Solution

a. $\left(\frac{5+(-1)}{2}, \frac{1+3}{2}\right) = (2, 2)$

b. $\left(\frac{3+(-6)}{2}, \frac{0+11}{2}\right) = \left(-\frac{3}{2}, \frac{11}{2}\right)$

In each case, we simply substitute the coordinates of each point into the midpoint formula. This has the effect of averaging both x-coordinates and both y-coordinates.

3.1 EXERCISES

PRACTICE

Plot the following sets of points in the Cartesian plane. See Example 1.

1. $\{(-3, 2), (5, -1), (0, -2), (3, 0)\}$
2. $\{(-4, 0), (0, -4), (-3, -3), (3, -3)\}$
3. $\{(3, 4), (-2, -1), (-1, -3), (-3, 0)\}$
4. $\{(2, 2), (0, 3), (4, -5), (-1, 3)\}$
5. $\{(0,5), (-3,2), (2,4), (1,1)\}$
6. $\{(8,3), (-3,4), (-4,-6), (3,-4)\}$
7. $\{(-5,-4), (3,2), (4,5), (-2,-1), (-4,-4), (1,1)\}$
8. $\{(-2,5), (0,1), (1,-1), (1,-3), (0,0), (-1,2), (0,-2)\}$

Identify the quadrant in which each point lies, if possible. If a point lies on an axis, specify which part (positive or negative) of which axis (x or y). See Example 1.

9. $(-2, -4)$
10. $(0, -12)$
11. $(4, -7)$
12. $(-2, 0)$
13. $(9, 0)$
14. $(3, 26)$
15. $(-4, -7)$
16. $(0, 1)$
17. $(17, -2)$
18. $\left(-\sqrt{2}, 4\right)$
19. $(-1, 1)$
20. $(-4, 0)$
21. $(3, -9)$
22. $(0, 0)$
23. $(4, 3)$
24. $(-3, -11)$
25. $(0, -97)$
26. $\left(\frac{1}{3}, 0\right)$

For each of the following equations, determine the value of the missing entries in the accompanying table of ordered pairs. Then plot the ordered pairs and sketch your guess of the complete graph of the equation. See Example 2.

27. $6x - 4y = 12$

x	y
0	?
?	0
3	?
?	3

28. $y = x^2 + 2x + 1$

x	y
?	0
1	?
?	1
2	?
−3	?

29. $x = y^2$

x	y
0	?
1	?
4	?
9	?
?	$-\sqrt{2}$

30. $5x - 2 = -y$

x	y
?	0
0	?
1	?
?	7
−2	?

31. $x^2 + y^2 = 9$

x	y
0	?
?	0
−1	?
1	?
?	2

32. $y = -x^2$

x	y
0	?
−1	?
1	?
−2	?
2	?

Determine **a.** the distance between the following pairs of points, and **b.** the midpoint of the line segment joining each pair of points. See Examples 3 and 4.

33. $(-2, 3)$ and $(-5, -2)$

34. $(-1, -2)$ and $(2, 2)$

35. $(0, 7)$ and $(3, 0)$

36. $\left(-\frac{1}{2}, 5\right)$ and $\left(\frac{9}{2}, -7\right)$

37. $(-2, 0)$ and $(0, -2)$

38. $(5, 6)$ and $(-3, -2)$

39. $(13, -14)$ and $(-7, -2)$

40. $(-8, 3)$ and $(2, 11)$

41. $(-3, -3)$ and $(5, -9)$

42. $(7, -7)$ and $(-7, -6)$

43. $(5, -4)$ and $(-1, 5)$

44. $(4, 6)$ and $(2, -7)$

45. $(8, 8)$ and $(-2, -2)$

46. $\left(3, \frac{26}{5}\right)$ and $\left(9, -\frac{14}{5}\right)$

47. Given $(10, 4)$ and $(x, -2)$, find x such that the distance between these two points is 10.

48. Given $(1, y)$ and $(13, -3)$, find y such that the distance between these two points is 15.

49. Given $(x, 3)$ and $(-6, y)$, find x and y such that the midpoint between these two points is $(2, 2)$.

Find the perimeter of the triangle whose vertices are the specified points in the plane.

50. $(-2, 3)$, $(-2, 1)$, and $(-5, -2)$

51. $(-1, -2)$, $(2, -2)$, and $(2, 2)$

52. $(6, -1)$, $(-6, 4)$, and $(9, 3)$

53. $(3, -4)$, $(-7, 0)$, and $(-2, -5)$

54. $(-3, 7)$, $(5, 1)$, and $(-3, -14)$

55. $(-12, -3)$, $(-7, 9)$, and $(9, -3)$

APPLICATIONS

56. Two college friends are taking a weekend road trip. Friday they leave home and drive 87 miles north for a night of dinner and dancing in the city. The next morning they drive 116 miles east to spend a day at the beach. If they drive straight home from the beach the next day, how far do they have to travel on Sunday?

57. Your backpacker's guide contains a grid map of Paris, with each unit on the grid representing 0.25 kilometers. If the Eiffel Tower is located at $(-8,-1)$ and the Arc de Triomphe is located at $(-8,4)$, what is the direct distance (not walking distance, which would have to account for bridges and roadways) between the two monuments in kilometers?

58. Your hotel, located at $(-1,-2)$ on the map from Exercise 57, is advertised as exactly halfway between the Eiffel Tower and Notre Dame. What are the grid coordinates of Notre Dame on your map? Find the direct distance from the Eiffel Tower to Notre Dame, rounded to the nearest hundredth of a kilometer.

59. The navigator of a submarine plots the position of the submarine and surrounding objects using a rectangular coordinate system, where each block is one square meter.

a. If his submarine is located at $(50,231)$ and the mobile base to which he is heading is located at $(83,478)$, how far is he from the mobile base?

b. Suppose there is another submarine located halfway between the first submarine and the mobile base. What is the position of the second sub?

60. At the entrance to Paradise Island Theme Park you are given a map of the park that is in the form of a grid, with the park entrance located at $(-5,-5)$. After walking past three rides and the restrooms, you arrive at the Tsunami Water Ride, which is located at $(-3,-1)$ on the grid. If you have traveled halfway along a straight line to your favorite ride, Thundering Tower, where on the grid is your favorite ride located? How far is Thundering Tower from the park entrance on the map?

WRITING & THINKING

61. Use the distance formula to prove that the triangle with vertices at the points $(1,1)$, $(-2,-5)$, and $(3,0)$ is a right triangle. Then determine the area of the triangle.

62. Use the distance formula to prove that the triangle with vertices at the points $(-2,2)$, $(1,-2)$, and $(2,5)$ is isosceles. Then determine the area of the triangle. (**Hint:** Make use of the midpoint formula.)

63. Use the distance formula to prove that the triangle with vertices at the points $(5,1)$, $(-3,7)$, and $(8,5)$ is a right triangle. Then determine the area of the triangle.

64. Use the distance formula to prove that the triangle with vertices at the points $(1,2)$, $(-2,0)$, and $(3,5)$ is isosceles. Then determine the area of the triangle. (**Hint:** Make use of the midpoint formula.)

65. Use the distance formula to prove that the triangle with vertices at the points $(2,2)$, $(6,3)$, and $(4,11)$ is a right triangle. Then determine the area of the triangle.

66. Use the distance formula to prove that the triangle with vertices at the points $(2,-1)$, $(4,3)$, and $(-2,-3)$ is isosceles. Then determine the area of the triangle. (**Hint:** Make use of the midpoint formula.)

67. Use the distance formula to prove that the polygon with vertices at the points $(-2,-1)$, $(6,5)$, $(-2,5)$, and $(6,-1)$ is a rectangle. Then determine the area of the rectangle. (**Hint:** It may help to plot the points before you begin.)

68. Plot the points $(-3,3)$, $(-5,-2)$, $(3,-2)$, and $(1,3)$ to demonstrate they are the vertices of a trapezoid. Then determine the area of the trapezoid.

TECHNOLOGY

Determine appropriate settings on a graphing utility so that each of the given points will lie within the viewing window. Answers will vary slightly.

69. $\{(-4,1),(2,8),(5,7)\}$

70. $\{(12,3),(5,-11),(-9,6)\}$

71. $\{(3,2),(-2,4),(5,-3)\}$

72. $\{(30,55),(40,25),(-80,-10)\}$

73. $\{(3.75,-8.5),(-5.25,6.0),(7.5,-2.25)\}$

74. $\{(63,99),(-87,34),(45,-22)\}$