Continuously Compounded Interest

Continuously compounded interest on a principal P invested at an annual interest rate r for t years can be calculated using the following formula, where A is the amount accumulated.

$$A = Pe^{rt}$$

FORMULA

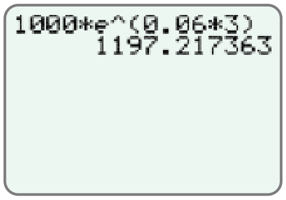
As illustrated in Example 6, a calculator is needed to use the formula for continuously compounded interest.

Example 6 Using a Graphing Calculator to Calculate Continuously Compounded Interest

Find the value of $1000 invested at 6% for 3 years if interest is compounded continuously. (In this case, $P = \$1000$, $r = 6\% = 0.06$, and $t = 3$.)

Solution

To find the value of $A = Pe^{rt} = 1000e^{0.06 \cdot 3}$ enter the numbers as shown and press ENTER to get the result.



The entire exponent must be in parentheses.

Note: Press 2nd and LN and `e^(` will appear on the display.)

Thus, the value of $1000 compounded continuously at 6% for 3 years will be $1197.22. (Notice that from Example 4 there is only a 54 cent gain in A when $1000 is compounded continuously instead of monthly at 6% for 3 years.)

Now work margin exercise 6.

6. Find the value of $1500 invested at 5.5% for 4 years if interest is compounded continuously.

Margin Exercise Answers

1. $y = 2{,}621{,}440{,}000$ or 2.62144×10^9 **2.** $y = 7000 \cdot 2^t$ **3.** $A = \$2205$ **4.** $A = \$2209.88$ **5.** $A = \$2210.33$ **6.** $1869.12

11.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. In an exponential function, the base is a/an ______ and the exponent is a/an ______.

2. Exponential growth is faster if the base is ______.

3. Exponential decay functions have a base between ____ and ____.

4. The y-intercept of any exponential function is _____.

5. The formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ is used to calculate ________ interest.

6. The formula $A = Pe^{rt}$ is used to calculate _______ compounded interest.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so that the statement will be true. (**Note:** There may be more than one acceptable change.)

7. For all exponential functions $f(x) = x^b,\ b < 0$.

8. The function $f(x) = 5^x$ is an example of an exponential growth model.

9. In an exponential decay function, b^x approaches the x-axis for positive values of x.

10. The number e is defined to be approximately 3.14159.

Practice

Sketch the graph of each exponential function and label three points on each graph. (Note that some of the graphs are shifts, horizontal or vertical, of the basic exponential functions. These are similar to the shifts performed on parabolas in Chapter 10.)

1. $y = 4^x$

2. $y = 5^x$

3. $y = \left(\frac{1}{3}\right)^x$

4. $y = \left(\frac{1}{5}\right)^x$

5. $y = \left(\frac{2}{3}\right)^x$

6. $y = \left(\frac{5}{2}\right)^x$

7. $y = \left(\frac{1}{2}\right)^{-x}$

8. $y = \left(\frac{3}{4}\right)^{-x}$

9. $y = 2^{x-1}$

10. $y = 3^{x+1}$

11. $f(x) = 2^x + 1$

12. $f(x) = 3^x - 1$

13. $f(x) = -4^{-x}$

14. $g(x) = -2^{-x}$

15. $f(x) = 2^{0.5x}$

16. $g(x) = 10^{0.5x}$

17. $f(x) = 4^{-x} - 1$

18. $g(x) = 10^{-x} - 3$

19. $f(x) = 3\cdot\left(\frac{1}{2}\right)^{x+1}$

20. $y = -4\cdot\left(\frac{1}{3}\right)^{x-1}$

Find the following function values.

21. If $f(t) = 3\cdot 4^t$, what is the value of $f(2)$?

22. For $f(x) = 3\cdot 10^{2x}$, find the value of $f(0.5)$.

Use your calculator to find each value as indicated. Round your answer to the nearest hundredth.

23. Find $f(2)$ if $f(x) = 27.3\cdot e^{-0.4x}$.

24. Find $f(3)$ if $f(x) = 41.2\cdot e^{-0.3x}$.

25. Find $f(9)$ if $f(t) = 2000\cdot e^{0.08t}$.

26. Find $f(22)$ if $f(t) = 2000\cdot e^{0.05t}$.

Solve.

27. Use a graphing calculator to graph each of the following functions. In each case the x-axis is a horizontal asymptote.

a. $y = e^{x}$ **b.** $y = e^{-x}$ **c.** $y = e^{-x^2}$

Applications

Solve.

28. A biologist knows that, in the laboratory, bacteria in a culture grow according to the function $y = y_0 \cdot 5^{0.2t}$, where y_0 is the initial number of bacteria present and t is time measured in hours. How many bacteria will be present in a culture at the end of 5 hours if there were 5000 present initially?

29. Referring to Exercise 28, how many bacteria were present initially if, at the end of 15 hours, there were 2,500,000 bacteria present?

30. Four thousand dollars is deposited into a savings account with a rate of 8% per year. Find the total amount A on deposit at the end of 5 years if the interest is compounded

a. annually.
b. semiannually.
c. quarterly.
d. daily.
e. continuously.

31. Find the amount A in a savings account if \$2000 is invested at 7% for 4 years and the interest is compounded

a. annually.
b. semiannually.
c. quarterly.
d. daily.
e. continuously.

32. Find the value of \$1800 invested at 6% for 3 years if the interest is compounded continuously.

33. Find the value of \$2500 invested at 5% for 5 years if the interest is compounded continuously.

34. The revenue function is given by $R(x) = x \cdot p(x)$ dollars, where x is the number of units sold and $p(x)$ is the unit price. If $p(x) = 25(2)^{\frac{-x}{5}}$, find the revenue if 15 units are sold.

35. Referring to Exercise 34, if $p(x) = 40(3)^{\frac{-x}{6}}$, find the revenue if 12 units are sold.

36. A radio station knows that during an intense advertising campaign, the number of people N who will hear a commercial is given by $N = A\left(1 - 2^{-0.05t}\right)$, where A is the number of people in the broadcasting area and t is the number of hours the commercial has been run. If there are 500,000 people in the area, how many will hear a commercial during the first 20 hours?

37. Bethany invested \$45,000 in a retirement fund that earns 8% interest and is compounded continuously. How much money will the account be worth after:

a. 10 years
b. 20 years
c. 40 years

38. Statistics show that the fractional part of flashlight batteries f that are still good after t hours of use is given by $f = 4^{-0.02t}$. What fractional part of the batteries are still operating after 150 hours of use?

39. If a principal P is invested at a rate r compounded continuously, the interest earned is given by $I = A - P$.

- **a.** Find the interest earned in 20 years on \$10,000 invested at 10% and compounded continuously.
- **b.** Find the interest earned in 20 years on \$10,000 invested at 5% and compounded continuously.
- **c.** Explain why the interest earned at 5% is not just one-half of the interest earned at 10% in parts a. and b.

40. The value V of a machine at the end of t years is given by $V = C(1-r)^t$, where C is the original cost and r is the rate of depreciation. Find the value of a machine at the end of 4 years if the original cost was \$1200 and $r = 0.20$.

41. Referring to Exercise 40, find the value of a machine at the end of 3 years if the original cost was \$2000 and $r = 0.15$.

42. A cancer patient is given a dose of 50 mg of a particular drug. In five days, the amount of the drug in her system is reduced to 1.5625 mg. If the drug decays (or is absorbed) at an exponential rate, find the function that represents the amount of the drug at a given time. (**Hint:** Use the formula $y = y_0 b^{-t}$ and solve for b.)

43. Determine the exponential function that fits the following information concerning exponential growth of cancer cells: $y_0 = 10,000$ cancer cells, and there are 160,000 cancer cells present after 4 days. (**Hint:** Use the formula $y = y_0 b^t$ and solve for b.)

Writing & Thinking

44. Discuss, in your own words, the symmetrical relationship of the graphs of the two exponential functions $y = 10^x$ and $y = 10^{-x}$.

45. Discuss, in your own words, the symmetrical relationship of the graphs of the two exponential functions $y = 10^x$ and $y = -10^x$.

Collaborative Learning

46. The following formula can be used to calculate monthly mortgage payments:

$$A = \frac{P\left(1+\frac{r}{12}\right)^{n} \cdot \frac{r}{12}}{\left(1+\frac{r}{12}\right)^{n} - 1}$$

where

A = the monthly payment,

P = amount initially borrowed (the mortgage),

r = the annual interest rate (in decimal form), and

n = the total number of monthly payments (12 times the number of years).

With the class divided into teams of 3 or 4 students, each team should complete one table (using different values for r and for P). Discuss the results as a class. Explain what this might mean for you personally.

For annual rate r = _____ and initial mortgage P = _________

Length of Mortgage (in years)	Monthly Payment *A*	Total Cost of Mortgage *n* times *A*
15		
20		
25		
30		