

11.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- To evaluate a function at an algebraic expression, replace the _____ with the expression everywhere the _____ appears.
- Given two functions $f(x)$ and $g(x)$ a new function $f(g(x))$, called the composition of f and g , is found by substituting the _____ for $g(x)$ into the place of x in the _____ f .
- In general, $f(g(x))$ _____ $g(f(x))$.
- The domain of $f \circ g$ consists of those values of x in the _____ of g for which $g(x)$ is in the _____ of f .
- Functions that have only one x -value for each y -value in the range are said to be _____ functions.
- In general, every _____ function has a/an _____ function.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- The vertical line test is used to determine whether a graph represents a vertical line.
- In a one-to-one function, each x -value corresponds to exactly one y -value.
- The horizontal line test is used to determine whether a graph of a function is one-to-one.
- The notation $f^{-1}(x)$ means $\frac{1}{f(x)}$.

Practice

Find the indicated function values for each function given. See Example 1.

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|----------------------|---------------------------|
| 1. $f(x) = 8x - 5$ | 4. $h(y) = y^4 + 8$ |
| a. $f(r)$ | a. $h(3p)$ |
| b. $f(3a - 1)$ | b. $h(2s^2)$ |
| 2. $r(x) = 4x - 6$ | 5. $f(c) = 3c^2 + 6c - 9$ |
| a. $r(g - 5)$ | a. $f(n - 2)$ |
| b. $r(h^2 + 8)$ | b. $f(4y^3)$ |
| 3. $g(y) = 5y^2 + 4$ | 6. $b(t) = t^2 - 2t + 7$ |
| a. $g(x - 2)$ | a. $b(5k)$ |
| b. $g(3n^2)$ | b. $b(x + 1)$ |

Find the following function compositions.


7. $f(x) = 3x + 5$, $g(x) = \frac{x+4}{2}$ Find **a.** $f(g(2))$ and **b.** $g(f(2))$.
8. $f(x) = \frac{1}{4}x + 1$, $g(x) = 6x - 7$ Find **a.** $f(g(4))$ and **b.** $g(f(4))$.
9. $f(x) = x^2$, $g(x) = 2x + 3$ Find **a.** $(f \circ g)(-5)$ and **b.** $(g \circ f)(-1)$.
10. $f(x) = x^2 + 1$, $g(x) = x - 6$ Find **a.** $(f \circ g)(3)$ and **b.** $(g \circ f)(-2)$.

Form the compositions $f(g(x))$ and $g(f(x))$ for each pair of functions. See Examples 2 through 4.

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| 11. $f(x) = \sqrt{x}$, $g(x) = x^2$ | 19. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x^2$ |
| 12. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x}$ | 20. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x^2 - 4$ |
| 13. $f(x) = \sqrt{x}$, $g(x) = x - 2$ | 21. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 7x - 8$ |
| 14. $f(x) = \sqrt{x}$, $g(x) = x^2 - 9$ | 22. $f(x) = \frac{1}{x+1}$, $g(x) = x^2 + x - 3$ |
| 15. $f(x) = x - 1$, $g(x) = \frac{1}{x^2}$ | 23. $f(x) = x^{3n}$, $g(x) = 2x - 6$ |
| 16. $f(x) = \frac{1}{x^2}$, $g(x) = x^2 + 1$ | 24. $f(x) = x^{\frac{1}{3}}$, $g(x) = 4x + 7$ |
| 17. $f(x) = x^3 + x + 1$, $g(x) = x + 1$ | 25. $f(x) = x^3$, $g(x) = \sqrt{x - 8}$ |
| 18. $f(x) = x^3$, $g(x) = 2x - 1$ | 26. $f(x) = x^3 + 1$, $g(x) = \frac{1}{x}$ |

Solve.

27. For the functions $f(x) = 6x - 3$ and $g(x) = \frac{1}{3}x + 3$, find:
- $f(g(3))$
 - $g(f(0))$
 - Does it appear that f and g are inverses of each other? Explain.
28. For the functions $h(x) = -2x + 4$ and $g(x) = \frac{4-x}{2}$, find:
- $h(g(6))$
 - $g(h(-4))$
 - Does it appear that h and g are inverses of each other? Explain.
29. Given $f(x) = \frac{1}{2x+1}$ and $g(x) = -\frac{1}{x}$, find:
- $g(f(4))$
 - $f(g(2))$
 - Explain the different results from parts a. and b.
30. Given $f(x) = \sqrt{x-9}$ and $g(x) = x-9$, find:
- $g(f(109))$
 - $f(g(9))$
 - Explain the different results from parts a. and b.

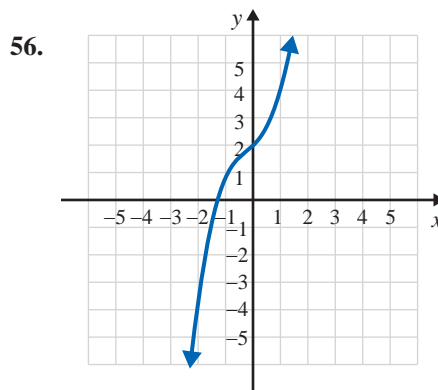
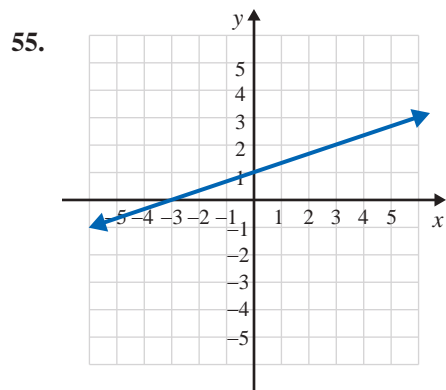
 Show that the given one-to-one functions are inverses of each other. Then graph both functions on the same set of axes and show the line $y = x$ as a dotted line on each graph. (You may use a calculator as an aid in finding the graphs.) See Examples 6 and 7.

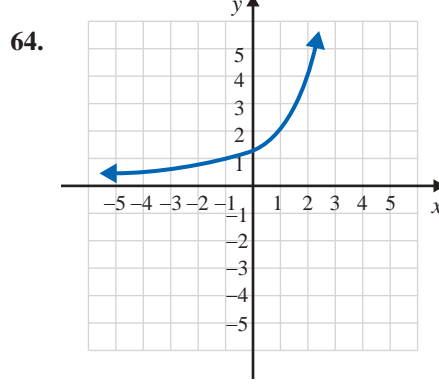
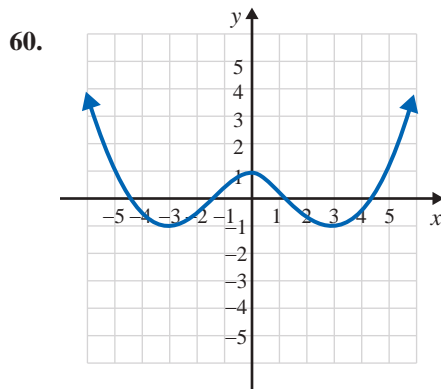
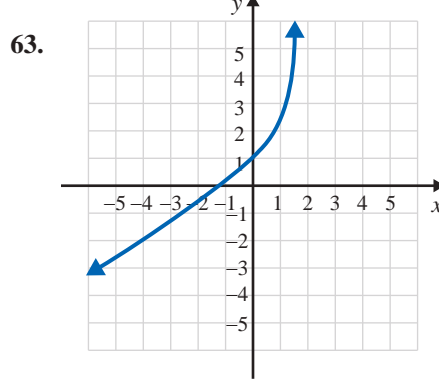
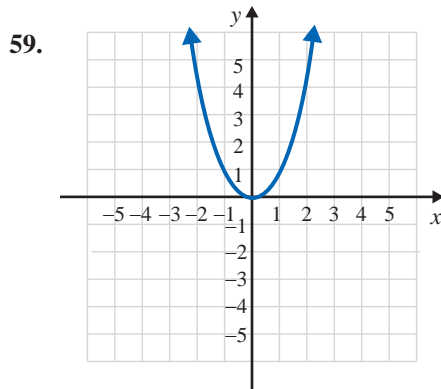
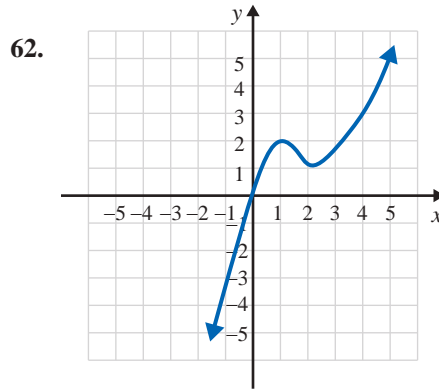
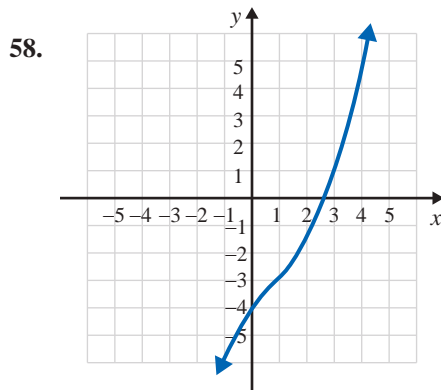
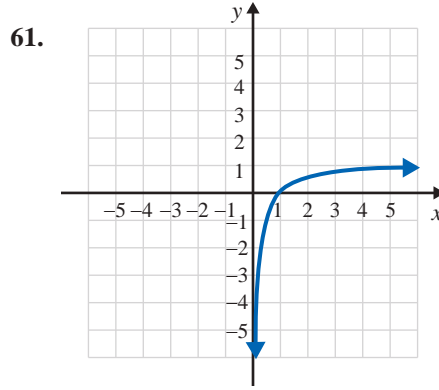
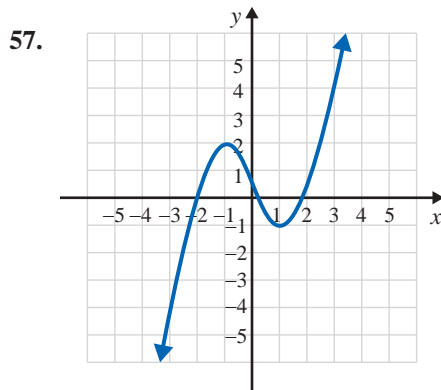
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| 31. $f(x) = 3x + 1$ and $g(x) = \frac{x-1}{3}$ | 37. $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ |
| 32. $f(x) = -2x + 3$ and $g(x) = \frac{3-x}{2}$ | 38. $f(x) = \sqrt[5]{x+6}$ and $g(x) = x^5 - 6$ |
| 33. $f(x) = \sqrt[3]{x-1}$ and $g(x) = x^3 + 1$ | 39. $f(x) = \frac{2}{x}$ and $g(x) = \frac{2}{x}$ |
| 34. $f(x) = x^3 - 4$ and $g(x) = \sqrt[3]{x+4}$ | 40. $f(x) = \frac{3}{x}$ and $g(x) = \frac{3}{x}$ |
| 35. $f(x) = x^2$ for $x \geq 0$ and
$g(x) = \sqrt{x}$ | |
| 36. $f(x) = \sqrt{x+3}$ and
$g(x) = x^2 - 3$ for $x \geq 0$ | |


Find the inverse of the given function. Then graph both functions on the same set of axes and show the line $y = x$ as a dotted line on the graph. See Examples 9 and 10.

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|-------------------------------|------------------------------------|
| 41. $f(x) = 2x - 3$ | 48. $f(x) = -\frac{1}{2}x - 3$ |
| 42. $f(x) = 2x - 5$ | 49. $f(x) = -x - 2$ |
| 43. $g(x) = x$ | 50. $f(x) = -2x + 4$ |
| 44. $g(x) = 1 - 4x$ | 51. $f(x) = x^2 + 1, x \geq 0$ |
| 45. $f(x) = 5x + 1$ | 52. $f(x) = x^2 - 1, x \geq 0$ |
| 46. $g(x) = -3x + 1$ | 53. $f(x) = -\sqrt{x}, x \geq 0$ |
| 47. $g(x) = \frac{2}{3}x + 2$ | 54. $f(x) = -\sqrt{x-2}, x \geq 2$ |

Using the horizontal line test, determine which of the graphs are graphs of one-to-one functions. If the graph represents a one-to-one function, graph its inverse by reflecting the graph of the function across the line $y = x$. (**Hint:** If a function is one-to-one, label a few points on the graph and use the fact that the x - and y -coordinates are interchanged on the graph of the inverse.) See Example 5.





 Use a graphing calculator to graph each of the functions and determine which of the functions are one-to-one by inspecting the graph and using the horizontal line test.

65. $f(x) = 2x + 3$

66. $f(x) = 7 - 4x$

67. $g(x) = x^2 - 2$

68. $g(x) = 9 - x^2$

69. $f(x) = 4 - x^3$

70. $f(x) = x^3 + 2$

71. $f(x) = \frac{4}{x}$


72. $g(x) = \frac{1}{x}$

73. $g(x) = \sqrt{x-3}$

74. $f(x) = \sqrt{x+5}$

75. $f(x) = |x+1|$

76. $f(x) = |x-5|$

 Find the inverse of the given function. Then use a graphing calculator to graph both the function and its inverse. Set the WINDOW so that it is "square."

77. $f(x) = x^3$

78. $f(x) = (x+1)^3$

79. $f(x) = \frac{1}{x-3}$

80. $f(x) = \frac{1}{x}$

81. $f(x) = x^2, x \geq 0$

82. $f(x) = x^2 + 2, x \geq 0$

83. $g(x) = x^3 + 2$

84. $g(x) = 6 - x^3$

85. $f(x) = \sqrt{x+5}, x \geq -5$

86. $g(x) = \sqrt{x-3}, x \geq 3$

87. $f(x) = -x^2 + 1, x \geq 0$

88. $g(x) = -x^2 - 2, x \geq 0$

Writing & Thinking

89. Explain in your own words why the domains of the two composite functions $f(g(x))$ and $g(f(x))$ might not be the same. Give an example of two functions that illustrate this possibility.
90. Explain briefly why a function must be one-to-one to have an inverse.