points is an equilateral triangle (all sides equal).

53. $A(1,0),\ B\left(3,\sqrt{12}\right),\ C(5,0)$

54. $A(0,5),\ B(0,-3),\ C\left(\sqrt{48},1\right)$

Show that the lengths of the diagonals (*AC* and *BD*) of the rectangle *ABCD* are equal.

55. $A(2,-2),\ B(2,3),\ C(8,3),\ D(8,-2)$

56. $A(-1,1),\ B(-1,4),\ C(4,4),\ D(4,1)$

Find the perimeter of the triangle determined by the given points.

57. $A(-5,0),\ B(3,4),\ C(0,0)$

58. $A(-6,-1),\ B(-3,3),\ C(6,4)$

Use a graphing calculator to graph the circles. Be sure to set a square window.

59. $x^2+y^2=16$

60. $x^2+y^2=25$

61. $(x+3)^2+y^2=49$

62. $x^2+(y+2)^2=36$

63. $(x-2)^2+(y-5)^2=100$

64. $(x-1)^2+(y+3)^2=64$

Applications

Solve.

65. The roadways in Descartesville are laid out such that streets run east to west and avenues run north to south. The north to south avenues and the east to west streets are numbered sequentially, beginning with 1. For example, a person standing on the corner of 1st Street and 1st Avenue may be considered to be at standing at $(1, 1)$. Suppose a person begins at the corner of 1st Street and 2nd Avenue and walks to the corner of 9th Street and 8th Avenue. If the person were able to walk directly from the beginning corner to the ending corner, the distance traveled would be the same as the distance of how many blocks?

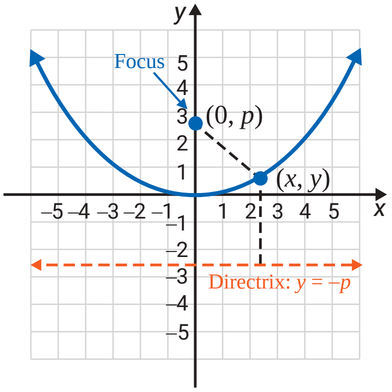
66. A bored greens keeper at the Descartesville Golf Course decides to sketch the entire golf course on a coordinate grid, where each unit on the grid corresponds to 100 yards. He notices that the first hole's tee box is located at the point $(-8, -10)$ and the hole's cup is located at the point $(-11, -6)$. What is the straight-line distance in yards between the first hole's tee box and the hole's cup?

67. A group of math majors at Homestate University decide to make a map of the campus on a coordinate grid. Each unit on the grid corresponds to 100 yards. If the calculus class is held in the Math building located at $(5, 12)$ on the graph and the cafeteria is located at $(9, 14)$ on the graph, how many hundreds of yards do students have to walk if they follow a straight line from the calculus class to the cafeteria?

68. A conservation society used a grant to purchase a 1000-acre tract of land in Descartesville and plans to turn the land into a camping ground. The society uses a coordinate grid to plan the design of the camp where each unit on the grid corresponds to 1000 yards. The plans include a dock on the lake to be located at $(6, 15)$ on the grid and a picnic pavilion to be located at $(10, 8)$. How many thousands of yards will campers have to walk if they follow a straight line from the dock to the picnic pavilion?

Writing & Thinking

69. For a given line and a point not on the line, a parabola is defined as the set of all points that are the same distance from the point and the line. The point is called the focus and the line is called the directrix. See the figure provided.



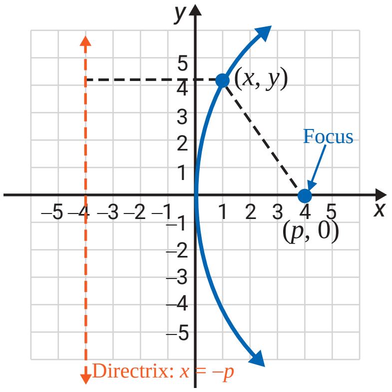
a. Suppose that (x, y) is any point on a parabola and $(0, p)$ is the focus. Find the distance from (x, y) to the focus.

b. Suppose that (x, y) is any point on the same parabola in part a. and the line $y = -p$ is the directrix. Find the distance from (x, y) to the directrix.

c. Show that the equation of the parabola is $x^2 = 4py$.

70. Using the equation developed in Exercise 69, find the equation of the parabola with focus at $(0, 2)$ and line $y = -2$ as directrix. Draw the graph.

71. For a given line and a point not on the line, a parabola is defined as the set of all points that are the same distance from the point and the line. The point is called the focus and the line is called the directrix. See the figure provided.



a. Suppose that (x, y) is any point on a parabola and $(p, 0)$ is the focus. Find the distance from (x, y) to the focus.

b. Suppose that (x, y) is any point on the same parabola in part a. and the line $x = -p$ is the directrix. Find the distance from (x, y) to the directrix.

c. Show that the equation of the parabola is $y^2 = 4px$.

72. Using the equation developed in Exercise 71, find the equation of the parabola with focus at $(-3, 0)$ and line $x = 3$ as directrix. Draw the graph.