

**Example 4: Solving Equations by Factoring**

Solve the equation  $x^{-2} - 7x^{-1} + 12 = 0$  by factoring.

**Solution**

While the given equation is not quadratic, it certainly bears a strong resemblance to one that is. In fact, if we make the substitution  $y = x^{-1}$ , we obtain the quadratic equation

$$y^2 - 7y + 12 = 0,$$

which we can solve by factoring the trinomial as  $(y - 3)(y - 4)$ . This gives us  $y = 3$  or  $y = 4$ . Translating this back to the variable  $x$ , we have  $x^{-1} = 3$  or  $x^{-1} = 4$ , so  $x = \frac{1}{3}$  or  $x = \frac{1}{4}$ .

**2.4 EXERCISES****PRACTICE**

Solve the following quadratic-like equations. See Example 1.

1.  $(x-1)^2 + (x-1) - 12 = 0$

2.  $(z-8)^2 - 7(z-8) + 12 = 0$

3.  $(y-5)^2 - 11(y-5) + 24 = 0$

4.  $(x^2-1)^2 + (x^2-1) - 12 = 0$

5.  $(x^2+1)^2 + (x^2+1) - 12 = 0$

6.  $(x^2-13)^2 + (x^2-13) - 12 = 0$

7.  $(x^2-2x+1)^2 + (x^2-2x+1) - 12 = 0$

8.  $2y^{\frac{2}{3}} + y^{\frac{1}{3}} - 1 = 0$

9.  $2x^{\frac{2}{3}} - 7x^{\frac{1}{3}} + 3 = 0$

10.  $(x^2-6x)^2 + 4(x^2-6x) - 5 = 0$

11.  $(y^2-5)^2 + 5(y^2-5) - 36 = 0$

12.  $(x^2+7)^2 + 8(x^2+7) + 12 = 0$

13.  $(t^2-t)^2 - 8(t^2-t) + 12 = 0$

14.  $2x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 2 = 0$

15.  $3x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0$

16.  $y^{\frac{1}{2}} - 5y^{\frac{1}{4}} + 6 = 0$

17.  $(z^2+4z)^2 + 7(z^2+4z) + 12 = 0$

18.  $5y^{\frac{2}{3}} + 33y^{\frac{1}{3}} + 18 = 0$

Solve the following polynomial equations by factoring. See Example 2.

19.  $a^3 - 3a^2 = a - 3$

20.  $2x^3 + x^2 + 2x + 1 = 0$

21.  $2x^3 - x^2 = 15x$

22.  $x^4 + 5x^2 - 36 = 0$

23.  $y^4 + 21y^2 - 100 = 0$

24.  $y^3 + 8 = 0$

25.  $5s^3 + 6s^2 - 20s = 24$

26.  $8a^3 - 27 = 0$

27.  $16a^4 = 81$

28.  $6x^3 + 8x^2 = 14x$

29.  $14x^3 + 27x^2 - 20x = 0$

30.  $5z^3 + 28z^2 = 49z$

31.  $27x^3 + 64 = 0$

32.  $x^3 - 4x^2 + x = 4$

33.  $x^3 + 27 = 0$

Solve the following equations by factoring. See Examples 3 and 4.

34.  $3x^{\frac{11}{3}} + 2x^{\frac{8}{3}} - 5x^{\frac{5}{3}} = 0$

35.  $(x-3)^{\frac{-1}{2}} + 2(x-3)^{\frac{1}{2}} = 0$

36.  $(y-6)^{\frac{-5}{2}} + 7(y-6)^{\frac{-3}{2}} = 0$

37.  $y^{-2} - 2y^{-1} + 1 = 0$

38.  $2x^{\frac{13}{5}} - 5x^{\frac{8}{5}} + 2x^{\frac{3}{5}} = 0$

39.  $(2x-5)^{\frac{1}{3}} - 3(2x-5)^{\frac{-2}{3}} = 0$

40.  $x^{-4} - 13x^{-2} + 36 = 0$

41.  $y^{\frac{7}{2}} - 5y^{\frac{5}{2}} + 6y^{\frac{3}{2}} = 0$

42.  $(t+4)^{\frac{2}{3}} + 2(t+4)^{\frac{8}{3}} = 0$

43.  $y^{-2} - 2y^{-1} - 35 = 0$

44.  $x^{\frac{11}{2}} - 6x^{\frac{9}{2}} + 9x^{\frac{7}{2}} = 0$

45.  $5y^{\frac{11}{3}} + 3y^{\frac{8}{3}} - 2y^{\frac{5}{3}} = 0$

46.  $5y^{\frac{12}{5}} - 43y^{\frac{7}{5}} + 24y^{\frac{2}{5}} = 0$

47.  $(3x-3)^{\frac{-1}{3}} - 5(3x-3)^{\frac{-4}{3}} = 0$

48.  $x^{-2} + 8x^{-1} + 15 = 0$

49.  $(y+3)^{\frac{2}{5}} + 4(y+3)^{\frac{7}{5}} = 0$



### WRITING & THINKING

50. Find  $b$ ,  $c$ , and  $d$  so the equation  $x^3 + bx^2 + cx + d = 0$  has solutions of  $-3$ ,  $-1$ , and  $5$ .

51. Find  $b$ ,  $c$ , and  $d$  so the equation  $x^3 + bx^2 + cx + d = 0$  has solutions of  $-2$ ,  $0$ , and  $6$ .

52. Find  $b$  and  $c$  so the equation  $x^3 + bx^2 + cx = 0$  has solutions of  $0$ ,  $1$ , and  $-7$ .

53. Find  $a$ ,  $c$ , and  $d$  so the equation  $ax^3 + 4x^2 + cx + d = 0$  has solutions of  $-4$ ,  $6$ , and  $-6$ .

54. Find  $a$ ,  $b$ , and  $d$  so the equation  $ax^3 + bx^2 + 3x + d = 0$  has solutions of  $-3$ ,  $-\frac{1}{2}$ , and  $0$ .

55. Find  $a$ ,  $b$ , and  $c$  so the equation  $ax^3 + bx^2 + cx + 6 = 0$  has solutions of  $-\frac{3}{5}$ ,  $\frac{2}{3}$ , and  $1$ .