10.3 EXERCISES

PRACTICE

Find the explicit formula for the general n^{th} term of each geometric sequence. See Examples 1 and 2.

1. $-3,-6,-12,-24,-48,\ldots$

2. $7,\frac{7}{2},\frac{7}{4},\frac{7}{8},\frac{7}{16},\ldots$

3. $2,-\frac{2}{3},\frac{2}{9},-\frac{2}{27},\frac{2}{81},\ldots$

4. $a_1=5$ and $a_4=40$

5. $a_2=-\frac{1}{4}$ and $a_5=\frac{1}{256}$

6. $a_1=1$ and $a_4=-0.001$

7. $a_2=\frac{1}{7}$ and $r=\frac{1}{7}$

8. $a_3=\frac{9}{16}$ and $r=-\frac{3}{4}$

9. $a_3=9,\ a_5=81,$ and $r<0$

10. $-3,9,-27,81,-243,\ldots$

11. $3,2,\frac{4}{3},\frac{8}{9},\frac{16}{27},\ldots$

12. $-5,\frac{5}{4},-\frac{5}{16},\frac{5}{64},-\frac{5}{256},\ldots$

13. $a_3=28$ and $a_6=-224$

14. $a_2=-24$ and $a_5=-81$

15. $a_5=1$ and $a_6=2$

16. $a_4=\frac{343}{3}$ and $r=7$

17. $a_2=\frac{13}{17}$ and $r=\frac{4}{3}$

18. $a_4=8,\ a_8=128,$ and $r>0$

Determine if each of the following sequences is geometric. If so, find the common ratio.

19. The sequence of odd numbers

20. $4,4,4,4,4,4,\ldots$

21. $100,50,25,12.5,6.25,\ldots$

22. $2,5,11,23,47,\ldots$

23. $\frac{7}{8},\frac{7}{4},\frac{7}{2},7,14,\ldots$

24. The sequence of numbers called out at a Bingo game

25. $7,49,343,2401,\ldots$

26. $10,15,22.5,33.75,\ldots$

Given the two terms of a geometric sequence, find the common ratio and first five terms of the sequence.

27. $a_1=8$ and $a_2=24$

28. $a_6=\frac{1}{2}$ and $a_9=\frac{1}{54}$

29. $a_7=16$ and $a_{11}=256$

30. $a_4=108$ and $a_8=8748$

31. $a_5=100$ and $a_9=\frac{4}{25}$

32. $a_8=100$ and $a_{10}=1$

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Use the given information about each geometric sequence to answer the question.

33. Given that $a_2 = -\frac{5}{2}$ and $a_5 = \frac{5}{16}$, what is a_{15}?

34. Given that $a_1 = 1$ and $a_4 = \frac{8}{27}$, what is the common ratio r?

35. Given that $a_3 = -2$ and $a_4 = -16$, what is a_{13}?

36. Given that $a_2 = 24$ and $a_5 = 375$, what is the common ratio r?

37. Given that $a_1 = -1$ and $a_3 = -4$, what is the common ratio r?

38. Given that $a_3 = 108$ and $a_4 = -648$, what is the common ratio r?

39. Given that $a_3 = -\frac{4}{25}$ and $a_7 = -\frac{4}{15,625}$, what is the common ratio r?

Each of the following sums is a partial sum of a geometric sequence. Use this fact to evaluate the sums. See Examples 3 and 4.

40. $\sum_{i=1}^{10} 3\left(-\frac{1}{2}\right)^i$

41. $\sum_{i=5}^{20} 5\left(\frac{3}{2}\right)^i$

42. $\sum_{i=10}^{40} 2^i$

43. $1 - \frac{1}{2} + \cdots + \frac{1}{16,384}$

44. $2 + 6 + \cdots + 39,366$

45. $5 - \frac{5}{3} + \cdots - \frac{5}{19,683}$

46. $1 - 3 + \cdots + 59,049$

47. $\sum_{i=4}^{15} 5(-2)^i$

48. $1 + \frac{3}{5} + \cdots + \frac{243}{3125}$

Determine if each of the following infinite geometric series converges. If so, find the sum. See Example 5.

49. $\sum_{i=0}^{\infty} -\frac{1}{2}\left(\frac{2}{3}\right)^i$

50. $\sum_{i=1}^{\infty} \left(\frac{4}{5}\right)^i$

51. $\sum_{i=0}^{\infty} \left(-\frac{9}{8}\right)^i$

52. $\sum_{i=0}^{\infty} \left(-\frac{8}{9}\right)^i$

53. $\sum_{i=5}^{\infty} \left(\frac{19}{20}\right)^i$

54. $\sum_{i=0}^{\infty} (-1)^i$

55. $\sum_{i=1}^{\infty} \frac{1}{3}(2)^{i-1}$

56. $\sum_{i=0}^{\infty} 5\left(\frac{6}{11}\right)^i$

57. $\sum_{i=4}^{\infty} \left(\frac{13}{24}\right)^i$

Write each of the following repeating decimal numbers as a fraction. See Example 5b.

58. $1.\overline{65}$

59. $0.\overline{123}$

60. $-0.\overline{5}$

61. $-3.\overline{8}$

62. $0.\overline{029}$

63. $9.\overline{98}$

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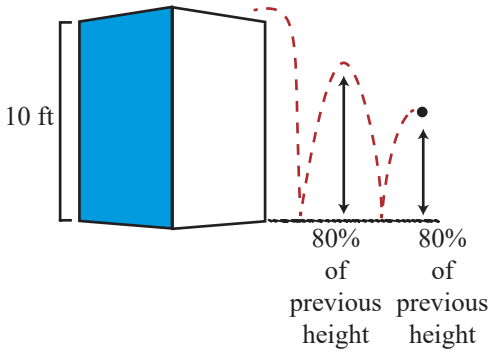
APPLICATIONS

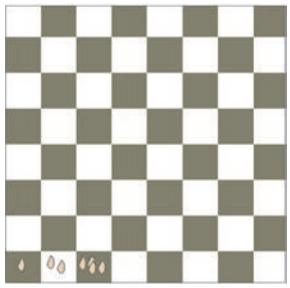
64. A rubber ball is dropped from a height of 10 feet, and on each bounce it rebounds up to 80% of its previous height. How far has it traveled vertically at the moment when it hits the ground for the tenth time? If we assume it bounces an infinite number of times, what is the total vertical distance traveled?



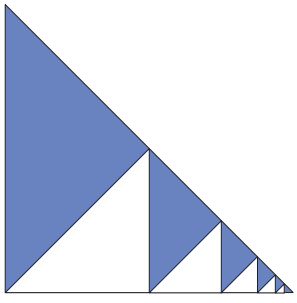
65. If $10,000 is invested in a simple savings account with an annual interest rate of 4% compounded once a year, what is the value of the account after ten years?

66. If $10,000 is invested in a simple savings account with an annual interest rate of 4% compounded once a month, what is the value of the account after ten years?

67. An ancient story about the game of chess tells of a king who offered to grant the inventor of the game a wish. The inventor replied, "Place a grain of wheat on the first square of the board, 2 grains on the second square, 4 grains on the third, and so on. The wheat will be my reward." How many grains of wheat would the king have had to come up with? (There are 64 squares on a chessboard.)



68. An isosceles right triangle is divided into two similar triangles, one of the new triangles is divided into two similar triangles, and this process is continued without end. If the shading pattern seen below is continued indefinitely, what fraction of the original triangle is shaded?



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69. Each year the university admissions committee accepts 3% more students than they accepted in the previous year. If 2130 students were admitted in the first year of this trend, how many total students will have been admitted after 6 years?

70. On the day you were born, your parents deposited $15,000 in a simple savings account for your college education. If the annual interest rate is 6.8%, compounded quarterly, how much money will be in the account when you begin college at the age of 18? What is the common ratio of this series?

71. Spamway, an internet advertising agency, uses e-mail forwards to collect addresses to which they send advertisements. They begin with an e-mail chain letter that they send to 10 people. According to the letter, each of those 10 people has to forward the e-mail to 10 more people or they will have 7 years of bad luck. Assuming the e-mails are received and forwarded only once a day and all the recipients are superstitious and follow the rules, how many e-mail addresses will Spamway have collected after 30 days? Is this series geometric? If so, find the common ratio.

72. Last summer you were a camp counselor for Sunny Days day camp. An arts and crafts project required you to distribute pieces of string (of different lengths) among the children. If you began with a piece of string 18 feet long and gave each child exactly half of your remaining string as you distributed, how long (in inches) was the seventh child's string?

WRITING & THINKING

73. Is it possible for a geometric sequence to also be an arithmetic sequence? If so, give an example and if not, explain your reasoning.

TECHNOLOGY

Use a graphing utility to evaluate each of the following sums.

74. $1.2+1.44+1.728+2.0736+\cdots+38.3376$

75. $5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\cdots+\frac{5}{256}$

76. $\sum_{i=53}^{92}(4.21)^i$

77. $\sum_{i=13}^{54}\left(\frac{19}{436}\right)^i$

78. $\sum_{i=23}^{79}\frac{25}{81}\left(\frac{57}{397}\right)^i$

79. $\sum_{i=16}^{71}3.42(5.26)^i$

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