

- For the logarithmic function  $y = \log_b x$  (or  $x = b^y$ ),  
the domain is all  $x > 0$ , and (The graph is to the right of the  $y$ -axis.)  
the range is all real  $y$ .  
There is a vertical asymptote at  $x = 0$ .

**Margin Exercise Answers**

1. a.  $\log_4 4 = 1$  b.  $\log_4 64 = 3$  c.  $\log_4 \left(\frac{1}{4}\right) = -1$  d.  $3^2 = 9$  e.  $3^4 = 81$  f.  $3^{-1} = \frac{1}{3}$  2. a. 0 b. 1  
c. 30 d. 6 e.  $-3$  3. 4 4.  $\frac{5}{2}$

## 11.4 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- The function  $x = b^y$  is equivalent to  $y = \underline{\hspace{2cm}}$ .
- The line  $y = 0$  is the  $\underline{\hspace{2cm}}$  asymptote of  $y = b^x$ .
- The inverse of an exponential function is a/an  $\underline{\hspace{2cm}}$  function.
- Regardless of the base, the logarithm of 1 is  $\underline{\hspace{2cm}}$ .
- The graph of a logarithmic function can be found by  $\underline{\hspace{2cm}}$  the corresponding exponential function across the line  $y = x$ .
- The points on the graph of the inverse function can be found by  $\underline{\hspace{2cm}}$  the coordinates of the ordered pairs.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so that the statement will be true. (**Note:** There may be more than one acceptable change.)

- Exponential functions of the form  $y = b^x$  are one-to-one functions and have inverses.
- The exponent of an exponential function is the base of its inverse logarithmic function.
- Exponents are logarithms.
- The logarithm of the base is always 1.

### Practice

Express each equation in logarithmic form. See Example 1.

- |                            |                            |                 |
|----------------------------|----------------------------|-----------------|
| 1. $7^2 = 49$              | 4. $2^{-5} = \frac{1}{32}$ | 7. $10^2 = 100$ |
| 2. $3^3 = 27$              | 5. $1 = \pi^0$             | 8. $10^1 = 10$  |
| 3. $5^{-2} = \frac{1}{25}$ | 6. $6^0 = 1$               | 9. $10^k = 23$  |

10.  $4^k = 11.6$

11.  $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

12.  $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Express each equation in exponential form. See Example 1.

13.  $\log_3 9 = 2$

17.  $\log_7 \frac{1}{7} = -1$

21.  $\log_b 18 = 4$

14.  $\log_5 125 = 3$

18.  $\log_{1/2} 8 = -3$

22.  $\log_b 39 = 10$

15.  $\log_9 3 = \frac{1}{2}$

19.  $\log_{10} N = 1.74$

23.  $\log_n y^2 = x$

16.  $\log_b 4 = \frac{2}{3}$

20.  $\log_2 42.3 = x$

24.  $\log_b a = x^2$

Use the four basic properties of logarithms to evaluate each expression. See Example 2.

25.  $\log_3 81$

27.  $\log_7 1$

29.  $\log_4 \frac{1}{64}$

26.  $7^{\log_7 15}$

28.  $5^{\log_5 25}$

30.  $\log_{12} 12$

Solve by first changing each equation to exponential form. See Examples 3 and 4.

31.  $\log_4 x = 2$

37.  $\log_{36} x = -\frac{1}{2}$

43.  $\log_8 8^{3.7} = x$

32.  $\log_3 x = 4$

38.  $\log_{81} x = -\frac{3}{4}$

44.  $\log_{10} 10^{1.52} = x$

33.  $\log_{14} 196 = x$

39.  $\log_x 32 = 5$

45.  $\log_5 5^{\log_5 25} = x$

34.  $\log_{25} 125 = x$

40.  $\log_x 121 = 2$

46.  $\log_4 4^{\log_2 8} = x$

35.  $\log_5 \frac{1}{125} = x$

41.  $\log_8 x = \frac{5}{3}$

36.  $\log_3 \frac{1}{9} = x$

42.  $\log_{16} x = \frac{3}{4}$

Graph each function and its inverse on the same set of axes. Label two points on each graph.

47.  $f(x) = 6^x$

50.  $y = \left(\frac{1}{4}\right)^x$

53.  $y = \log_{1/2} x$

48.  $f(x) = 2^x$

51.  $f(x) = \log_4 x$

54.  $y = \log_{1/3} x$

49.  $y = \left(\frac{2}{3}\right)^x$

52.  $f(x) = \log_5 x$

55.  $y = \log_8 x$

56.  $y = \log_7 x$

57. Consider the function  $y = c(3^x)$  where  $c$  is a constant greater than zero. List the following:

- The domain of the function.
- The range of the function.
- Any asymptotes of the graph of the function.
- Give  $c$  two different values and sketch the graphs of both functions.

58. Consider the function  $y = c(3^{-x})$  where  $c$  is a constant greater than zero. List the following:
- The domain of the function.
  - The range of the function.
  - Any asymptotes of the graph of the function.
  - Give  $c$  two different values and sketch the graphs of both functions.

### Writing & Thinking

59. Discuss, in your own words, the symmetrical relationship of the graphs of the two functions  $y = 10^x$  and  $y = \log_{10} x$ .
60. Discuss, in your own words, the symmetrical relationship of the graphs of the two logarithmic functions  $y = \log_{10} x$  and  $y = -\log_{10} x$ .