4.5 EXERCISES

PRACTICE

Mathematical modeling is the process of finding a function that describes how quantities or variables relate to one another. The function is called the mathematical model. (Mathematical modeling will be studied in far greater detail in Section 4.6.) Find the mathematical model for each of the following verbal statements.

1. A varies directly as the product of b and h.

2. V varies directly as the product of four-thirds and r cubed.

3. W varies inversely as d squared.

4. P varies inversely as V.

5. r varies inversely as t.

6. S varies directly as the product of four and r squared.

7. x varies jointly as the cube of y and the square of z.

8. a varies jointly as the square of b and inversely as c.

Find the mathematical model for each of the following verbal statements and then use it to solve for the unknown variable. See Examples 1, 2, and 3.

9. Suppose that y varies directly as the square root of x and that $y = 36$ when $x = 16$. What is y when $x = 20$?

10. Suppose that y varies inversely as the cube of x and that $y = 0.005$ when $x = 10$. What is y when $x = 5$?

11. Suppose that y varies directly as the cube root of x and that $y = 75$ when $y = 125$. What is y when $y = 128$?

12. Suppose that y is proportional to the 5^{th} power of x and that $y = 96$ when $x = 2$. What is y when $x = 5$?

13. Suppose that y varies inversely as the square of x and that $y = 3$ when $x = 4$. What is y when $x = 8$?

14. Suppose that y varies inversely as the square of x and that $y = 8$ when $x = 6$. What is y when $x = 20$?

15. Suppose that y is inversely proportional to the 4^{th} power of x and that $y = 15$ when $x = 4$. What is y when $x = 20$?

16. Suppose that z varies jointly as the square of x and the cube of y and that $z = 768$ when $x = 4$ and $y = 2$. What is z when $x = 3$ and $y = 2$?

17. Suppose that z is jointly proportional to x and y and that $z = 90$ when $x = 1.5$ and $y = 3$. What is z when $x = 0.8$ and $y = 7$?

18. Suppose that z is jointly proportional to x and the cube of y and that $z = 9828$ when $x = 13$ and $y = 6$. What is z when $x = 7$ and $y = 8$?

19. Suppose that z varies directly as the square of x and inversely as y. If $z = 36$ when $x = 6$ and $y = 7$, what value does z have when $x = 12$ and $y = 21$?

20. The quantity F is jointly proportional to a and b and varies inversely as c. If $F = 10$ when $a = 6$, $b = 5$, and $c = 2$, what is the value of F when $a = 12$, $b = 6$, and $c = 3$?

21. The variable a is proportional to $\sqrt{b}$. If $a = 15$ when $b = 9$, what is a when $b = 12$?

22. The variable a varies directly as b. If $a = 3$ when $b = 9$, what is a when $b = 7$?

23. The variable a varies directly as the square of b. If $a = 9$ when $b = 2$, what is a when $b = 4$?

24. The variable a is proportional to the square of b and varies inversely as the square root of c. If $a = 108$ when $b = 6$ and $c = 4$, what is a when $b = 4$ and $c = 9$?

25. The variable a varies jointly as b and c. If $a = 210$ when $b = 14$ and $c = 5$, what is the value of a when $b = 6$ and $c = 6$?

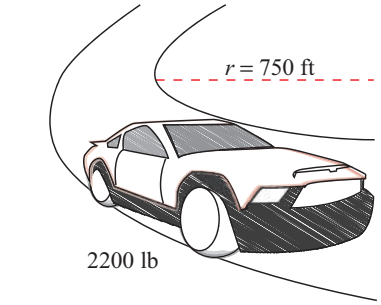
26. The variable a varies directly as the cube of b and inversely as c. If $a = 9$ when $b = 6$ and $c = 7$, what is the value of a when $b = 3$ and $c = 21$?

APPLICATIONS

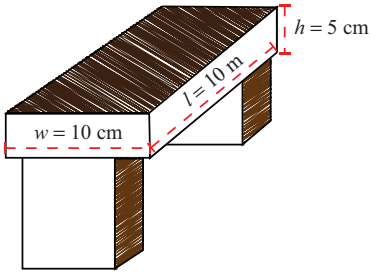
27. The distance that an object falls from rest, when air resistance is negligible, varies directly as the square of the time. A stone dropped from rest travels 144 feet in the first 3 seconds. How far does it travel in the first 4 seconds?

28. A record store manager observes that the number of records sold seems to vary inversely as the price per record. If the store sells 840 records per week when the price per record is \$15.99, how many does he expect to sell if he lowers the price to \$14.99?

29. A person's Body Mass Index (BMI) is used by physicians to determine if a patient's weight falls within reasonable guidelines relative to the patient's height. The BMI varies directly as a person's weight in pounds and inversely as the square of a person's height in inches. Given that a 6-foot-tall man weighing 180 pounds has a BMI of 24.41, what is the BMI of a woman weighing 120 pounds with a height of 5 feet 4 inches?



30. The force necessary to keep a car from skidding as it travels along a circular arc varies directly as the product of the weight of the car and the square of the car's speed, and inversely as the radius of the arc. If it takes 241 pounds of force to keep a 2200-pound car moving 35 miles per hour on an arc whose radius is 750 feet, how many pounds of force would be required if the car were to travel 40 miles per hour?



31. If a beam of width w, height h, and length l is supported at both ends, the maximum load that the beam can hold varies directly as the product of the width and the square of the height, and inversely as the length. A given beam 10 meters long with a width of 10 centimeters and a height of 5 centimeters can hold a load of 200 kilograms when the beam is supported at both ends. If the supports are moved inward so that the effective length of the beam is shorter, the beam can support more load. What should the distance between the supports be if the beam has to hold a load of 300 kilograms?

32. In a simple electric circuit connecting a battery and a light bulb, the current I varies directly as the voltage V but inversely as the resistance R. When a 1.5-volt battery is connected to a light bulb with resistance 0.3 ohms (Ω), the current that travels through the circuit is 5 amps. Find the current if the same light bulb is connected to a 6-volt battery.

33. The amount of time it takes for water to flow down a drainage pipe is inversely proportional to the square of the radius of the pipe. If a pipe of radius 1 cm can empty a sink in 25 seconds, find the radius of a pipe that would allow the sink to drain completely in 16 seconds.

34. The perimeter of a square varies directly as the length of the side of a square. If the perimeter of a square is 308 inches when one side is 77 inches, what is the perimeter of a square when the side is 133 inches?

35. The circumference of a circle varies directly as the diameter. A circular pizza slice has a length of 6.5 inches when the circumference of the pizza is 40.82 inches. What would the circumference of a pizza be if the pizza slice has a length of 5.5 inches?

36. The volume of a cylinder varies jointly as its height and the square of its radius. If a cylinder has the measurements $V = 301.44$ cubic inches, $r = 4$ inches, and $h = 6$ inches, what is the volume of a cylinder that has a radius of 6 inches and a height of 8 inches?

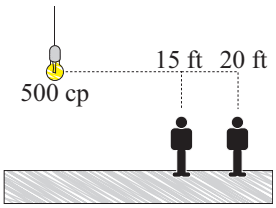
37. The surface area of a right circular cylinder varies directly as the sum of the radius times the height and the square of the radius. With a height of 18 in. and a radius of 7 in., the surface area of a right circular cylinder is 1099 in.^{2} What would the surface area be if the height equaled 5 in. and the radius equaled 3.2 in.?

38. The gravitational force, F, between an object and Earth is inversely proportional to the square of the distance from the object to the center of Earth. If an astronaut weighs 193 pounds on the surface of Earth, what will this astronaut weigh 1000 miles above Earth? Assume that the radius of Earth is 4000 miles.

39. In an electrical schematic, the voltage across a load is directly proportional to the power used by the load but inversely proportional to the current through the load. If a computer is connected to a wall outlet and the computer needs 18 volts to run and absorbs 54 watts of power, the current through the computer is 3 amps. Find the power absorbed by the computer if the same 18-volt computer is attached to a circuit with a loop current of 0.5 amps.

40. A hot dog vendor has determined that the number of hot dogs she sells a day is inversely proportional to the price she charges. The vendor wants to decide if increasing her price by 50 cents will drive away too many customers. On average, she sells 80 hot dogs a day at a price of \$3.50. How many hot dogs can she expect to sell if the price is increased by 50 cents?

41. The price of gasoline purchased varies directly with the number of gallons of gas purchased. If 16 gallons of gas are purchased for \$34.40, what is the price of purchasing 20 gallons?



42. The illumination I of a light source varies directly as the intensity i and inversely as the square of the distance d. If a light source with an intensity of 500 cp (candlepower) has an illumination of 20 fc (foot-candles) at a distance of 15 feet, what is the illumination at a distance of 20 feet?

43. The resistance of a wire varies directly as its length and inversely as the square of the diameter. When a wire is 500 feet long and has a diameter of 0.015 in., it has a resistance of 20 ohms. What is the resistance of a wire that is 1200 feet long and has a diameter of 0.025 in.?

In Exercises 44–45, use Hooke's Law, which says that the force exerted on a spring varies directly with the distance that the spring is stretched.

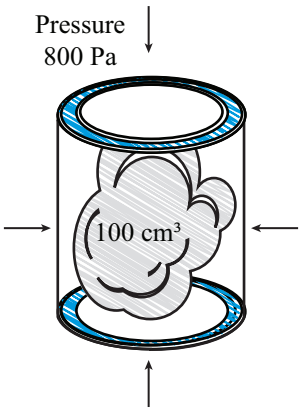
44. A hanging spring will stretch 9 cm if a weight of 15 g is placed on the end of the spring. How far will the spring stretch if the weight is increased to 20 g?

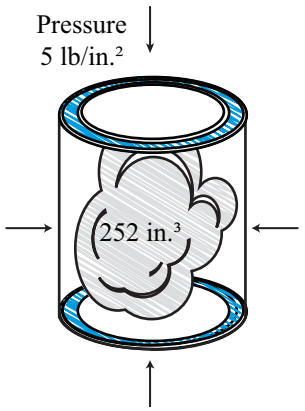
45. If a 32-pound weight suspended on a spring scale stretches the spring 17 inches, how far will a 37-pound weight stretch the spring?

In Exercises 46–47, use Boyle's Law, which says that at a constant temperature, the volume of a gas in a container varies inversely as the pressure on the gas.

46. If the volume is 100 cubic centimeters under a pressure of 800 pascals, what would be the volume of the gas if the pressure was decreased to 400 pascals?

47. If a gas has a volume of 252 cubic inches under a pressure of 5 pounds per square inch, what will its volume be if the pressure is increased to 6 pounds per square inch?





In Exercises 48–50, express the indicated quantities as functions of the other variables. See Example 4.

48. A person's body mass index (BMI) varies directly as a person's weight in pounds and inversely as the square of a person's height in inches. Given that a 6-foot-tall man weighing 180 pounds has a BMI of 24.41, express BMI as a function of weight (w) and height (h).

49. The electric pressure varies directly as the square of the surface charge density (σ) and inversely as the permittivity (ε). If the surface charge density is 6 coulombs per unit area and the free space permittivity equals 3, the pressure is equal to 6 N/m^2. Express the electric pressure as a function of surface charge density and permittivity.

50. The volume of a right circular cylinder varies directly as the radius squared times the height of the cylinder. If the radius is 7 and the height is 4, the volume is equal to 615.44. Determine the expression of the volume of a right circular cylinder.