The final step is to evaluate which intervals satisfy each inequality. The solution to the first inequality is the union of the two intervals where *f* is positive.

$$(-\infty,-3)\cup(-2,\infty)$$

For the second inequality, we have to decide which endpoints to include. We include $x=-3,$ since this is a zero of the rational function, but we do not include $x=-2,$ since the value is not in the domain of *f*. Thus, the solution to the second inequality is

$$(-\infty,-3]\cup(-2,\infty).$$

6.5 EXERCISES

PRACTICE

Find equations for the vertical asymptotes, if any, for each of the following rational functions. See Example 1.

1. $f(x)=\dfrac{5}{x-1}$

2. $f(x)=\dfrac{x^2+3}{x+3}$

3. $f(x)=\dfrac{x^2-4}{x+2}$

4. $f(x)=\dfrac{-3x+5}{x-2}$

5. $f(x)=\dfrac{3x^2+1}{x-2}$

6. $f(x)=\dfrac{x^2+2x}{x+1}$

7. $f(x)=\dfrac{x^2-4}{2x-x^2}$

8. $f(x)=\dfrac{x+2}{x^2-9}$

9. $f(x)=\dfrac{x^2-2x-3}{2x^2-5x-3}$

10. $f(x)=\dfrac{2x^2+2x-4}{x^2+2x+1}$

11. $f(x)=\dfrac{x^3-27}{x^2+5}$

12. $f(x)=\dfrac{x^2+5}{x^3-27}$

13. $f(x)=\dfrac{x^2-1}{x^2-8x+7}$

14. $f(x)=\dfrac{2x^2+7x-14}{2x^2+7x-15}$

15. $f(x)=\dfrac{x^3-6x^2+11x-6}{x^3+8}$

16. $f(x)=\dfrac{x^2-2x-15}{x-5}$

17. $f(x)=\dfrac{x^2-16}{x^2-4}$

18. $f(x)=\dfrac{x^2+4x+4}{x^2+x-2}$

Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions. See Example 2.

19. $f(x)=\dfrac{5}{x-1}$

20. $f(x)=\dfrac{x^2+3}{x+3}$

21. $f(x)=\dfrac{x^4-4}{x^2+2}$

22. $f(x)=\dfrac{x^2-4}{2x-x^2}$

23. $f(x)=\dfrac{x+2}{x^2-9}$

24. $f(x)=\dfrac{x^2-2x-3}{2x^2-5x-3}$

25. $f(x)=\dfrac{2x^2+2x-4}{x^2+2x+1}$

26. $f(x)=\dfrac{-3x+5}{x-2}$

27. $f(x)=\dfrac{3x^2+1}{x-2}$

28. $f(x)=\dfrac{x^3-27}{x^2+5}$

29. $f(x)=\dfrac{x^2+5}{x^3-27}$

30. $f(x)=\dfrac{x^2+2x}{x+1}$

31. $f(x)=\dfrac{x^2-81}{x^3+7x-12}$

32. $f(x)=\dfrac{x^3-3x^2+2x}{x-7}$

33. $f(x)=\dfrac{x^2-9x+4}{x+2}$

34. $f(x)=\dfrac{-x^5+2x^2}{5x^5+3x^3-7}$

35. $f(x)=\dfrac{5x^2-x+12}{x-1}$

36. $f(x)=\dfrac{2x^2-5x+6}{x-3}$

Sketch the graphs of the following rational functions, making use of your work in the problems above and additional information about intercepts and any other points that may be useful. See Example 3.

37. $f(x)=\dfrac{5}{x-1}$

38. $f(x)=\dfrac{x^2+3}{x+3}$

39. $f(x)=\dfrac{x^2-4}{x+2}$

40. $f(x)=\dfrac{x^2-4}{2x-x^2}$

41. $f(x)=\dfrac{x+2}{x^2-9}$

42. $f(x)=\dfrac{x^2-2x-3}{2x^2-5x-3}$

43. $f(x)=\dfrac{2x^2+2x-4}{x^2+2x+1}$

44. $f(x)=\dfrac{-3x+5}{x-2}$

45. $f(x)=\dfrac{3x^2+1}{x-2}$

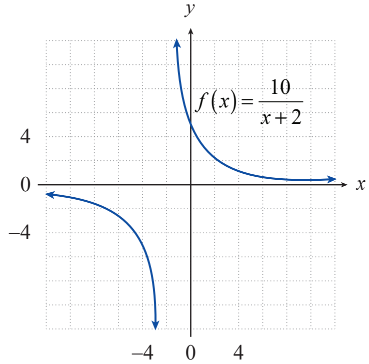
46. $f(x)=\dfrac{x^3-27}{x^2+5}$

47. $f(x)=\dfrac{x^2+5}{x^3-27}$

48. $f(x)=\dfrac{x^2+2x}{x+1}$

For each graph, find any **a.** vertical asymptotes, **b.** horizontal asymptotes, **c.** oblique asymptotes, **d.** visible *x*-intercepts, or **e.** visible *y*-intercepts.

49.



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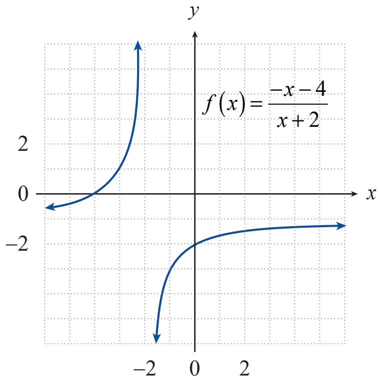

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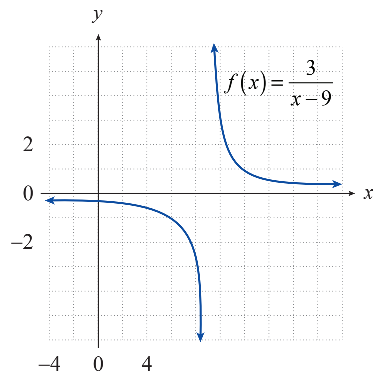

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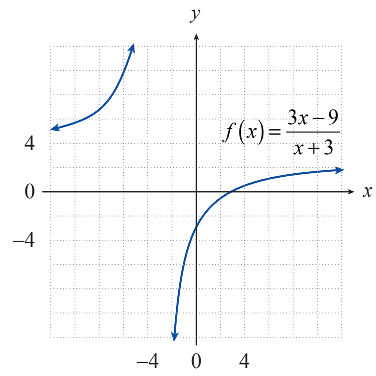

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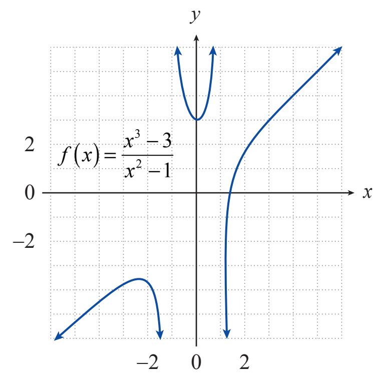

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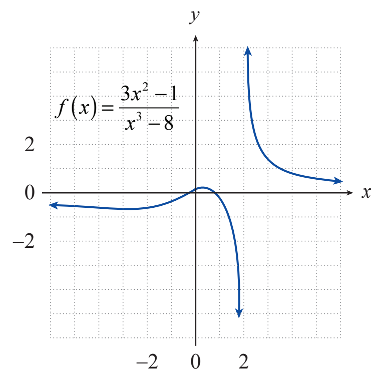

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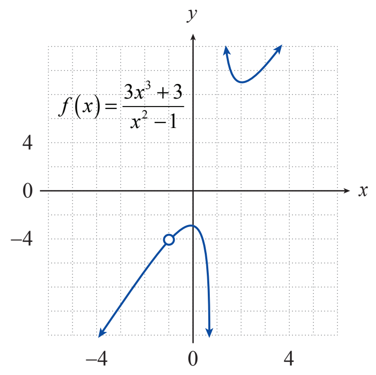
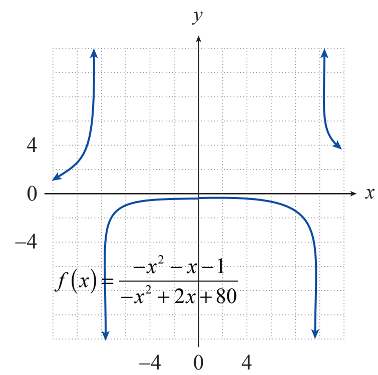


56.



Solve the following rational inequalities. See Examples 4 and 5.

57. $2x < \dfrac{4}{x+1}$

58. $\dfrac{5}{x-2} \geq \dfrac{3x}{x-2}$

59. $\dfrac{5}{x-2} > \dfrac{3}{x+2}$

60. $\dfrac{x}{x^2-x-6} \leq \dfrac{-1}{x^2-x-6}$

61. $\dfrac{x}{x^2-x-6} \leq \dfrac{-2}{x^2-x-6}$

62. $x > \dfrac{1}{x}$

63. $\dfrac{4}{x-3} \leq \dfrac{4}{x}$

64. $\dfrac{x-7}{x-3} \geq \dfrac{x}{x-1}$

65. $\dfrac{x}{x^2+3x+2} > \dfrac{1}{x^2+3x+2}$

66. $\dfrac{1}{x-4} \geq \dfrac{1}{x+1}$

67. $\dfrac{x}{x+1} \geq \dfrac{x+1}{x}$

68. $\dfrac{x}{x^2-2x-3} > \dfrac{3}{x^2-2x-3}$

APPLICATIONS

69. April raises a species of aquarium fish, and the total number of fish she has follows the formula

$$p(t) = \frac{200t}{t+1},$$

where $t \geq 0$ represents the number of months since she began.

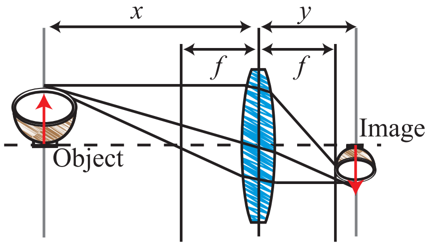
a. Sketch the graph of $p(t)$ for $t \geq 0$.

b. What happens to April's fish population in the long run?

70. If an object is placed a distance x from a lens with a focal length of f, the image of the object will appear a distance y on the opposite side of the lens, where x, f, and y are related by the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{f}$.

a. Express y as a function of x and f.

b. Graph your function for a lens with a focal length of 30 mm $(f = 30)$. What happens to y as the distance x increases?



71. At t minutes after injection, the concentration (in mg/L) of a certain drug in the bloodstream of a patient is given by the formula

$$c(t) = \frac{20t}{t^2+1}.$$

a. Sketch the graph of $c(t)$ for $t \geq 0$.

b. What happens to the concentration of the drug in the long run?