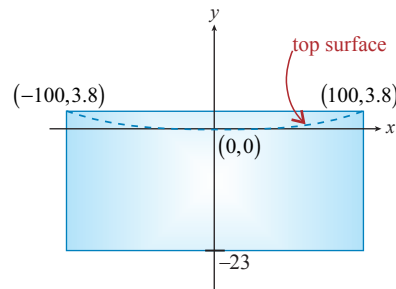


NOTE

The same concept can be used to focus sunlight, intensely heating a small area at the focus. This is called a parabolic furnace.

Solution

First, we need to draw a picture of the situation. In order to make the math as easy as possible, we can locate the origin of our coordinate system at the vertex of a parabolic cross section of the mirror, and we can assume the parabola opens upward.

**FIGURE 9**

Since we placed the vertex at $(0,0)$, we know the equation $x^2 = 4py$ describes the shape of the cross section for some value p . If we can determine p , we can find the focus of the parabola.

To find p , we need the coordinates of another point on the parabola. The difference in thickness of the mirror between the center and the outer rim is 3.8 inches, and the mirror has a diameter of 200 inches, so the two points $(-100, 3.8)$ and $(100, 3.8)$ must lie on the graph. Plugging a point into the equation $x^2 = 4py$, we can solve for p .

$$\begin{aligned}(100)^2 &= 4p(3.8) \\ 10000 &= 15.2p \\ p &\approx 657.9 \text{ inches} \\ p &\approx 54.8 \text{ feet}\end{aligned}$$

We know that the focus of a parabola is p units from the vertex, so the focus of the Hale Telescope is nearly 55 feet from the mirror.

8.2 EXERCISES**PRACTICE**

Graph the following parabolas and determine the focus and directrix of each. See Examples 1 and 2.

1. $(x+1)^2 = 4(y-3)$

2. $y^2 - 4y = 8x + 4$

3. $(y-4)^2 = -2(x-1)$

4. $(y-1)^2 = 8(x+3)$

5. $(x-2)^2 = 4(y+1)$

6. $(y+1)^2 = -12(x+1)$

7. $y^2 = 6x$

8. $x^2 = 2y$

9. $x^2 = 7y$

10. $x^2 = -5y$

11. $y = -12x^2$

12. $x = -4y^2$

13. $x = \frac{1}{6}y^2$

14. $\frac{1}{5}x = -y^2$

15. $y^2 + 16x = 0$

16. $-6x - 2y^2 = 0$

17. $4y + 2x^2 = 4$

18. $2y^2 - 10x = 10$

19. $y^2 + 2y + 12x + 37 = 0$

20. $x^2 - 8y = 6x - 1$

21. $x^2 + 6x + 8y = -17$

22. $x^2 + 2x + 8y = 31$

23. $y^2 + 6y - 2x + 13 = 0$

24. $y^2 - 2y - 4x + 13 = 0$

Match the following equations to their graphs.

25. $(x+2)^2 = 3(y-1)$

26. $(y-1)^2 = 2(x+2)$

27. $y^2 = 4(x+1)$

28. $x^2 = 2(y+1)$

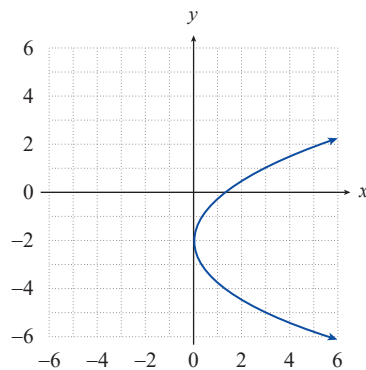
29. $(x-1)^2 = -(y-2)$

30. $(y+2)^2 = 3x$

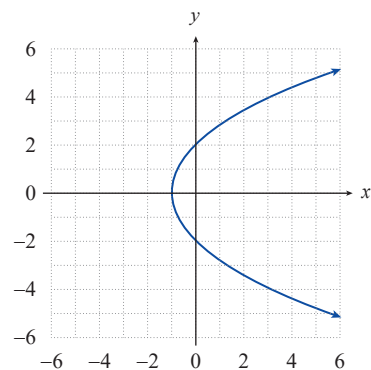
31. $(x-2)^2 = 4y$

32. $y^2 = -2(x+1)$

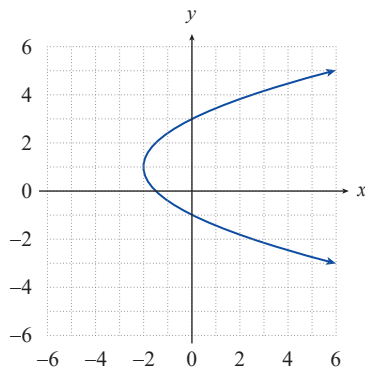
a.



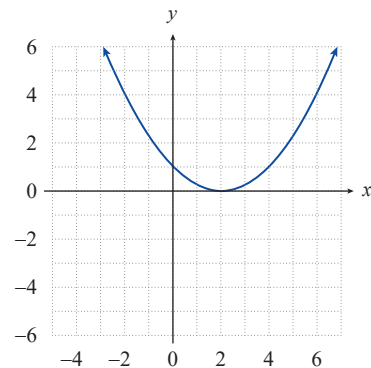
b.

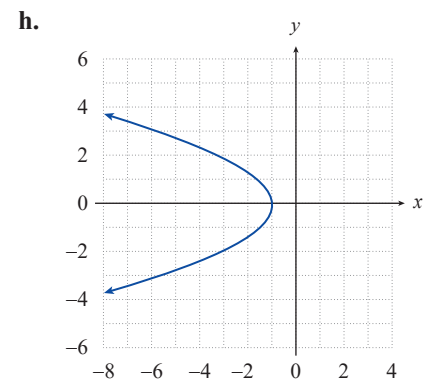
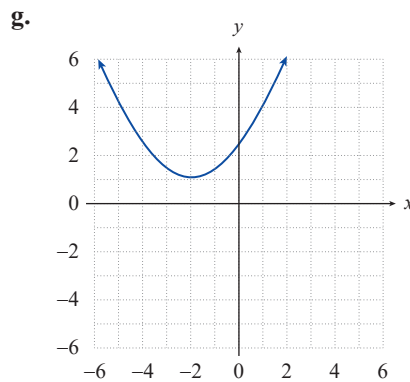
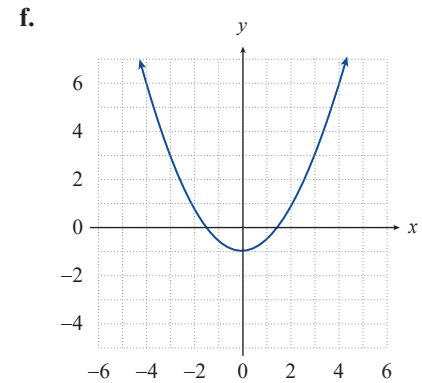
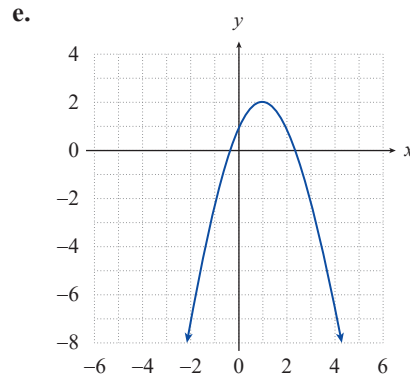


c.



d.

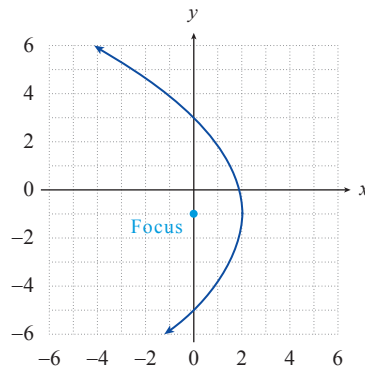




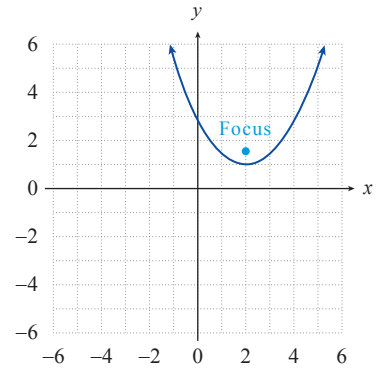
Find the equation, in standard form, for the parabola with the given properties or with the given graph. See Example 3.

33. Focus at $(-2, 1)$, directrix is the y -axis.
34. Focus at $(-2, 1)$, directrix is the x -axis.
35. Vertex at $(3, -1)$, focus at $(3, 1)$.
36. Symmetric with respect to the line $y = 1$, directrix is the line $x = 2$, and $p = -3$.
37. Vertex at $(3, -2)$, directrix is the line $x = -3$.
38. Vertex at $(7, 8)$, directrix is the line $x = \frac{27}{4}$.
39. Focus at $\left(-3, -\frac{3}{2}\right)$, directrix is the line $y = -\frac{1}{2}$.
40. Vertex at $(3, 16)$, focus at $(3, 11)$.
41. Vertex at $(-4, 3)$, focus at $\left(-\frac{3}{2}, 3\right)$.
42. Symmetric with respect to the x -axis, focus at $(-3, 0)$, and $p = 2$.

43.



44.



APPLICATIONS

45. One design for a solar furnace is based on the paraboloid formed by rotating the parabola $x^2 = 8y$ around its axis of symmetry. The object to be heated in the furnace is then placed at the focus of the paraboloid (assume that x and y are in units of feet). How far from the vertex of the paraboloid is the hottest part of the furnace?
46. A certain brand of satellite dish antenna is a paraboloid with a diameter of 6 feet and a depth of 1 foot. How far from the vertex of the dish should the receiver of the antenna be placed given that the receiver should be located at the focus of the paraboloid?
47. A spotlight is made by placing a strong lightbulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 6y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

TECHNOLOGY

Use a graphing utility to graph the following equations.

48. $3x^2 - 4y + 24x = -56$

49. $y^2 + 2y = 8x - 41$

50. $y^2 - 6y + 4x = -17$

51. $x^2 - 6x + 12y + 21 = 0$