TOPIC 3: Solving Equations for One Variable

The procedure we use to solve radical equations can also be used to solve a given equation for a specified variable. We illustrate the process with one last example.

Example 4: Escape Speed

The speed required for an object to escape from the gravitational pull of a planet is called the **escape speed** of the planet. The escape speed is given by the equation $v_e = \sqrt{\frac{2GM}{r}}$, where v_e is the escape speed, G is the universal gravitation constant, M is the mass of the planet, and r is the radius of the planet. Solve this equation for r.

Solution

We follow the same procedure for solving radical equations.

$v_e = \sqrt{\frac{2GM}{r}}$	The radical expression is already isolated.
$v_e^2 = \frac{2GM}{r}$	Square both sides to eliminate the radical.
$r = \frac{2GM}{v_e^2}$	Solve for r.

2.6 EXERCISES

PRACTICE

Solve the following radical equations. See Example 2.

1. $\sqrt{4-x} - x = 2$
2. $\sqrt{3y+4} + \sqrt{5y+6} = 2$
3. $\sqrt{3-3x} - 3 = \sqrt{3x+2}$
4. $\sqrt{x^2-4x+5} - x + 2 = 0$
5. $\sqrt{x^2-4x+4} + 2 = 3x$
6. $\sqrt{50+7s} - s = 8$
7. $\sqrt[3]{3-2x} - \sqrt[3]{x+1} = 0$
8. $\sqrt[4]{x^2-x} = \sqrt[4]{x-1}$
9. $\sqrt[4]{2x+3} = -1$
10. $\sqrt{11x+3} + 4x = 18$
11. $\sqrt{2b-1} + 3 = \sqrt{10b-6}$
12. $\sqrt{5x+5} = \sqrt{4x-7} + 2$
13. $\sqrt{x+10} + 1 = x - 1$
14. $\sqrt{x+1} + 10 = x - 1$
15. $\sqrt{x^2-10} - 1 = x + 1$
16. $\sqrt[3]{5x^2-14x} = -2$
17. $\sqrt[5]{7t^2+2t} = \sqrt[5]{5t^2+4}$
18. $\sqrt[3]{y^3-7y+2} = \sqrt[3]{2-3y}$
19. $\sqrt{14y^2-18y+4} + 2 = 2y$
20. $\sqrt{9x+4} = \sqrt{7x+1} + 1$

21. $\sqrt{4z+41}+3=z+2$

Solve the following equations. See Example 3.

22. $(x+3)^{\frac{1}{4}}+2=0$

23. $(2x-5)^{\frac{1}{4}}=(x-1)^{\frac{1}{4}}$

24. $(2x-1)^{\frac{2}{3}}=x^{\frac{1}{3}}$

25. $(3y^2+9y-5)^{\frac{1}{2}}=y+3$

26. $(3x-5)^{\frac{1}{5}}=(x+1)^{\frac{1}{5}}$

27. $w^{\frac{3}{5}}+8=0$

28. $z^{\frac{4}{3}}-\frac{16}{81}=0$

29. $x^{\frac{2}{3}}-\frac{25}{49}=0$

30. $(x^2+21)^{\frac{-3}{2}}=\frac{1}{125}$

31. $(x-2)^{\frac{2}{3}}=(14-x)^{\frac{1}{3}}$

32. $(x^2+7)^{\frac{-3}{2}}=\frac{1}{64}$

33. $(y-2)^{\frac{2}{3}}=(13y-66)^{\frac{1}{3}}$

Solve the following formulas for the indicated variable. See Example 4.

34. The formula $T=2\pi\sqrt{\frac{l}{g}}$ gives the period T of a pendulum of length l. Solve this formula for l.

35. The formula $c=\sqrt{a^2+b^2}$ gives the length of the hypotenuse c of a right triangle. Solve this formula for a.

36. Einstein's Theory of Relativity states that $E=mc^2$. Solve this equation for c.

37. The formula $\omega=\sqrt{\frac{k}{m}}$ gives the angular frequency ω of a mass m suspended from a spring of spring constant k. Solve this formula for m.

38. The formula $V=\frac{4}{3}\pi r^3$ gives the volume of a sphere with radius r. Solve the equation for r.

39. The formula $F=\frac{mv^2}{r}$ gives the force on an object in circular motion. Solve the equation for v.

40. The formula for lateral acceleration, used in automotives, is $a=\frac{1.227r}{t^2}$. Solve this equation for t.

41. According to one guideline regarding body mass index, a healthy mass for an adult male can be found using the formula $m=23h^2$, where m is expressed in kilograms and h in meters. Solve this equation for h.

42. Kepler's Third Law is $T^2=\frac{4\pi^2r^3}{GM}$. It relates the period T of a planet to the radius r of its orbit and the sun's mass M. Solve this formula for r.

43. The equation $r = \frac{2gm}{c^2}$ is the Schwarzschild Radius Formula used to find the radius of a black hole in space. Solve the equation for c.

44. The total mechanical energy of an object with mass m at height h in a closed system can be written as $ME = \frac{1}{2}mv^2 + mgh$. Solve for v, the velocity of the object, in terms of the given quantities.

45. Recall, the formula for the Pythagorean Theorem states that $a^2 + b^2 = c^2$. Solve this formula for b.

46. In a circuit with an AC power source, the total impedance Z depends on the resistance R, the capacitance C, the inductance L, and the frequency of the current ω according to $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$. Solve this equation for the inductance L.

47. The formula used to find the orbital period for circular Keplerian orbits is $P = \frac{2\pi}{\sqrt{\frac{u}{a^3}}}$. Solve this equation for a.