Substituting back into our first equation, we have $c \approx 94.56 - 92.98$, so $c \approx 1.58$.

The closest approach to the sun must be a distance of $a - c$, which is approximately $92.98 - 1.58 = 91.40$ million miles.

8.1 EXERCISES

PRACTICE

Find the center, foci, and vertices of the ellipse that each equation describes. See Examples 1, 2 and 3.

1. $\dfrac{(x-5)^2}{4} + \dfrac{(y-2)^2}{25} = 1$

2. $\dfrac{(x+3)^2}{9} + \dfrac{(y+1)^2}{16} = 1$

3. $(x+2)^2 + 3(y+5)^2 = 9$

4. $4(x-4)^2 + (y-2)^2 = 8$

5. $x^2 + 6x + 2y^2 - 8y + 13 = 0$

6. $2x^2 + y^2 - 4x + 4y - 10 = 0$

7. $4x^2 + y^2 + 40x - 2y + 85 = 0$

8. $x^2 + 2y^2 - 6x + 16y + 37 = 0$

9. $x^2 + 3y^2 + 8x - 12y + 1 = 0$

10. $4x^2 + 3y^2 - 8x + 18y + 19 = 0$

11. $x^2 - 4x + 5y^2 - 1 = 0$

12. $x^2 + 4y^2 + 24y + 28 = 0$

Match the following equations to their graphs.

13. $\dfrac{(x-1)^2}{4} + \dfrac{y^2}{81} = 1$

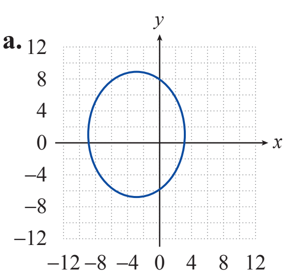
14. $\dfrac{(x-3)^2}{49} + \dfrac{(y-2)^2}{25} = 1$

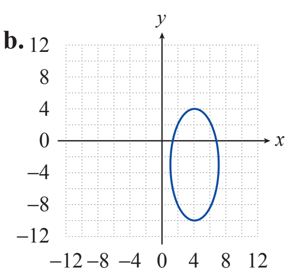
15. $\dfrac{(x+4)^2}{9} + \dfrac{(y-3)^2}{4} = 1$

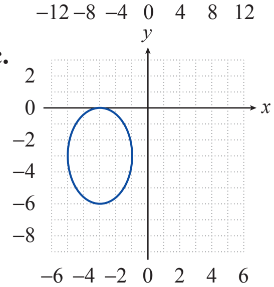
16. $\dfrac{(x+3)^2}{36} + \dfrac{(y-1)^2}{64} = 1$

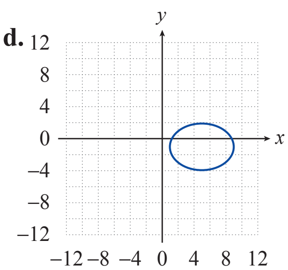
17. $\dfrac{(x+3)^2}{4} + \dfrac{(y+3)^2}{9} = 1$

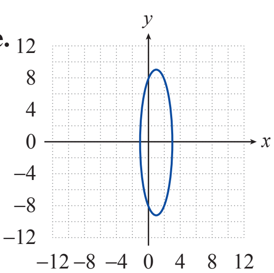
18. $\dfrac{x^2}{16} + \dfrac{(y+5)^2}{4} = 1$

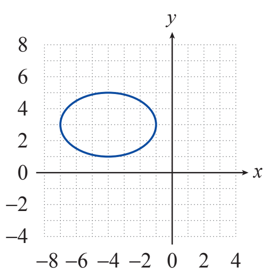
a.


b.


c.


d.


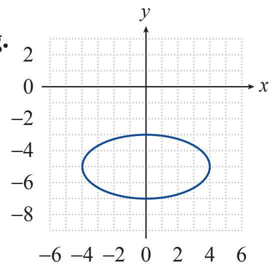
e.


f.


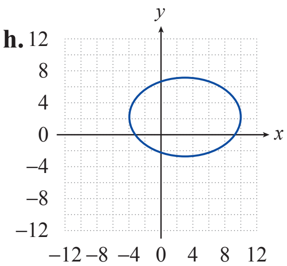
19. $\dfrac{(x-4)^2}{9}+\dfrac{(y+3)^2}{49}=1$

20. $\dfrac{(x-5)^2}{16}+\dfrac{(y+1)^2}{9}=1$

g.



h.



Sketch the graphs of the following ellipses and determine the coordinates of the foci. See Examples 1, 2, and 3.

21. $\dfrac{(x-3)^2}{9}+\dfrac{(y+1)^2}{1}=1$

22. $\dfrac{(x+5)^2}{4}+\dfrac{(y+2)^2}{16}=1$

23. $\dfrac{(x-3)^2}{9}+\dfrac{(y-4)^2}{4}=1$

24. $\dfrac{x^2}{25}+\dfrac{(y-3)^2}{16}=1$

25. $(x-1)^2+\dfrac{(y-4)^2}{4}=1$

26. $\dfrac{(x-4)^2}{16}+\dfrac{(y-4)^2}{4}=1$

27. $\dfrac{(x+1)^2}{25}+\dfrac{(y+5)^2}{4}=1$

28. $\dfrac{(x-2)^2}{9}+\dfrac{(y+1)^2}{9}=1$

29. $\dfrac{(x+2)^2}{16}+\dfrac{(y+1)^2}{9}=1$

30. $\dfrac{x^2}{25}+(y+2)^2=1$

31. $9x^2+16y^2+18x-64y=71$

32. $9x^2+4y^2-36x-24y+36=0$

33. $16x^2+y^2+160x-6y=-393$

34. $25x^2+4y^2-100x+8y+4=0$

35. $4x^2+9y^2+40x+90y+289=0$

36. $16x^2+y^2-64x+6y+57=0$

37. $4x^2+y^2+4y=0$

38. $9x^2+4y^2+108x-32y=-352$

In each of the following exercises, an ellipse is described by either a picture or by the properties it possesses. Find the equation, in standard form, for each ellipse. See Example 4.

39. Center at the origin, major axis of length 10 on the y-axis, foci 3 units from the center.

40. Center at $(-2,3)$, major axis of length 8 oriented horizontally, minor axis of length 4.

41. Vertices at $(1,4)$ and $(1,-2)$, foci $2\sqrt{2}$ units from the center.

42. Vertices at $(5,-1)$ and $(1,-1)$, minor axis of length 2.

43. Foci at $(0,0)$ and $(6,0)$, $e=\dfrac{1}{2}$.

44. Vertices at $(-1,4)$ and $(-1,0)$, $e=0$.

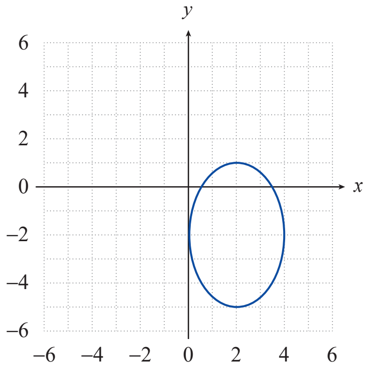
45. Vertices at $(-2,-1)$ and $(-2,-5)$, minor axis of length 2.

46. Vertices at $(-4,6)$ and $(-14,6)$, $e=\frac{2}{5}$.

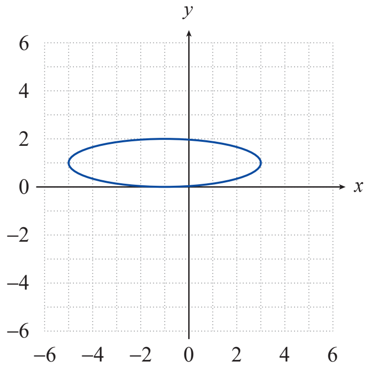
47. Vertices at $(1,3)$ and $(9,3)$, one of the foci at $(6,3)$.

48. Foci at $(2,-4)$ and $(2,-8)$, minor axis of length 6.

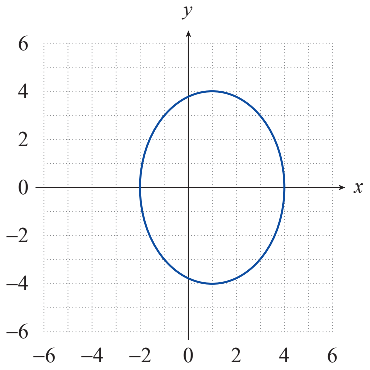
49.



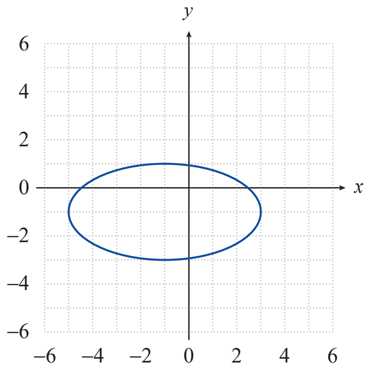
50.



51.



52.



Find the eccentricity and the lengths of the minor and major axes of the following ellipses.

53. $\frac{x^2}{100}+\frac{y^2}{144}=1$

54. $\frac{x^2}{64}+\frac{y^2}{9}=1$

55. $x^2+9y^2=36$

56. $25x^2+4y^2=100$

57. $4x^2+16y^2=16$

58. $5x^2+8y^2=40$

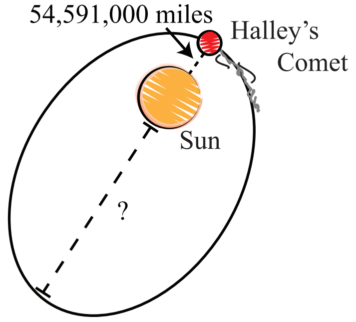
59. $20x^2+10y^2=40$

60. $\frac{1}{4}x^2+\frac{1}{12}y^2=\frac{1}{2}$

61. $x^2=49-7y^2$

APPLICATIONS

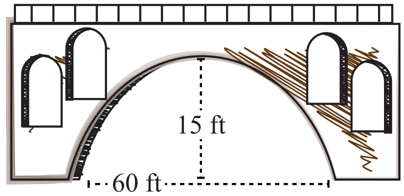
62. The orbit of Halley's Comet is an ellipse with an eccentricity of 0.967. Its closest approach to the sun is approximately 54,591,000 miles. What is the farthest Halley's Comet ever gets from the sun?



63. Pluto's closest approach to the sun is approximately 4.43×10^9 kilometers, and its maximum distance from the sun is approximately 7.37×10^9 kilometers. What is the eccentricity of Pluto's orbit?

64. Use the information given in Example 5 to determine the length of the minor axis of the ellipse formed by Earth's orbit around the sun.

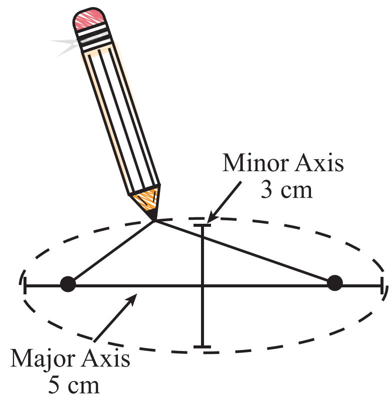
65. The archway supporting a bridge over a river is in the shape of half an ellipse. The archway is 60 feet wide and is 15 feet tall at the middle. A boat is 10 feet wide and 14 feet, 9 inches tall. Is the boat capable of passing under the archway?



66. *The Whispering Gallery* in Chicago's Museum of Science and Industry is a giant ellipsoid that transmits the slightest whisper from one focus to the other focus. This giant ellipse is known to have a length of about 568 inches and a width of about 162 inches. Find the eccentricity of *The Whispering Gallery*. About how far apart are two whisperers when communicating in this gallery? Round your answers to four decimal places.

WRITING & THINKING

67. Since the sum of the distances from each of the two foci to any point on an ellipse is constant, we can draw an ellipse using the following method. Tack the ends of a length of string at two points (the foci) and, keeping the string taut by pulling outward with the tip of a pencil, trace around the foci to form an ellipse (the total length of the string remains constant). If you want to create an ellipse with a major axis of length 5 cm and a minor axis of length 3 cm, how long should your string be and how far apart should you place the tacks? Use the relationships of distances and formulas that you have learned in this section.



68. Using the method described in Exercise 67, describe the change in your ellipse when you move the two foci closer together. What happens when you move them farther apart?

TECHNOLOGY

Use a graphing utility to graph the following equations.

69. $15x^2 + 9y^2 + 150x - 36y = -276$

70. $5x^2 + 12y^2 - 20x + 144y + 392 = 0$

71. $3x^2 + 2y^2 = 3 - 18x$

72. $2x^2 + 5y^2 = 70y - 205$