

11.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- Operating algebraically with functions, as well as understanding and finding the _____ and _____ of functions, relies heavily on function notation.
- Logarithms are _____.
- If two or more functions have the same _____, then we can perform the operations of addition, subtraction, multiplication, and division with these functions.
- In the case of finding the quotient of functions, no denominator can be _____.
- When operating with functions, the operations are performed with the _____ for each value of _____ in the common domain.
- In general, graphing the sum of two functions will involve a/an _____ number of points.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- One way to find the sum of two functions is to find the algebraic sum of the two expressions.
- The function $(f + g)(x)$ means the same as $f(x) + g(x)$.
- If functions do not have the same domain, any algebraic sums, differences, products, and quotients are restricted to portions of the range that are in common.
- If two functions have graphs that consist of line segments, the sum of the two functions will produce a graph that is a continuous line.

Practice

For the following pairs of functions find, **a.** $(f + g)(x)$, **b.** $(f - g)(x)$, **c.** $(f \cdot g)(x)$, and **d.** $\left(\frac{f}{g}\right)(x)$. See Examples 1 and 2.

- $f(x) = x + 2$, $g(x) = x - 5$
- $f(x) = 2x$, $g(x) = x + 4$
- $f(x) = x^2$, $g(x) = 3x - 4$
- $f(x) = x - 3$, $g(x) = x^2 + 1$
- $f(x) = x^2 - 9$, $g(x) = x - 3$
- $f(x) = x^2 - 25$, $g(x) = x + 5$
- $f(x) = 2x^2 + x$, $g(x) = x^2 + 2$
- $f(x) = x^3 + 6x$, $g(x) = x^2 + 6$
- $f(x) = x^2 + 4x + 1$, $g(x) = x^2 - 4x + 1$
- $f(x) = x^3 - x^2$, $g(x) = 6 - x^2$

Let $f(x) = x^2 + 4$ and $g(x) = -x + 3$. Find the values of the indicated expressions. See Examples 1 and 2.

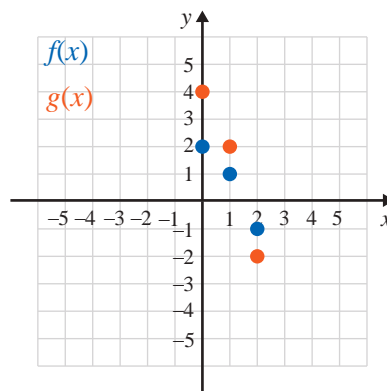
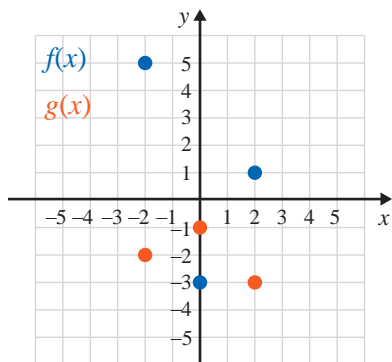
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| 11. $f(2) + g(2)$ | 16. $(f - g)(0.5)$ |
| 12. $f(2) \cdot g(2)$ | 17. $\left(\frac{f}{g}\right)(-2)$ |
| 13. $g(a) - f(a)$ | 18. $(f \cdot g)(-3)$ |
| 14. $\frac{g(a)}{f(a)}$ | 19. $(g - f)(-6)$ |
| 15. $(f + g)(-4)$ | 20. $\left(\frac{g}{f}\right)(-1)$ |

Find the indicated functions and state their domains in interval notation. See Example 3.

21. If $f(x) = \sqrt{2x - 6}$ and $g(x) = x + 4$, find $(f + g)(x)$.
22. If $f(x) = x^2 - 2x + 1$ and $g(x) = x - 1$, find $\left(\frac{f}{g}\right)(x)$.
23. Find $f(x) \cdot g(x)$ given that $f(x) = 3x + 2$ and $g(x) = x - 7$.
24. Find $f(x) - g(x)$ given that $f(x) = x^2$ and $g(x) = x^2 - 2$.
25. For $f(x) = x - 5$ and $g(x) = \sqrt{x + 3}$, find $\frac{f(x)}{g(x)}$.
26. For $f(x) = 2x - 8$ and $g(x) = \sqrt{2 - x}$, find $f(x) \cdot g(x)$.
27. If $f(x) = -\sqrt{x - 3}$ and $g(x) = 3x$, find $(f \cdot g)(x)$.
28. If $f(x) = -\sqrt{4 - x}$ and $g(x) = 5 - x$, find $(g - f)(x)$.
29. If $f(x) = \sqrt[3]{x + 3}$ and $g(x) = \sqrt{5 + x}$, find $f(x) + g(x)$.
30. If $f(x) = \sqrt{x - 1}$ and $g(x) = \sqrt[3]{2x + 1}$, find $f(x) - g(x)$.

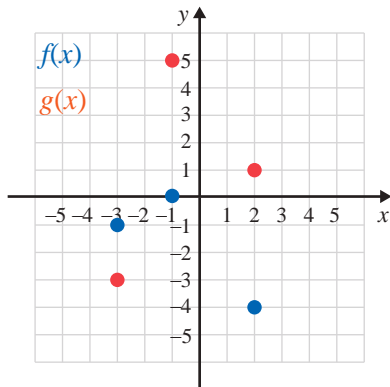
For the following pairs of functions, graph **a.** the sum $(f + g)$ and **b.** the difference $(f - g)$ on two different graphs.

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| 31. $f = \{(-2, 5), (0, -3), (2, 1)\}$ | 32. $f = \{(0, 2), (1, 1), (2, -1)\}$ |
| $g = \{(-2, -2), (0, -1), (2, -3)\}$ | $g = \{(0, 4), (1, 2), (2, -2)\}$ |



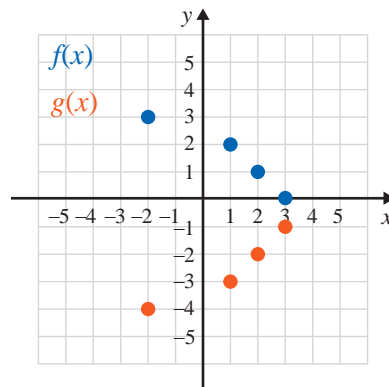
$$33. f = \{(-3, -1), (-1, 0), (2, -4)\}$$

$$g = \{(-3, -3), (-1, 5), (2, 1)\}$$



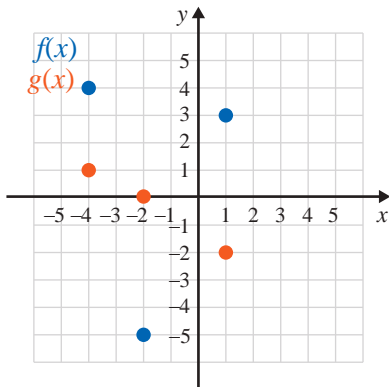
$$36. f = \{(-2, 3), (1, 2), (2, 1), (3, 0)\}$$

$$g = \{(-2, -4), (1, -3), (2, -2), (3, -1)\}$$



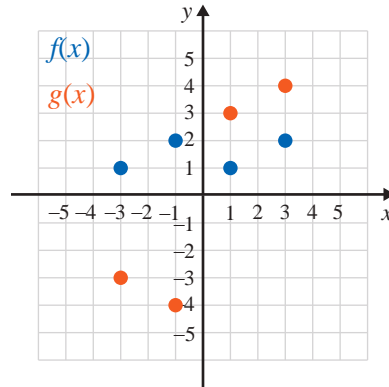
$$34. f = \{(-4, 4), (-2, -5), (1, 3)\}$$

$$g = \{(-4, 1), (-2, 0), (1, -2)\}$$



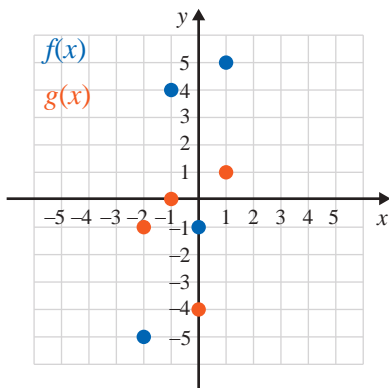
$$37. f = \{(-3, 1), (-1, 2), (1, 1), (3, 2)\}$$

$$g = \{(-3, -3), (-1, -4), (1, 3), (3, 4)\}$$



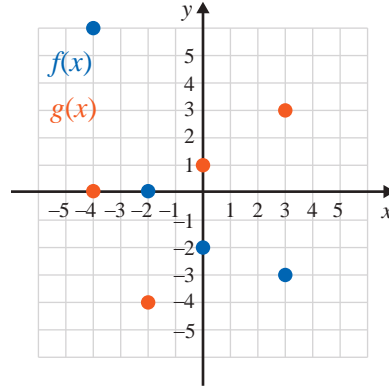
$$35. f = \{(-2, -5), (-1, 4), (0, -1), (1, 5)\}$$

$$g = \{(-2, -1), (-1, 0), (0, -4), (1, 1)\}$$

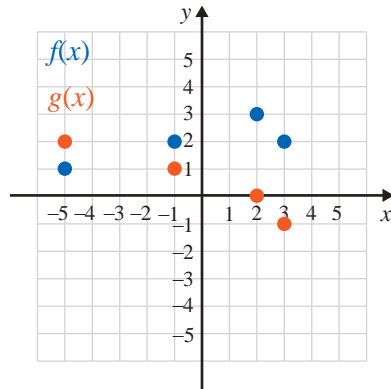


$$38. f = \{(-4, 6), (-2, 0), (0, -2), (3, -3)\}$$

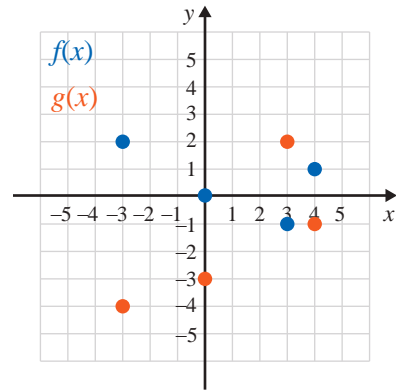
$$g = \{(-4, 0), (-2, -4), (0, 1), (3, 3)\}$$



39. $f = \{(-5, 1), (-1, 2), (2, 3), (3, 2)\}$
 $g = \{(-5, 2), (-1, 1), (2, 0), (3, -1)\}$



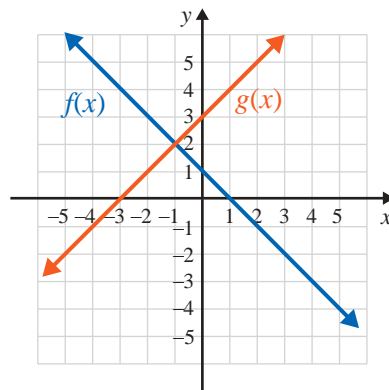
40. $f = \{(-3, 2), (0, 0), (3, -1), (4, 1)\}$
 $g = \{(-3, -4), (0, -3), (3, 2), (4, -1)\}$



Graph each pair of functions and the sum of these functions on the same set of axes.

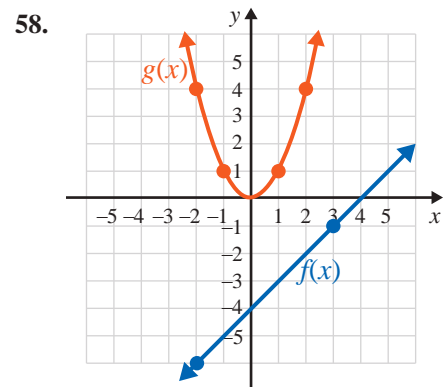
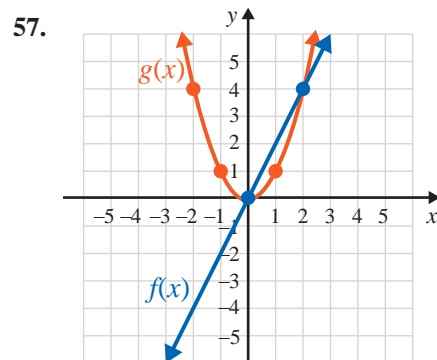
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|---------------------------------------|---|
| 41. $f(x) = x^2$ and $g(x) = -1$ | 46. $f(x) = 2 - x$ and $g(x) = x$ |
| 42. $f(x) = x^2$ and $g(x) = 2$ | 47. $f(x) = x + 1$ and $g(x) = x^2 - 1$ |
| 43. $f(x) = x + 1$ and $g(x) = 2x$ | 48. $f(x) = x^2 + 2$ and $g(x) = x^2 - 2$ |
| 44. $f(x) = x + 5$ and $g(x) = x - 5$ | 49. $f(x) = \sqrt{x - 6}$ and $g(x) = 2$ |
| 45. $f(x) = x + 4$ and $g(x) = -x$ | 50. $f(x) = \sqrt{3 - x}$ and $g(x) = -1$ |

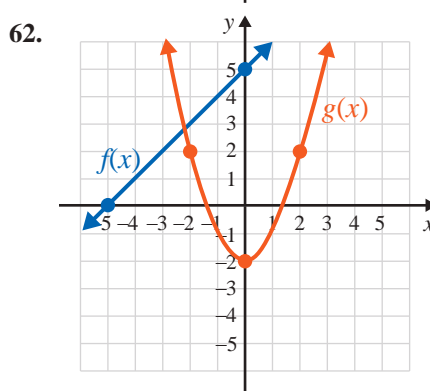
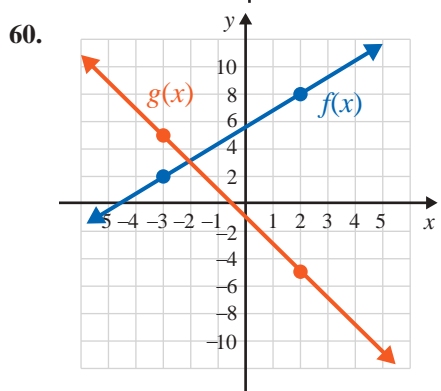
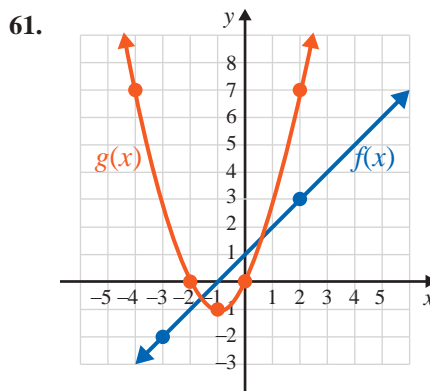
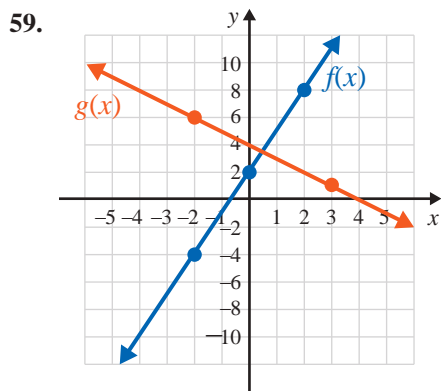
Use the graph shown here to find the values indicated.




51. $(f + g)(-2)$
 52. $(f - g)(2)$
 53. $(f \cdot g)(3)$
 54. $(g - f)(0)$
 55. $\left(\frac{f}{g}\right)(4)$
 56. $(g \cdot f)(4)$

Graph the sum of each function.





 Use a graphing calculator to graph each pair of functions and the sum of these functions on the same set of axes.

63. $f(x) = x^2$ and $h(x) = 2x + 1$

67. $f(x) = \sqrt[3]{x+5}$ and $h(x) = 2x$

64. $g(x) = x^2 + x$ and $h(x) = 3x + 4$

68. $h(x) = \sqrt[3]{x-1}$ and $g(x) = x - 1$

65. $f(x) = \sqrt{x+4}$ and $g(x) = -2$

69. $g(x) = 7 - x^2$ and $h(x) = x^2 - 3$

66. $f(x) = -\sqrt{x-1}$ and $g(x) = 3$

70. $f(x) = x^2 + 5$ and $g(x) = 4 - x^2$

Writing & Thinking

71. Explain why, in general, $(f - g)(x) \neq (g - f)(x)$ if $f(x) \neq g(x)$.

72. Given the two functions f and g ,

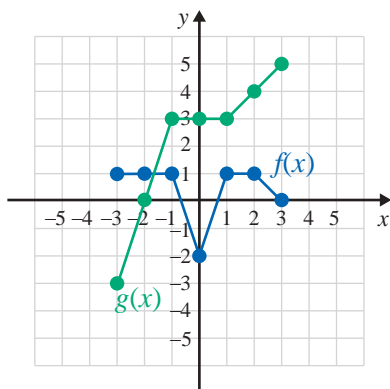
$$f = \{(-2, 0), (-1, 1), (0, 4), (2, 4), (3, 5), (4, 1)\}$$

$$g = \{(-2, 3), (-1, 4), (0, 1), (2, -1), (3, 2), (4, 6)\},$$

find and graph the following.

a. $f - g$ b. $f \cdot g$ c. $\frac{f}{g}$

73. Use the graphs of the two functions f and g shown in the graph.



- a. Sketch the graph of $f - g$.
- b. Sketch the graph of $f \cdot g$.
- c. Is $\frac{f}{g}$ defined on the entire interval $[-3, 3]$? Briefly explain your reasoning.