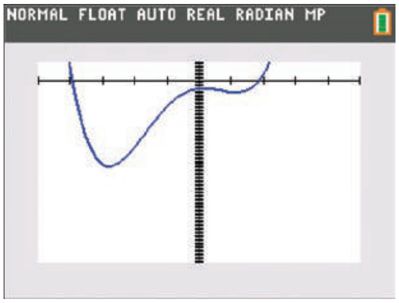


FIGURE 19

TECHNOLOGY: Finding the Maximum or Minimum of a Function

As we've seen, finding the maximum or minimum possible values of some function $f(x)$ can be extremely important, and we have a method for doing so when the function is quadratic. But what if we wanted to find the minimum of the function $f(x)=x^4+2x^3-7x^2+2x-4$? One way is to graph it on a TI-84 Plus, shown in Figure 19 with the following window settings: Xmin=-5, Xmax=5, Ymin=-100, Ymax=10.

To find the minimum, press 2nd trace to access the CALCULATE menu and select minimum. (If we were trying to find the maximum, we would select maximum.) The screen should now display the graph with LeftBound? shown at the bottom. Use the left arrow to move the cursor anywhere to the left of where the minimum appears to be and press enter. The screen should now say RightBound? at the bottom of the graph. Use the right arrow to move the cursor to the right of where the minimum appears to be and press enter again. The text at the bottom of the graph should now read Guess? (see Figure 20). Press enter a third time and the *x*- and *y*-values of the minimum will appear at the bottom of the screen (see Figure 21).

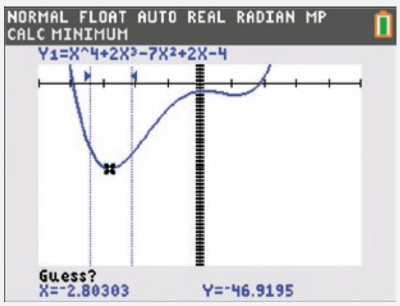


FIGURE 20

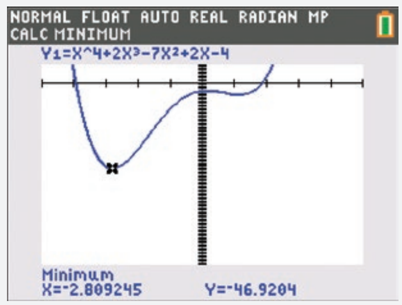


FIGURE 21

So the minimum is approximately $(-2.809,-46.920)$.

4.3 EXERCISES

PRACTICE

Graph the following quadratic functions, accurately locating the vertices and *x*-intercepts (if any). See Example 1.

1. $f(x)=(x-2)^2+3$
2. $g(x)=-(x+2)^2-1$
3. $h(x)=x^2+6x+7$
4. $F(x)=3x^2+2$
5. $G(x)=x^2-x-6$
6. $p(x)=-2x^2+2x+12$
7. $q(x)=2x^2+4x+3$
8. $r(x)=-3x^2-1$
9. $s(x)=\frac{(x-1)^2}{4}$
10. $m(x)=x^2+2x+4$
11. $n(x)=(x+2)(2-x)$
12. $p(x)=-x^2+2x-5$

13. $f(x) = 4x^2 - 6$

14. $k(x) = 2x^2 - 4x$

15. $q(x) = (x+10)(x-2) + 36$

Match the following functions with their graphs.

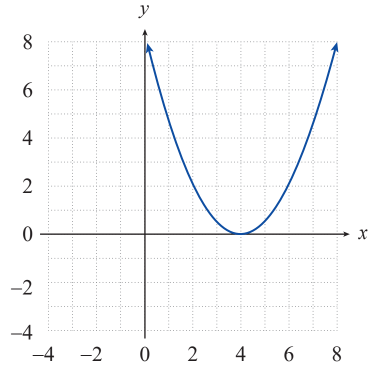
16. $f(x) = -x^2 + 2x$

17. $f(x) = x^2 + 7x + 6$

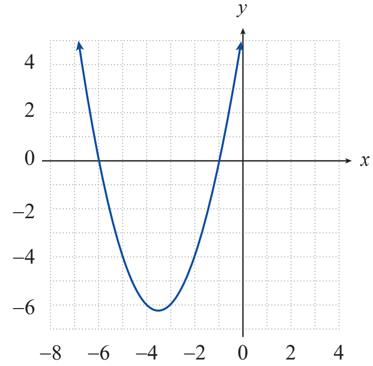
18. $f(x) = \dfrac{x^2 - 8x + 16}{2}$

19. $f(x) = (x-5)(x+3) + 16$

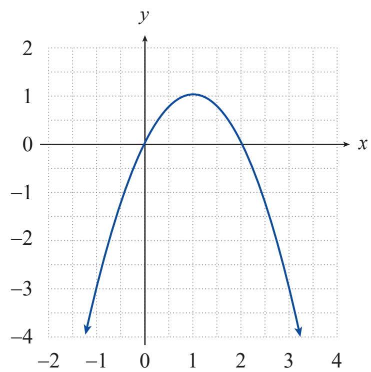
a.



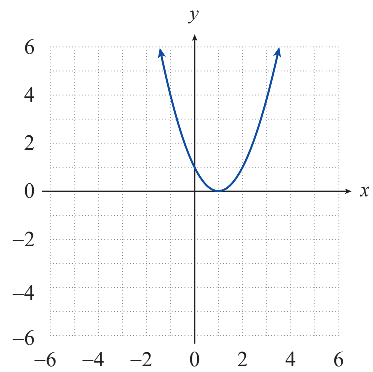
b.



c.

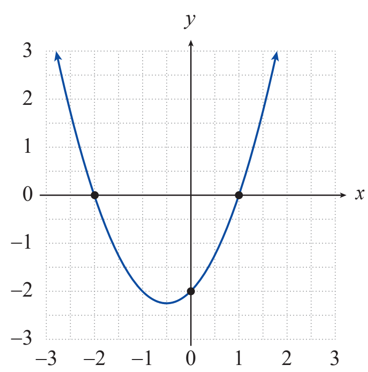


d.

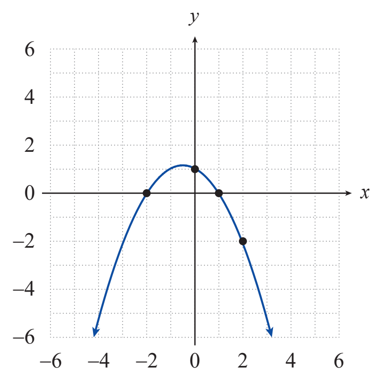


For each of the following parabolic graphs, **a.** find a formula for the corresponding quadratic function, and **b.** use the formula to determine the coordinates of the parabola's vertex. See Example 3.

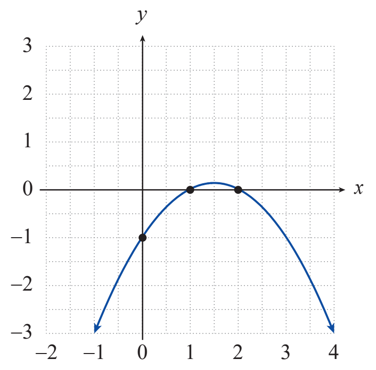
20.



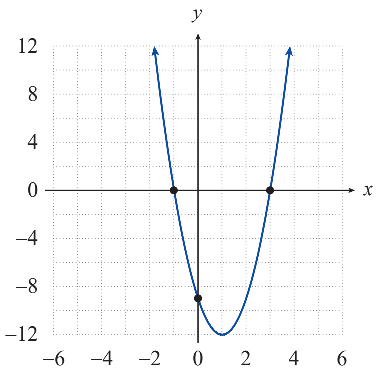
21.



22.

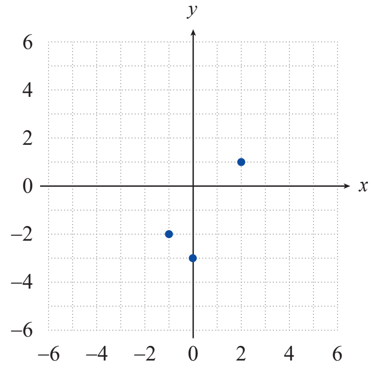


23.

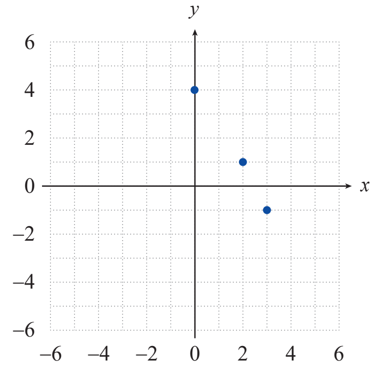


Given the points graphed in each of the following figures, use quadratic regression to find and graph each quadratic function of best fit. See Example 4.

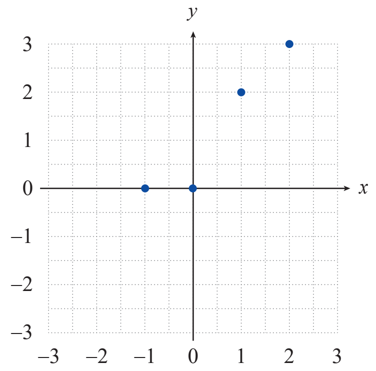
24.



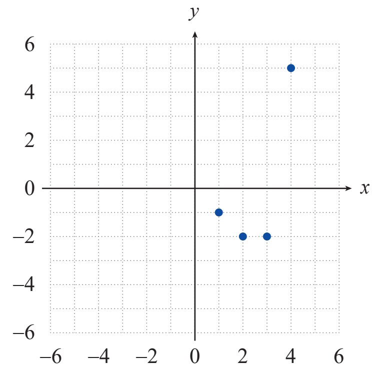
25.



26.

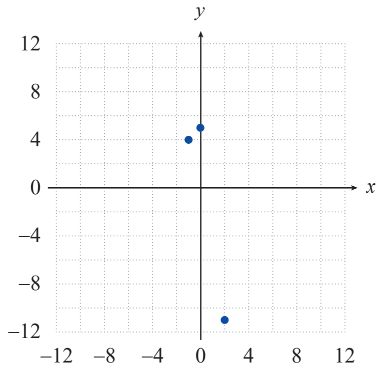


27.

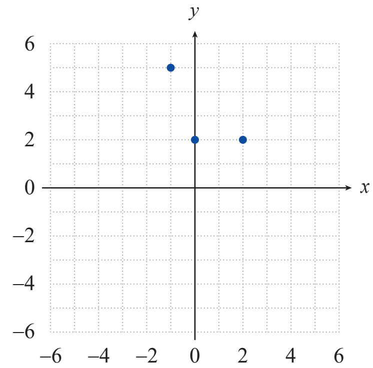


Given the points graphed in each of the following figures, **a.** find the quadratic function that best fits the points, and **b.** use your result to determine the coordinates of the vertex of the best-fitting parabola. See Example 5.

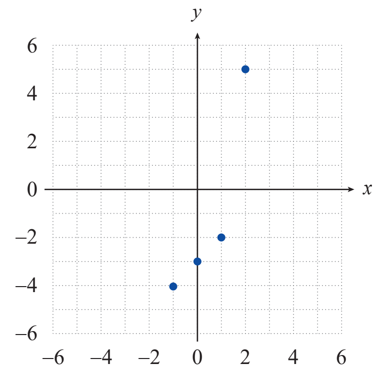
28.



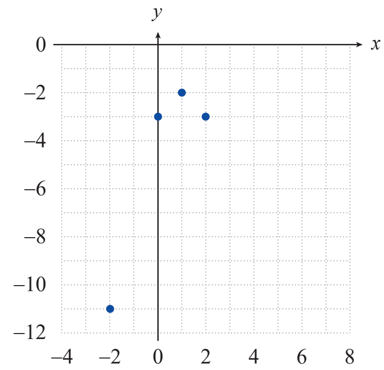
29.



30.

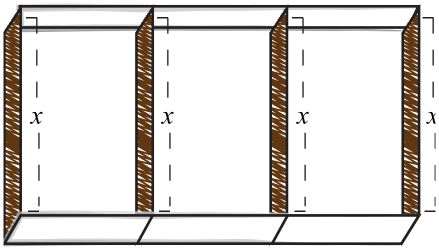


31.



APPLICATIONS

32. Cindy wants to construct three rectangular dog-training arenas side by side, as shown, using a total of 400 feet of fencing. What should the overall length and width be in order to maximize the area of the three combined arenas? (**Hint:** Let x represent the width, as shown, and find an expression for the overall length in terms of x.)



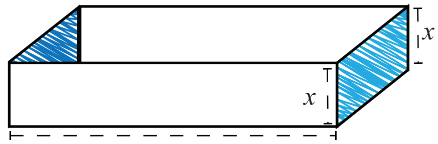
33. Among all the pairs of numbers with a sum of 10, find the pair whose product is maximum.

34. Among all rectangles that have a perimeter of 20, find the dimensions of the one whose area is largest.

35. Find the point on the line $2x + y = 5$ that is closest to the origin. (**Hint:** Instead of trying to minimize the distance between the origin and points on the line, minimize the square of the distance.)

36. Among all the pairs of numbers (x, y) such that $2x + y = 20$, find the pair for which the sum of the squares is minimum.

37. A rancher has a rectangular piece of sheet metal that is 20 inches wide by 10 feet long. He plans to fold the metal to create a narrow three-sided channel and weld two other sheets of metal to the ends to form a watering trough 10 feet long, as shown. How should he fold the metal in order to maximize the volume of the resulting trough?



38. Find a pair of numbers whose product is maximum if the pair must have a sum of 16.

39. Search the Seas cruise ship has a conference room onboard that can hold up to 60 people. Companies can reserve the room for groups of 38 or more. If the group contains 38 people, the company pays \$60 per person. The cost per person is reduced by \$1 for each person in excess of 38. Find the size of the group that maximizes the income for the owners of the ship and find this income.

40. The back of George's property is a creek. George would like to enclose a rectangular area, using the creek as one side and fencing for the other three sides, to create a vegetable garden. If he has 300 feet of material, what is the maximum possible area of the garden?

41. Find a pair of numbers whose product is maximum if two times the first number plus the second number is 48.

42. The total revenue for Thompson's Studio Apartments is given by the function

$$R(x) = 100x - 0.1x^2,$$

where x is the number of rooms rented. What number of rooms rented produces the maximum revenue?

43. The total revenue of Tran's Machinery Rental is given by the function

$$R(x) = 300x - 0.4x^2,$$

where x is the number of units rented. What number of units rented produces the maximum revenue?

44. The total cost of producing a type of small car is given by

$$C(x) = 9000 - 135x + 0.045x^2,$$

where x is the number of cars produced. How many cars should be produced to incur minimum cost?

45. The total cost of manufacturing a set of golf clubs is given by

$$C(x) = 800 - 10x + 0.20x^2,$$

where x is the number of sets of golf clubs produced. How many sets of golf clubs should be manufactured to incur minimum cost?

46. The owner of a parking lot is going to enclose a rectangular area with fencing, using an existing fence as one of the sides. The owner has 220 feet of new fencing material (which is much less than the length of the existing fence). What is the maximum possible area that the owner can enclose?

In Exercises 47–49, use the formula $h(t) = -16t^2 + v_0 t + h_0$ for the height at time t of an object thrown vertically upward with velocity v_0 (in feet per second) from an initial height of h_0 (in feet).

47. Sitting in a tree, 48 feet above ground level, Sue shoots a pebble straight up with a velocity of 64 feet per second. What is the maximum height attained by the pebble?

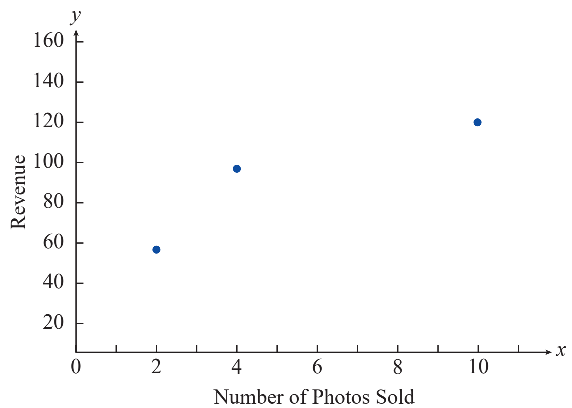
48. A ball is thrown upward with a velocity of 48 feet per second from the top of a 144-foot building. What is the maximum height of the ball?

49. A rock is thrown upward with a velocity of 80 feet per second from the top of a 64-foot-high cliff. What is the maximum height of the rock?

Use quadratic regression to answer the following questions. See Example 7.

50. Darlena has started taking photos at amateur dog racing events, later offering the photos for sale to the dog owners by email. The prices she has charged per photo at each of her first three events, and the corresponding number of photos sold and total revenue raised, appear in the following table. Treating revenue as a function of the number of photos sold, a graph of the three data points is also shown. If she uses quadratic regression to fit a curve to the data, what number of photos sold and what price per photo will maximize her revenue?

Price per Photo	Number of Photos Sold	Revenue
\$28	2	\$56
\$24	4	\$96
\$12	10	\$120



51. Joe makes a video of his friend Zach throwing a baseball as hard as he can straight up in the air. Looking at the video frame by frame later, they estimate that Zach released the ball, at a time they designate as $t = 0$ seconds, at a height of 7 feet. The ball appears to be the same height as the top of a nearby 20-foot-tall billboard at time $t = 0.23$ seconds on the way up and again at time $t = 3.56$ seconds on the way down. If they use quadratic regression to fit a curve to these three points, what maximum height did the baseball reach?

WRITING & THINKING

52. Without graphing, state the number of x-intercepts for each of the following functions and describe the location of the vertex in relation to the x-axis.

a. $y=(x-2)^2$

b. $y=(x-2)(x+2)$

c. $y=-(x-3)(x-1)$

d. $y=-(x-\sqrt{3})(x+\sqrt{3})$

e. $y=x(x+1)$

f. $y=-(x^2+1)$

TECHNOLOGY

Use a graphing utility to graph each of the following quadratic functions. Then determine the vertex and x-intercepts.

53. $f(x)=2x^2-16x+31$

54. $f(x)=-x^2-2x+3$

55. $f(x)=x^2-8x-20$

56. $f(x)=x^2-4x$

57. $f(x)=25-x^2$

58. $f(x)=3x^2+18x$

59. $f(x)=x^2+2x+1$

60. $f(x)=3x^2-8x+2$

61. $f(x)=-x^2+10x-4$

62. $f(x)=\frac{1}{2}x^2+x-1$