Margin Exercise Answers

1. $x = 2, 8$ **2.** $x = -6 \pm 2\sqrt{3}$ **3.** $\pm 4i$ **4.** $x = 4 \pm 2\sqrt{3}$

5. a. $y^2 - 14y + \underline{49} = \left(\underline{y-7}\right)^2$ **b.** $x^2 + 9x + \underline{\frac{81}{4}} = \underline{\left(x + \frac{9}{2}\right)^2}$ **6.** $x = 5 \pm 2\sqrt{14}$

7. $x = -2 \pm i\sqrt{7}$ **8.** $x = 1 \pm \sqrt{6}$ **9.** $y^2 - 8y + 25 = 0$ **10.** $x^2 - 2x + 6 = 0$

10.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The zero-factor law states that if the product of two or more factors is ___________, then at least one of the factors must be ________.
2. When solving quadratic equations, if one side of the equation is a/an ___________, expression and the other side is constant, it can be solved by taking the square root of both sides.
3. The two equations $x = \sqrt{c}$ and $x = -\sqrt{c}$ can be written as ____________.
4. When using the square root method, if the squared expression is set equal to a/an _______ number, then the solution will be nonreal.
5. Completing the square is the process of adding terms to binomials so that the result will be a perfect square _______.
6. When solving by completing the square, the quadratic equation should have a leading coefficient of _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. It's possible for the roots of a quadratic equation to be nonreal numbers.
8. The square of a real number can be negative.
9. The last step of solving a quadratic equation by completing the square is to use the square root property.

Practice

Solve the following quadratic equations by factoring. See Example 1.

1. $x^2 = 11x$
2. $x^2 - 10x + 16 = 0$
3. $x^2 = -15x - 36$
4. $2x^2 + 36x + 34 = 0$
5. $9x^2 + 6x - 15 = 0$
6. $5x^2 + 17x = -6$
7. $(x+3)(x-1) = 4x$
8. $(x-7)(x-2) = 6$

9. $(2x-3)(2x+1)=3x-6$

10. $(x-2)(5x+4)=3x^2-15x-12$

Solve the following quadratic equations by using the square root method. Write each radical in simplest form. See Examples 2 through 4.

11. $x^2=121$

12. $x^2=81$

13. $3x^2=108$

14. $5x^2=245$

15. $x^2=35$

16. $x^2=42$

17. $x^2+25=0$

18. $x^2+81=0$

19. $x^2-62=0$

20. $x^2-75=0$

21. $3x^2=54$

22. $5x^2=60$

23. $9x^2=4$

24. $4x^2=25$

25. $(x-1)^2=4$

26. $(x+3)^2=9$

27. $(x-3)^2=-4$

28. $(x+8)^2=-9$

29. $(x+1)^2=\frac{1}{4}$

30. $(x-9)^2=-\frac{9}{25}$

31. $(x+2)^2=-7$

32. $(x+8)^2=75$

33. $(5x-2)^2=63$

34. $(4x-3)^2=125$

35. $(3x+4)^2+3=30$

36. $(2x+1)^2+12=60$

37. $2(x-7)^2=24$

38. $3(x+11)^2=60$

39. $3(x-5)^2+5=-25$

40. $2(x-6)^2-11=25$

Add the correct constant to complete the square; then factor the trinomial as indicated. See Example 5.

41. $x^2-12x+\_\_\_=(\_\_\_\_)^2$

42. $y^2+14y+\_\_\_=(\_\_\_\_)^2$

43. $x^2-5x+\_\_\_=(\_\_\_\_)^2$

44. $x^2+7x+\_\_\_=(\_\_\_\_)^2$

45. $y^2+y+\_\_\_=(\_\_\_\_)^2$

46. $x^2+\frac{1}{2}x+\_\_\_=(\_\_\_\_)^2$

47. $x^2+\frac{1}{3}x+\_\_\_=(\_\_\_\_)^2$

48. $y^2+\frac{3}{4}y+\_\_\_=(\_\_\_\_)^2$

49. $2x^2+4x+\_\_\_=2(\_\_\_\_)^2$

50. $3x^2+18x+\_\_\_=3(\_\_\_\_)^2$

Solve the quadratic equations by completing the square. See Examples 6 through 8.

51. $x^2+4x-5=0$

52. $x^2+6x-7=0$

53. $y^2+2y=5$

54. $x^2+3=8x$

55. $x^2+3=10x$

56. $z^2+4z=2$

57. $x^2-4x-45=0$

58. $x^2-10x+21=0$

59. $3x^2+x-4=0$

60. $2x^2+x-6=0$

61. $x^2-6x+10=0$

62. $x^2-2x+5=0$

63. $y^2=10y-4$

64. $x^2=3-4x$

65. $z^2+3z-5=0$

66. $x^2-5x+5=0$

67. $x^2+x+2=0$

68. $y^2+3y+3=0$

69. $x^2+5x+2=0$

70. $4x^2+7x+2=0$

71. $3x^2-10x+5=0$

72. $3y^2+5y-3=0$

73. $3x^2+6x+18=0$

74. $4x^2+8x+16=0$

75. $2x-3=4x^2$

76. $2x+2=-6x^2$

77. $5y^2+15y+25=0$

78. $4x^2+20x+32=0$

79. $3y^2=4-y$

80. $2x^2+4=-9x$

81. $2x^2-8x+4=0$

82. $3x^2-18x+12=0$

Write a quadratic equation with integer coefficients that has the given roots. See Examples 9 and 10.

83. $x=\sqrt{7},\ x=-\sqrt{7}$

84. $x=\sqrt{6},\ x=-\sqrt{6}$

85. $x=1+\sqrt{3},\ x=1-\sqrt{3}$

86. $z=3+\sqrt{2},\ z=3-\sqrt{2}$

87. $y=-2+2\sqrt{5},\ y=-2-2\sqrt{5}$

88. $x=1+2\sqrt{3},\ x=1-2\sqrt{3}$

89. $x=4i,\ x=-4i$

90. $x=7i,\ x=-7i$

91. $y=i\sqrt{6},\ y=-i\sqrt{6}$

92. $y=i\sqrt{5},\ y=-i\sqrt{5}$

93. $x=2+i,\ x=2-i$

94. $x=-3+2i,\ x=-3-2i$

95. $x=1+i\sqrt{2},\ x=1-i\sqrt{2}$

96. $x=2+i\sqrt{3},\ x=2-i\sqrt{3}$

97. $x=-5+2i\sqrt{6},\ x=-5-2i\sqrt{6}$

98. $y=4+3i\sqrt{2},\ y=4-3i\sqrt{2}$

Applications

Solve.

99. A ball is dropped from the top of a building that is known to be 144 feet high. The formula for finding the height of the ball at any time is $h=144-16t^2$ where t is measured in seconds. How many seconds will it take for the ball to hit the ground?

100. A ball is dropped from the top of a building that is 784 feet high. The height of the ball above ground level is given by the polynomial function $h(t) = -16t^2 + 784$ where t is measured in seconds.

a. How high is the ball after 3 seconds? 5 seconds?

b. How far has the ball traveled in 3 seconds? 5 seconds?

c. When will the ball hit the ground? Explain your reasoning in terms of factors.

101. A tennis ball is dropped from a building. The position of the ball after t seconds is given by the polynomial function $s(t) = -4.9t^2 + 490$, where s is the height in meters of the ball.

a. Find $s(0)$ What does this value represent in the context of this problem?

b. How high is the tennis ball 2 seconds after it has been dropped?

c. How long before the tennis ball hits the ground?

102. A financial consultant is asked for advice about finances and savings plans. When a client invests money, they need to know which interest rate will meet their financial goals based on the amount invested. The financial consultant can use the formula $A = P(r+1)^n$ to find the future amount A, after n years, of an investment with a starting principal P invested at an interest rate of r.

a. A client has \$3000 to invest and would like to earn \$300 on his investment after 2 years. At what interest rate will the client need to invest his money? Round to the nearest hundredth of a percent.

b. Another client has \$5000 to invest and would like to earn \$750 on her investment after 2 years. At what interest rate will the client need to invest her money? Round to the nearest hundredth of a percent.

103. A local frame shop determines that the revenue function for their custom framing service is $R(p) = 360p - 4p^2$, where p is the base price in dollars for each custom framing job.

a. Set the function equal to 0 and solve for p using the method of completing the square.

b. What do the solutions from part a. mean?

104. The height of a golf ball that is hit from the ground at a speed of 128 feet per second can be modeled with the expression $h(t) = -16t^2 + 128t$, where t is the time in seconds after the ball is hit.

a. Set the function equal to 0 and solve for t using the method of completing the square.

b. What do the solutions from part a. mean?.

Writing & Thinking

105. Explain, in your own words, the steps involved in the process of solving a quadratic equation by completing the square..