

$$\begin{aligned}
 \text{b. } \frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}} &= \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} \\
 &= \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} \cdot \frac{x^2 y^2}{x^2 y^2} \\
 &= \frac{xy^2 - x^2 y}{y^2 - x^2} \\
 &= \frac{xy \cancel{(y-x)}}{\cancel{(y-x)}(y+x)} \\
 &= \frac{xy}{y+x}
 \end{aligned}$$

This expression is a complex rational expression, a fact that is more clear once we rewrite the terms that have negative exponents as fractions.

The LCD in this case is  $x^2 y^2$ , so we multiply top and bottom by this and factor the resulting polynomials.

We cancel the common factor of  $(y-x)$  to obtain the final simplified expression.

## 1.7 EXERCISES

### PRACTICE

Simplify the following rational expressions, indicating which real values of the variable must be excluded. See Example 1.

1.  $\frac{2x^2 + 7x + 3}{x^2 - 2x - 15}$
2.  $\frac{x^2 + 5x - 6}{x^3 + 2x^2 - 3x}$
3.  $\frac{x^3 + 2x^2 - 3x}{x + 3}$
4.  $\frac{x^2 - 4x + 4}{x^2 - 4}$
5.  $\frac{x^2 + 5x - 6}{x^2 + 4x - 5}$
6.  $\frac{2x^2 + 7x - 15}{x^2 + 3x - 10}$
7.  $\frac{x + 1}{x^3 + 1}$
8.  $\frac{x^3 + x}{3x^2 + 3}$
9.  $\frac{2x^2 + 11x + 5}{x + 5}$
10.  $\frac{x^4 - x^3}{x^2 - 3x + 2}$
11.  $\frac{2x^2 + 11x - 21}{x + 7}$
12.  $\frac{8x^3 - 27}{2x - 3}$

Add or subtract the rational expressions, as indicated, and simplify your answer. See Example 2.

13.  $\frac{x-3}{x+5} + \frac{x^2+3x+2}{x-3}$
14.  $\frac{x^2-1}{x-2} - \frac{x-1}{x+1}$
15.  $\frac{x+2}{x-3} - \frac{x-3}{x+5} - \frac{1}{x^2+2x-15}$
16.  $\frac{x+1}{x-3} + \frac{x^2+3x+2}{x^2-x-6} - \frac{x^2-2x-3}{x^2-6x+9}$
17.  $\frac{x^2+1}{x-3} + \frac{x-5}{x+3}$
18.  $\frac{x-37}{(x+3)(x-7)} + \frac{3x+6}{(x-7)(x+2)} - \frac{3}{x+3}$
19.  $\frac{x^2+2x-35}{x-5} + \frac{x-4}{x+3}$
20.  $\frac{y+2}{y-2} + \frac{y-6}{y+4} + \frac{4}{y^2+2y-8}$

$$21. \frac{x+2}{x-6} + \frac{x^2+5x+6}{x^2-3x-18} - \frac{x^2-4x-12}{x^2-12x+36} \quad 22. \frac{y^2+2}{y+3} - \frac{y-4}{y-3}$$

Multiply or divide the rational expressions, as indicated, and simplify your answer. See Example 3.

$$23. \frac{y-2}{y+1} \cdot \frac{y^2-1}{y-2} \quad 24. \frac{a^2-3a-4}{a-2} \div \frac{a^2-2a-8}{a-2}$$

$$25. \frac{2x^2-5x-12}{x-3} \cdot \frac{x^2-x-6}{x-4} \quad 26. \frac{z^2+2z+1}{2z^2+3z+1} \cdot \frac{2z^2-5z-3}{z+1}$$

$$27. \frac{y^2-11y+24}{y+6} \div \frac{y^2+5y-24}{y+6} \quad 28. \frac{y^2+8y+16}{5y^2+22y+8} \cdot \frac{5y^2-13y-6}{y+4}$$

$$29. \frac{5y^2-27y-18}{y-5} \cdot \frac{y^2-6y+5}{y-6} \quad 30. \frac{4z^2+20z-56}{z^2-8z+12} \div \frac{5z^2+43z+56}{15z^2-66z-144}$$

$$31. \frac{3b^2+9b-84}{b^2-5b+4} \div \frac{5b^2+37b+14}{-10b^2+6b+4}$$

$$32. \frac{3x^2-x-10}{x-1} \cdot \frac{x^2-1}{6x^2+x-15} \div \frac{x^2-x-2}{2x^2+5x-12}$$

Simplify the complex rational expressions. See Example 4.

$$33. \frac{\frac{3}{x} + \frac{x}{3}}{2 - \frac{1}{x}} \quad 34. \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}} \quad 35. \frac{6x-6}{3 - \frac{3}{x^2}}$$

$$36. \frac{x^{-2} - y^{-2}}{y-x} \quad 37. \frac{\frac{1}{r} - \frac{1}{s}}{r + \frac{1}{r}} \quad 38. \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{y^3} - \frac{1}{xy^2}}$$

$$39. \frac{\frac{m}{n} - \frac{n}{m}}{m-n} \quad 40. \frac{\frac{1}{y} - \frac{1}{x+3}}{\frac{1}{x} - \frac{y}{x^2+3x}} \quad 41. \frac{x+y^{-1}}{x^{-1}+y}$$

$$42. \frac{1+xy}{x^{-2}-y^2} \quad 43. \frac{x^2-y^2}{y^{-2}-x^{-2}} \quad 44. \frac{xy^{-1} + \left(\frac{x}{y}\right)^{-1}}{x^{-2} + y^{-2}}$$

$$45. \frac{\frac{1}{7y} + \frac{1}{x-2}}{\frac{1}{11x} + \frac{7y}{11x^2-22x}} \quad 46. \frac{8z+8}{2 - \frac{2}{z^2}} \quad 47. \frac{25x^{-2} - 9z^{-2}}{\frac{5z+3x}{x^2}}$$

$$48. \frac{\frac{3y}{5} - \frac{5}{3y}}{3 - \frac{5}{y}}$$

Perform the indicated operations on the following rational expressions, and simplify your answer.

$$49. \left( \frac{x^2 - 3x}{x^2 + 6x - 27} - \frac{2}{x+9} \right) \cdot \frac{x+9}{x+2}$$

$$50. \frac{2y(y-1)}{y^2 + 6y - 16} \div \frac{2}{y+8} - \frac{2}{y-2}$$

$$51. \left( \frac{z^2 - 17z + 30}{z^2 + 2z - 8} + \frac{6}{z-2} \right) \div \frac{1}{z^2 - 5z - 36}$$

$$52. \frac{y+3}{2y+18} + \frac{y^2 + 2y + 4}{y^2 + 3y - 54} \cdot \frac{y-6}{y+3}$$

$$53. \frac{y^2 + 2y - 15}{y+1} \cdot \left( \frac{y^2 + 3y + 4}{y^2 + 3y - 10} + \frac{y+4}{y+5} \right) \div \frac{y-3}{y-2}$$

$$54. \frac{y+6}{y-3} \left( \frac{y+5}{y-3} + \frac{y-3}{y+6} - \frac{y^2 + 4}{y^2 + 3y - 18} \right)$$