$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -2 & -1 & -2 \\ -6 & -3 & -7 \end{bmatrix} \begin{bmatrix} -17 \\ 2 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -2 & -1 & -2 \\ -6 & -3 & -7 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ -5 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$$

TECHNOLOGY: Inverting Matrices

We can also use a graphing utility to find the inverse of a matrix. When using a TI-84 Plus, first define the matrix whose inverse we want to find. Then, enter the matrix on the home screen, press x^{-1}, and press enter.

To show the answer in fraction form, press math and select ▸Frac. If we defined matrix A to be $\begin{bmatrix} 7 & 4 \\ 1 & 2 \end{bmatrix}$, we would find its inverse as shown in Figure 1.

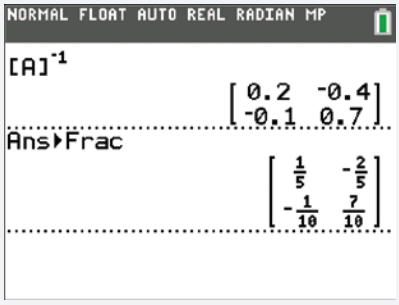

FIGURE 1

9.5 EXERCISES

PRACTICE

Write each of the following systems of equations as a single matrix equation. See Example 1.

1. $\begin{cases} 14x - 5y = 7 \\ x + 9y = 2 \end{cases}$

2. $\begin{cases} x - 5 = 9y \\ 3y - 2x = 8 \end{cases}$

3. $\begin{cases} -6 - 2y = x \\ 9x + 14 = 3y \end{cases}$

4. $\begin{cases} x - y = 5 \\ 2 - z = x \\ z - 3y = 4 \end{cases}$

5. $\begin{cases} 3x_1 - 7x_2 + x_3 = -4 \\ x_1 - x_2 = 2 \\ 8x_2 + 5x_3 = -3 \end{cases}$

6. $\begin{cases} x_3 = x_2 \\ x_2 = x_1 \\ x_1 = x_3 \end{cases}$

7. $\begin{cases} \dfrac{3x - 8y}{5} = 2 \\ y - 2 = 0 \end{cases}$

8. $\begin{cases} x - 7y = 5 \\ \dfrac{6 + x}{2} = 3y - 2 \end{cases}$

9. $\begin{cases} 4x = 3y - 9 \\ 13 - 2x = -4y \end{cases}$

10. $\begin{cases} -\frac{7}{3}y = \frac{5-x}{6} \\ x-5(y-3) = -2 \end{cases}$ **11.** $\begin{cases} 2x-y=-3z \\ y-x=17 \\ 2+z+4x=5y \end{cases}$ **12.** $\begin{cases} 2x_1-3x_3=7 \\ x_2-10x_3=0 \\ 2x_1-x_2+x_3=1 \end{cases}$

Find the inverse of each of the following matrices, if possible. See Examples 2 and 3.

13. $\begin{bmatrix} 0 & 4 \\ -5 & -1 \end{bmatrix}$ **14.** $\begin{bmatrix} -2 & -2 \\ -1 & 2 \end{bmatrix}$ **15.** $\begin{bmatrix} 3 & 4 \\ -4 & -5 \end{bmatrix}$

16. $\begin{bmatrix} -1 & -1 \\ -\frac{1}{4} & -\frac{1}{2} \end{bmatrix}$ **17.** $\begin{bmatrix} -\frac{1}{5} & 0 \\ \frac{1}{5} & \frac{1}{2} \end{bmatrix}$ **18.** $\begin{bmatrix} -7 & 2 \\ 7 & -2 \end{bmatrix}$

19. $\begin{bmatrix} -2 & -4 & -2 \\ 1 & -4 & 1 \\ 4 & -3 & 4 \end{bmatrix}$ **20.** $\begin{bmatrix} -3 & 0 & -4 \\ 2 & 5 & 4 \\ 1 & -5 & -2 \end{bmatrix}$ **21.** $\begin{bmatrix} -\frac{5}{11} & -\frac{8}{11} & 1 \\ \frac{13}{11} & \frac{12}{11} & -2 \\ -\frac{2}{11} & -\frac{1}{11} & 0 \end{bmatrix}$

22. $-\frac{1}{31}\begin{bmatrix} 17 & -8 & -2 \\ 1 & 5 & 9 \\ -6 & 1 & 8 \end{bmatrix}$ **23.** $\begin{bmatrix} -1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 4 & -1 \end{bmatrix}$ **24.** $\begin{bmatrix} -1 & 0 & -1 \\ -\frac{3}{2} & \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \end{bmatrix}$

25. $\begin{bmatrix} -\frac{6}{5} & -\frac{2}{5} & -1 \\ \frac{3}{5} & \frac{1}{5} & 1 \\ 1 & 0 & 1 \end{bmatrix}$ **26.** $\begin{bmatrix} 2 & -2 & 1 \\ -2 & 2 & -3 \\ 1 & 0 & 2 \end{bmatrix}$ **27.** $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

28. $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ **29.** $\begin{bmatrix} \frac{2}{3} & \frac{8}{9} & \frac{1}{9} \\ -\frac{1}{3} & \frac{2}{9} & -\frac{2}{9} \\ -\frac{1}{3} & -\frac{7}{9} & -\frac{2}{9} \end{bmatrix}$ **30.** $\begin{bmatrix} -3 & -3 & -4 \\ 0 & \frac{1}{4} & \frac{1}{2} \\ 2 & 2 & 3 \end{bmatrix}$

For each pair of matrices, determine if either matrix is the inverse of the other.

31. $\begin{bmatrix} -5 & -2 \\ -7 & 4 \end{bmatrix}, \begin{bmatrix} 10 & 4 \\ 14 & -8 \end{bmatrix}$ **32.** $\begin{bmatrix} 9 & -18 \\ 3 & 12 \end{bmatrix}, \begin{bmatrix} -3 & -6 \\ -1 & -4 \end{bmatrix}$

33. $\begin{bmatrix} -6 & -1 & 1 \\ 4 & -1 & -2 \\ 1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -2 & 3 \\ 2 & 5 & -8 \\ -3 & -7 & 10 \end{bmatrix}$ **34.** $\begin{bmatrix} -1 & 4 & 5 \\ 3 & -11 & -17 \\ 4 & -17 & -19 \end{bmatrix}, \begin{bmatrix} -80 & -9 & -13 \\ -11 & -1 & -2 \\ -7 & -1 & -1 \end{bmatrix}$

35. $\begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 2 \\ 1 & 1 & -3 \end{bmatrix}, \begin{bmatrix} 4 & 0 & -2 \\ 6 & 8 & 4 \\ 2 & 2 & -6 \end{bmatrix}$ **36.** $\begin{bmatrix} -7 & 0 & -2 \\ -10 & -1 & -2 \\ -7 & -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 2 & -2 \\ 4 & -7 & 6 \\ 3 & -7 & 7 \end{bmatrix}$

Solve the following systems by the inverse matrix method, if possible. If the inverse matrix method doesn't apply, use any other method to determine if the system is inconsistent or dependent. See Example 4.

37. $\begin{cases} -2x-2y=9 \\ -x+2y=-3 \end{cases}$ **38.** $\begin{cases} 3x+4y=-2 \\ -4x-5y=9 \end{cases}$ **39.** $\begin{cases} -2x+3y=1 \\ 4x-6y=-2 \end{cases}$

40. $\begin{cases} -2x+4y=5 \\ x-4y=-3 \end{cases}$ **41.** $\begin{cases} -5x=10 \\ 2x+2y=-4 \end{cases}$ **42.** $\begin{cases} -3x+y=2 \\ 9x-3y=5 \end{cases}$

43. $\begin{cases} 8x+2y=26 \\ -16x-2y=-90 \end{cases}$ **44.** $\begin{cases} 3x-7y=-2 \\ -6x+14y=4 \end{cases}$ **45.** $\begin{cases} 3y=15 \\ 8x+4y=20 \end{cases}$

46. $\begin{cases} 4y+3z=-254 \\ 2x-2y-z=100 \\ -x+y-2z=155 \end{cases}$ **47.** $\begin{cases} 2x-y-3z=-10 \\ 2y-z=11 \\ -x+4z=0 \end{cases}$ **48.** $\begin{cases} 3y-4z=15 \\ x+2y-3z=9 \\ -x-y+2z=-5 \end{cases}$

Solve the following sets of systems by the inverse matrix method. See Example 5.

49. $\begin{cases} x+2y-z=2 \\ 3x+3y-z=-5 \\ 4x+4y-z=1 \end{cases}$ $\begin{cases} x+2y-z=1 \\ 3x+3y-z=1 \\ 4x+4y-z=1 \end{cases}$ $\begin{cases} x+2y-z=0 \\ 3x+3y-z=1 \\ 4x+4y-z=1 \end{cases}$

50. $\begin{cases} -x-y-2z=4 \\ x+3y+3z=0 \\ -3y-2z=9 \end{cases}$ $\begin{cases} -x-y-2z=1 \\ x+3y+3z=0 \\ -3y-2z=0 \end{cases}$ $\begin{cases} -x-y-2z=-2 \\ x+3y+3z=-3 \\ -3y-2z=1 \end{cases}$

51. $\begin{cases} -x+z=6 \\ -x+3y+2z=-11 \\ 2x-4y-3z=13 \end{cases}$ $\begin{cases} -x+z=-2 \\ -x+3y+2z=2 \\ 2x-4y-3z=-1 \end{cases}$ $\begin{cases} -x+z=-4 \\ -x+3y+2z=2 \\ 2x-4y-3z=0 \end{cases}$

TECHNOLOGY

Using a graphing utility, find the inverse of each of the following matrices, if possible. Round your answers to three decimal places if necessary.

52. $\begin{bmatrix} -7 & 3 \\ -1 & 2 \end{bmatrix}$ **53.** $\begin{bmatrix} -6 & 2 \\ -5 & 5 \end{bmatrix}$ **54.** $\begin{bmatrix} -2 & 0 & 2 \\ 2 & -3 & 1 \\ 1 & -2 & 3 \end{bmatrix}$

55. $\begin{bmatrix} 2.3 & 7.8 \\ -3.4 & 1.6 \end{bmatrix}$ **56.** $\begin{bmatrix} 4.5 & -9.4 & 6.9 \\ 8.6 & -2.8 & 1.2 \\ 3.1 & 0.3 & -7.0 \end{bmatrix}$ **57.** $\begin{bmatrix} 38 & -44 & 72 \\ -93 & 16 & 29 \\ 65 & 23 & -19 \end{bmatrix}$