Example 5 Using the Change-of-Base Formula

Use the change-of-base formula to find the value of x (accurate to the nearest ten-thousandth) in the equation $5^x = 16$.

Solution

Because the base is 5, we can take $\log_5$ of both sides.

(This method is not necessary, but it does show how the change-of-base formula can be used.)

$$\begin{aligned} 5^x &= 16 \\ \log_5 5^x &= \log_5 16 \\ x &= \log_5 16 \\ x &= \frac{\ln 16}{\ln 5} && \text{Change-of-base formula} \\ x &\approx \frac{2.7726}{1.6094} \approx 1.7228 \end{aligned}$$

Now work margin exercise 5.

5. Use the change-of-base formula to find the value of x.

$8^x = 25$

Margin Exercise Answers

1. a. $x = 1$ or $x = 3$ **b.** $x = -1$ or $x = 2$ **2. a.** $x = \frac{\log 3}{\log 4} \approx 0.7925$ **b.** $x = \frac{\log 20}{\log 5} \approx 1.8614$

3. a. $x = 50$ **b.** $x = -1$ **4.** $\frac{\ln 4}{\ln 2} = 2$ **5.** $\frac{\ln 25}{\ln 8} \approx 1.5480$

11.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When solving exponential equations, if the bases are not the same, the equations are solved by taking the ______ of both sides.

2. Logarithms are defined only for ______ ______ numbers, so each answer should be checked in the original equation.

3. For any positive real number b, b^0 is equal to ______.

4. For any positive real number b and any real numbers x and y, $\left(b^x\right)^y$ is equivalent to ______.

5. For any positive real number b and any value of x, b^{-x} is equivalent to ______.

6. For $b > 0$ and $b \neq 1$, if $\log_b x = \log_b y$, then $x =$ ______.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The change of base formula is $\log_b x = \dfrac{\log_a b}{\log_a x}$, for $a, b, x > 0$ and $b \neq 1$.

8. If the terms in an exponential equation all have the same base, there is no need to use logarithms to solve the equation.

9. Exponential equations with different bases can be solved by taking either the log of both sides or the ln of both sides.

10. If $5^x = 5^y$, then x is equal to y.

Practice

Use the properties of exponents and logarithms to solve each of the equations. If necessary, use a calculator and round answers to the nearest ten-thousandth. See Examples 1 through 3.

1. $2^4 \cdot 2^7 = 2^x$

2. $3^7 \cdot 3^{-2} = 3^x$

3. $\left(3^5\right)^2 = 3^{x+1}$

4. $\left(2^x\right)^3 = 2^{x+4}$

5. $\dfrac{10^4 \cdot 10^{\frac{1}{2}}}{10^x} = 10$

6. $\left(10^2\right)^x = \dfrac{10 \cdot 10^{\frac{2}{3}}}{10^{\frac{1}{2}}}$

7. $2^{5x} = 4^3$

8. $7^{3x} = 49^4$

9. $(25)^x = 5^3 \cdot 5^4$

10. $10^x \cdot 10^8 = 100^3$

11. $8^{x+3} = 2^{x-1}$

12. $100^{2x+1} = 1000^{x-2}$

13. $27^x = 3 \cdot 9^{x-2}$

14. $16^x = 2 \cdot 8^{2x+3}$

15. $2^{3x+5} = 2^{x^2+1}$

16. $10^{x^2+x} = 10^{x+9}$

17. $10^{2x^2+3} = 10^{x+6}$

18. $3^{x^2+5x} = 3^{2x-2}$

19. $\left(3^{x+1}\right)^x = \left(3^{x+3}\right)^2$

20. $\left(10^x\right)^{x+3} = \left(10^{x+2}\right)^{-2}$

21. $3^{x+4} = 9$

22. $2^{5x-8} = 4$

23. $4^{x^2-x} = \left(\dfrac{1}{2}\right)^{5x}$

24. $25^{x^2+x} = \left(\dfrac{1}{5}\right)^{3x}$

25. $5^{2x-x^2} = \dfrac{1}{125}$

26. $10^{x^2-2x} = 1000$

27. $10^{3x} = 140$

28. $10^{2x} = 97$

29. $10^{0.32x} = 253$

30. $10^{-0.48x} = 88.6$

31. $4 \cdot 10^{-0.94x} = 126.2$

32. $3 \cdot 10^{-2.1x} = 83.5$

33. $e^{3x} = 2.1$

34. $e^{4t} = 184$

35. $e^{-0.5x} = 47$

36. $e^{-0.006t} = 50.3$

37. $3e^{-0.12t} = 3.6$

38. $5e^{2.4t} = 44$

39. $2^x = 10$

40. $3^{x-2} = 100$

41. $5^{2x} = \frac{1}{100}$

42. $7^{3x} = \frac{1}{10}$

43. $5^{1-x} = 1$

44. $12^{5x+2} = 1$

45. $4^{2x+5} = 0.01$

46. $4^{2-3x} = 0.1$

47. $14^{3x-1} = 10^3$

48. $12^{2x+7} = 10^4$

49. $7^x = 9$

50. $2^x = 20$

51. $3^{3x} = 23$

52. $5^{2x} = 23$

53. $6^{2x-1} = 14.8$

54. $4^{7-3x} = 26.3$

55. $5\log x = 7$

56. $3\log x = 13.2$

57. $4\log x - 6 = 0$

58. $2\log x - 15 = 0$

59. $4\log x^{\frac{1}{2}} + 8 = 0$

60. $\frac{2}{3}\log x^{\frac{2}{3}} + 9 = 0$

61. $5\ln x - 8 = 0$

62. $2\ln x + 3 = 0$

63. $\ln x^2 + 2.2 = 0$

64. $\ln x^2 - 41.6 = 0$

65. $\log x + \log 2x = \log 18$

66. $\log(x+4) + \log(x-4) = \log 9$

67. $\log x^2 - \log x = 2$

68. $\log x + \log x^2 = 3$

69. $\ln(x-3) + \ln x = \ln 18$

70. $\ln(x+5) + \ln(x-1) = \ln 16$

71. $\log(x-15) = 2 - \log x$

72. $\log(2x-17) = 2 - \log x$

73. $\log(3x-5) + \log(x-1) = 1$

74. $\log(2x-3) + \log(x+3) = 3$

75. $\log(x-3) - \log(x+1) = 1$

76. $\ln(x+1) + \ln(x-1) = 0$

77. $\log(x^2-9) - \log(x-3) = -2$

78. $\ln(x^2+4x-5) - \ln(x+5) = -2$

79. $\log(x^2-4x-5) - \log(x+1) = 2$

80. $\log(x^2-x-12) - \log(x-4) = -2$

81. $\ln(x^2-4) = 3 + \ln(x+2)$

82. $\ln(x^2+2x-3) = 1 + \ln(x-1)$

83. $\log\sqrt[3]{x^2+2x+20} = \frac{2}{3}$

84. $\log\sqrt{x^2-24} = \frac{3}{2}$

Use the change-of-base formula to evaluate each of the expressions or solve the equations. Round answers to the nearest ten-thousandth. See Examples 4 and 5.

85. $\log_3 12$

86. $\log_4 36$

87. $\log_5 1.68$

88. $\log_{11} 39.6$

89. $\log_8 0.271$

90. $\log_7 0.849$

91. $\log_{15} 739$

92. $\log_2 14.2$

93. $\log_{20} 0.0257$

94. $\log_9 2.384$

95. $2^x = 5$

96. $3^{2x} = 10$

97. $9^{2x-1} = 100$

98. $5^{x-1} = 30$

99. $4^{3-x} = 20$

100. $6^{4-3x} = 25$

Writing & Thinking

101. Solve the following equation for *x* two different ways: $a^{2x-1} = 1$.

102. Rewrite each of the following expressions as products.

a. 5^{x+2}

b. 3^{x-2}

103. Explain, in your own words, why $7 \cdot 7^x \neq 49^x$ when $x \neq 1$. Show each of the expressions $7 \cdot 7^x$ and 49^x as a single exponential expression with base 7.