Note that we have used the first property of determinants to factor x_1 out of the determinant. We can now solve this equation for x_1 (since $D \neq 0$) to obtain

$$x_1 = \frac{D_{x_1}}{D},$$

and we can then repeat the process for the remaining variables $x_2, x_3, \ldots, x_n$.

9.3 EXERCISES

PRACTICE

Evaluate each of the following determinants. See Example 1.

1. $\begin{vmatrix} 4 & -3 \\ 1 & 2 \end{vmatrix}$ **2.** $\begin{vmatrix} 5 & -2 \\ 5 & -2 \end{vmatrix}$ **3.** $\begin{vmatrix} 0 & 3 \\ -5 & 2 \end{vmatrix}$ **4.** $\begin{vmatrix} 34 & -2 \\ 17 & -1 \end{vmatrix}$

5. $\begin{vmatrix} a & x \\ x & b \end{vmatrix}$ **6.** $\begin{vmatrix} 5x & 2 \\ -x & 1 \end{vmatrix}$ **7.** $\begin{vmatrix} -2 & 2 \\ -2 & -2 \end{vmatrix}$ **8.** $\begin{vmatrix} ac & 2ad \\ bc & db \end{vmatrix}$

9. $\begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}$ **10.** $\begin{vmatrix} w & x \\ y & z \end{vmatrix}$ **11.** $\begin{vmatrix} -2 & 9 \\ 5 & -3 \end{vmatrix}$ **12.** $\begin{vmatrix} 2y & 3x \\ y-1 & x^2 \end{vmatrix}$

Solve for x by calculating the determinant.

13. $\begin{vmatrix} x-2 & 2 \\ 2 & x+1 \end{vmatrix} = 0$ **14.** $\begin{vmatrix} x+7 & -2 \\ 9 & x-2 \end{vmatrix} = 0$ **15.** $\begin{vmatrix} x+1 & 8 \\ 1 & x+3 \end{vmatrix} = 0$

16. $\begin{vmatrix} x-8 & 11 \\ -2 & x+5 \end{vmatrix} = 0$ **17.** $\begin{vmatrix} x+6 & 2 \\ -1 & x+3 \end{vmatrix} = 0$ **18.** $\begin{vmatrix} x-4 & -4 \\ 3 & x+9 \end{vmatrix} = 0$

19. $\begin{vmatrix} x+5 & 3 \\ 3 & x-3 \end{vmatrix} = 0$ **20.** $\begin{vmatrix} x+3 & 6 \\ 5 & x+7 \end{vmatrix} = 0$ **21.** $\begin{vmatrix} x-3 & 2 \\ 1 & x-4 \end{vmatrix} = 0$

Use the matrix $A = \begin{bmatrix} 2 & -1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & -2 \end{bmatrix}$ to evaluate the following. See Example 2.

22. The minor of a_{12}

23. The cofactor of a_{12}

24. The minor of a_{22}

25. The cofactor of a_{22}

26. The cofactor of a_{32}

27. The cofactor of a_{33}

28. The minor of a_{13}

29. The cofactor of a_{21}

30. The cofactor of a_{31}

Find the determinant by the method of expansion by cofactors along the given row or column. See Example 3.

31. $\begin{vmatrix} 4 & 5 & 3 \\ -1 & 2 & 7 \\ 11 & 6 & 2 \end{vmatrix}$ Expand along Row 3

32. $\begin{vmatrix} 8 & 2 & 0 \\ 3 & 4 & 7 \\ 1 & 0 & 2 \end{vmatrix}$ Expand along Column 3

33. $\begin{vmatrix} 5 & 8 & 5 \\ 0 & -6 & 3 \\ 2 & 4 & -1 \end{vmatrix}$ Expand along Row 2

34. $\begin{vmatrix} -4 & 2 & 1 \\ 9 & 12 & 8 \\ 0 & 6 & -3 \end{vmatrix}$ Expand along Column 1

35. $\begin{vmatrix} 13 & 0 & -7 \\ 4 & 2 & 3 \\ 1 & 4 & 0 \end{vmatrix}$ Expand along Row 2

36. $\begin{vmatrix} 7 & 0 & 1 \\ 2 & 5 & 3 \\ 8 & 6 & 2 \end{vmatrix}$ Expand along Column 3

37. $\begin{vmatrix} 8 & 0 & -7 & 5 \\ 4 & -2 & 3 & 3 \\ -1 & 1 & 0 & 2 \\ 2 & 0 & 6 & 0 \end{vmatrix}$ Expand along Row 4

38. $\begin{vmatrix} 4 & -2 & 9 & 2 \\ 7 & 0 & 1 & 7 \\ -6 & 3 & 0 & 1 \\ 3 & 1 & 2 & 0 \end{vmatrix}$ Expand along Column 2

Evaluate each of the following determinants. In each case, minimize the required number of computations by carefully choosing a row or column to expand along, and use the properties of determinants to simplify the process. See Examples 3 and 4.

39. $\begin{vmatrix} 2 & 0 & 1 \\ -5 & 1 & 0 \\ 3 & -1 & 1 \end{vmatrix}$

40. $\begin{vmatrix} 12 & 3 & 1 \\ 1 & 1 & -1 \\ 0 & 2 & 0 \end{vmatrix}$

41. $\begin{vmatrix} 12 & 3 & 6 \\ 2 & 2 & -4 \\ 0 & 2 & 0 \end{vmatrix}$

42. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

43. $\begin{vmatrix} 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \end{vmatrix}$

44. $\begin{vmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & x \end{vmatrix}$

45. $\begin{vmatrix} x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x \end{vmatrix}$

46. $\begin{vmatrix} 0 & 2 & 0 & 0 \\ -2 & -4 & 5 & 9 \\ 1 & 3 & -1 & 1 \\ 0 & 7 & 0 & 2 \end{vmatrix}$

47. $\begin{vmatrix} x & x & 0 & 0 \\ yz & x^3 & z & x^4 \\ z & xy & x & 0 \\ x^2 & 0 & 0 & 0 \end{vmatrix}$

Use Cramer's Rule to solve each system of equations. See Examples 5 and 6.

48. $\begin{cases} 2x - 3y = 8 \\ 8x + 5y = -2 \end{cases}$

49. $\begin{cases} 5x + 7y = 9 \\ 2x + 3y = -7 \end{cases}$

50. $\begin{cases} 5x - 10y = 9 \\ -x + 2y = -3 \end{cases}$

51. $\begin{cases} -2x - 2y = 4 \\ 3x + 3y = -6 \end{cases}$

52. $\begin{cases} \frac{2}{3}x + y = -3 \\ 3x + \frac{5}{2}y = -\frac{7}{2} \end{cases}$

53. $\begin{cases} \frac{2}{3}x + 2y = 1 \\ x + 3y = 0 \end{cases}$

54. $\begin{cases} x+2y=-1 \\ y+3z=7 \\ 2x+5z=21 \end{cases}$

55. $\begin{cases} 2x-y=0 \\ 5x-3y-3z=5 \\ 2x+6z=-10 \end{cases}$

56. $\begin{cases} 3x+8z=3 \\ -3x-7z=-3 \\ x+3z=1 \end{cases}$

57. $\begin{cases} 3w-x+5y+3z=2 \\ -4w-10y-2z=10 \\ w-x+2z=7 \\ 4w-2x+5y+5z=9 \end{cases}$

58. $\begin{cases} 2w+x-3y=3 \\ w-2x+y=1 \\ x+z=-2 \\ y+z=0 \end{cases}$

59. $\begin{cases} 3w-2x+y-5z=-1 \\ w+x-y+4z=2 \\ 4w-x-z=1 \\ 5w-x=9 \end{cases}$

60. $\begin{cases} -4x+y=1 \\ 7x+2y=407 \end{cases}$

61. $\begin{cases} 5x-4y=-49 \\ 24x-19y=179 \end{cases}$

62. $\begin{cases} 2w-3x+4y-z=21 \\ w+5x=2 \\ -2x+3y+z=12 \\ -3w+4z=-5 \end{cases}$

63. $\begin{cases} -5x+10y=3 \\ \frac{7}{2}x-7y=20 \end{cases}$

64. $\begin{cases} 23x+21y=-4 \\ x-3y=-8 \end{cases}$

65. $\begin{cases} w-x+y-z=2 \\ 2w-x+3y=-5 \\ x-2z=7 \\ 3w+4x=-13 \end{cases}$

APPLICATIONS

66. The three sides of a triangle are related as follows: the perimeter is 43 feet, the second side is 5 feet more than twice the first side, and the third side is 3 feet less than the sum of the other two sides. Find the lengths of the three sides of the triangle.

67. Eric's favorite candy bar and ice cream flavor have fat and calorie contents as follows: each candy bar has 5 grams of fat and 280 calories; each serving of ice cream has 10 grams of fat and 150 calories. How many candy bars and servings of ice cream did he eat during the weekend he consumed 85 grams of fat and 2300 calories from these two treats?

68. A farmer plants soybeans, corn, and wheat and rotates the planting each year on her 500-acre farm. In a particular year, the profits from her crops were \$120 per acre of soybeans, \$100 per acre of corn, and \$80 per acre of wheat. She planted twice as many acres of corn as soybeans. How many acres did she plant with each crop that year if she made a total profit of \$51,800?

TECHNOLOGY

Using a graphing utility find the determinant of the matrix.

69. $\begin{vmatrix} 0.1 & 0.4 & -0.7 \\ 0.3 & -0.1 & 0.2 \\ 0.5 & -0.2 & 0.3 \end{vmatrix}$

70. $\begin{vmatrix} 0.1 & 0.3 & 0.1 \\ 0.2 & -0.2 & -0.1 \\ -0.1 & -0.4 & 0.5 \end{vmatrix}$

71. $\begin{vmatrix} 2.2 & 0.3 & -1.7 \\ 0.4 & -0.2 & 0.1 \\ 0.2 & 0.3 & -1.6 \end{vmatrix}$

72. $\begin{vmatrix} 3.1 & 0.6 & -1.1 \\ 1.2 & 5.2 & -7.3 \\ -0.1 & -4.1 & 6.5 \end{vmatrix}$

73. $\begin{vmatrix} 13 & 23 & -21 \\ 17 & -32 & 14 \\ 15 & 12 & -16 \end{vmatrix}$

74. $\begin{vmatrix} 25 & 32 & 17 \\ -13 & 14 & -24 \\ 16 & 26 & 36 \end{vmatrix}$

Use a graphing utility and Cramer's Rule to solve each system of equations.

75. $\begin{cases} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{cases}$

76. $\begin{cases} 2x+4y+z=1 \\ x-2y-3z=2 \\ x+y-z=-1 \end{cases}$

77. $\begin{cases} w+x+y+z=6 \\ -w+2x+3y=0 \\ 2w-3x+4y+z=4 \\ w+x+2y-z=0 \end{cases}$