7. Solve each equation:

$x^3 - 216 = 0$

Example 7 Solving Higher-Degree Equations

Solve the equation: $x^3 - 27 = 0$

Solution

The polynomial is the difference of two cubes and can be factored. In this case, complex solutions can be found using the quadratic formula.

$$x^3 - 27 = 0$$

$$(x-3)(x^2+3x+9) = 0$$

$$x - 3 = 0 \qquad\qquad x^2 + 3x + 9 = 0$$

$$x = 3$$

Using the quadratic formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 9}}{2}$$

$$= \frac{-3 \pm \sqrt{-27}}{2}$$

$$= \frac{-3 \pm 3i\sqrt{3}}{2} \text{ or } -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$$

There are three solutions: $x = 3, \frac{-3+3i\sqrt{3}}{2}, \frac{-3-3i\sqrt{3}}{2}$.

Now work margin exercise 7.

Margin Exercise Answers

1. $\pm 1, \pm 2\sqrt{2}$ **2.** 512, –27 **3.** $\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{4}$ **4.** –4, 5 **5.** $\frac{2 \pm \sqrt{5}}{2}$ **6.** 0, $\pm\sqrt{5}$, $\pm i\sqrt{5}$
7. 6, $-3 \pm 3i\sqrt{3}$

10.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. An equation is in quadratic form when the degree of the ______ term is ______ the degree of the first term.

2. To solve an equation in quadratic form, a/an ______ can be made to clarify the problem.

3. When solving equations in quadratic form by substitution, substitute a/an ______-degree variable for the variable expression in the middle term.

4. When solving rational expressions, remember to check the ______ on the variables.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. Equations in quadratic form can be solved using the quadratic equation or by factoring.

6. When solving higher-degree equations, you shouldn't factor out any common monomials.

7. The LCM of the denominators is used to clear rational expressions of any fractions.

8. The degree of the first term of an equation in quadratic form must be 2.

Practice

Solve the equations.

1. $x^4 - 13x^2 + 36 = 0$
2. $x^4 - 29x^2 + 100 = 0$
3. $x^4 - 9x^2 + 20 = 0$
4. $y^4 - 11y^2 + 18 = 0$
5. $y^4 - 3y^2 - 28 = 0$
6. $y^4 + y^2 - 12 = 0$
7. $y^4 - 25 = 0$
8. $4x^4 - 100 = 0$
9. $2x - 9x^{\frac{1}{2}} + 10 = 0$
10. $2x - 3x^{\frac{1}{2}} + 1 = 0$
11. $x^3 - 9x^{\frac{3}{2}} + 8 = 0$
12. $y^3 - 28y^{\frac{3}{2}} + 27 = 0$
13. $2x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 2 = 0$
14. $2x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$
15. $3x^{\frac{5}{3}} + 15x^{\frac{4}{3}} + 18x = 0$
16. $2x^2 - 30x^{\frac{3}{2}} + 112x = 0$
17. $(x+7)^2 + 5(x+7) = 50$
18. $(x-1)^2 + (x-1) - 6 = 0$
19. $(2x+3)^2 + 7(2x+3) + 12 = 0$
20. $(5x-4)^2 + 2(5x-4) - 8 = 0$
21. $(x-3)^2 - 2(x-3) - 15 = 0$
22. $(x+4)^2 - 2(x+4) = 3$
23. $(2x+1)^2 + (2x+1) = 0$
24. $(3x-5)^2 + (3x-5) - 2 = 0$
25. $x^4 - 2x^2 + 2 = 0$
26. $x^4 - 4x^2 + 5 = 0$
27. $x^4 - 2x^2 + 10 = 0$
28. $x^4 - 6x^2 + 13 = 0$
29. $x^4 - 4x^2 + 7 = 0$
30. $x^4 - 6x^2 + 11 = 0$
31. $x^{-2} - 12x^{-1} + 35 = 0$
32. $z^{-2} - 2z^{-1} - 24 = 0$
33. $3x^{-2} + x^{-1} - 24 = 0$
34. $2x^{-2} - 7x^{-1} + 6 = 0$
35. $x^{-1} + 5x^{-\frac{1}{2}} - 50 = 0$
36. $3y^{-1} - 7y^{-\frac{1}{2}} + 2 = 0$
37. $x^{-4} - 6x^{-2} + 5 = 0$
38. $3x^{-4} - 5x^{-2} + 2 = 0$
39. $3x^{-4} + 25x^{-2} - 18 = 0$
40. $2x^{-4} + 3x^{-2} - 20 = 0$
41. $\frac{2}{4x-1} + \frac{1}{x+1} = \frac{-x}{x+1}$
42. $\frac{3x-2}{15} - \frac{16-3x}{x+6} = \frac{x+3}{5}$
43. $\frac{2x}{x-4} - \frac{12x}{x^2+x-20} = \frac{x-1}{x+5}$
44. $\frac{x+1}{x+3} + \frac{2x-1}{x-2} = \frac{12x-2}{x^2+x-6}$
45. $\frac{x+5}{3x+2} - \frac{4-2x}{3x^2+8x+4} = \frac{x+4}{x+2}$
46. $\frac{x+5}{3x+4} + \frac{16x^2+5x+6}{3x^2-2x-8} = \frac{4x}{x-2}$

47. $\frac{4x+1}{x-6}-\frac{3x^2-8x+20}{2x^2-13x+6}=\frac{3x+7}{2x-1}$

48. $\frac{3x+2}{x+3}+\frac{22x-31}{x^2-x-12}=\frac{3(x+4)}{x+3}$

49. $\frac{5(x-10)}{x-7}=\frac{2(x+1)}{x-4}+3$

50. $2+\frac{2-x}{x+2}=\frac{x-3}{x+5}$

51. $x^5-64x=0$

52. $x^5=36x$

53. $8x^3=64$

54. $x^3-125=0$

55. $x^5+1000x^2=0$

56. $2x^5+54x^2=0$

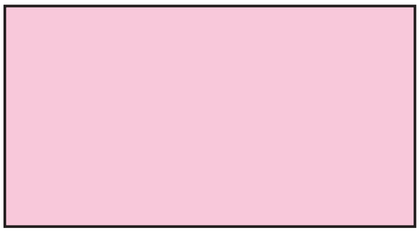
Writing & Thinking

57. One of the most studied and interesting visual and numerical concepts in algebra is the **Golden Ratio**. Ancient Greeks thought (and many people still do) that a rectangle was most aesthetically pleasing to the eye if the ratio of its length to its width is the Golden Ratio (about 1.618). In fact, the Parthenon, built by Greeks in the fifth century BC, utilizes the Golden Ratio. A rectangle is "golden" if its length l and width w satisfy the equation $\frac{l}{w}=\frac{w}{l-w}$.

a. By letting $w=1$ unit in the equation above, we get the equation $\frac{l}{1}=\frac{1}{l-1}$. Solve this equation for the positive value of l (which is the algebraic expression for the golden ratio).

b. Suppose that an architect is constructing a building with a rectangular front that is to be 60 feet high. About how long should the front be if he wants the appearance of a golden rectangle? (Assume $w=60$ feet and that you are looking for l.) Round to the hundredths place.

c. Consider rectangle A and rectangle B. Which seems most pleasing to your eye? Measure the length and width of each rectangle and see if you chose the golden rectangle.



Rectangle A

Rectangle B

58. Consider the following equation: $x-x^{\frac{1}{2}}-6=0$
In your own words, explain why, even though it is in quadratic form, this equation has only one solution.