1.4 EXERCISES

PRACTICE

Evaluate the following radical expressions. See Example 1.

1. $-\sqrt{9}$
2. $\sqrt[3]{-27}$
3. $\sqrt{-25}$
4. $\sqrt[6]{-64}$
5. $-\sqrt[6]{64}$
6. $-\sqrt{169}$
7. $\sqrt[3]{-125}$
8. $\sqrt{-49}$
9. $\sqrt[4]{-256}$
10. $-\sqrt[3]{-64}$
11. $\sqrt[3]{-\frac{27}{125}}$
12. $\sqrt{\frac{25}{121}}$
13. $\sqrt[3]{\frac{-8}{64}}$
14. $\sqrt{\frac{1}{4}}$
15. $-\sqrt[3]{-8}$
16. $\sqrt[4]{\sqrt{16}-\sqrt[3]{-27}+\sqrt{81}}$
17. $\sqrt{\frac{\sqrt[3]{-64}}{-\sqrt{144}-\sqrt{169}}}$
18. $\sqrt{\sqrt[3]{64}+\sqrt[4]{81}+\sqrt[5]{32}}$

Simplify the following radical expressions. See Example 3.

19. $\sqrt{9x^2}$
20. $\sqrt[3]{-8x^6y^9}$
21. $\sqrt[4]{\frac{x^8z^4}{16}}$
22. $\sqrt{2x^6y}$
23. $\sqrt[7]{x^{14}y^{49}z^{21}}$
24. $\sqrt{\frac{x^2}{4x^4y^6}}$
25. $\sqrt[3]{\frac{a^3b^{12}}{27c^6}}$
26. $\sqrt[3]{-125x^{12}y^9}$
27. $\sqrt[4]{\frac{x^{12}y^8}{16}}$
28. $\sqrt[3]{81m^4n^7}$
29. $\sqrt[5]{\frac{y^{30}z^{25}}{32x^{35}}}$
30. $\sqrt[5]{32x^7y^{10}}$

Simplify the following radicals by rationalizing the denominators. See Example 4.

31. $\sqrt[3]{\frac{4x^2}{3y^4}}$
32. $\frac{-\sqrt{3a^3}}{\sqrt{6a}}$
33. $\frac{3}{\sqrt{2}-\sqrt{5}}$
34. $\frac{10}{\sqrt{7}-\sqrt{2}}$
35. $\frac{3}{\sqrt{6}-\sqrt{3}}$
36. $\frac{5}{6-\sqrt{5}}$
37. $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{2}}$
38. $\frac{x-y}{\sqrt{x}+\sqrt{y}}$
39. $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$
40. $\frac{1}{2-\sqrt{x}}$
41. $\frac{\sqrt{y}}{\sqrt{y}+2}$
42. $-\frac{\sqrt{6y^7}}{\sqrt{5y}}$

Rationalize the numerators of the following expressions. See Example 5.

43. $\frac{\sqrt{5}-3}{-4}$
44. $\frac{\sqrt{7}-6}{7}$
45. $\frac{3+\sqrt{y}}{6}$
46. $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}}$
47. $\frac{\sqrt{13}+\sqrt{t}}{13-t}$
48. $\frac{2\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$
49. $\frac{\sqrt{6}+\sqrt{y}}{\sqrt{6}-\sqrt{y}}$
50. $\frac{4\sqrt{xy}+y}{x-y}$

Combine the radical expressions, if possible. See Example 6.

51. $\sqrt[3]{-16x^4} + 5x\sqrt[3]{2x}$

52. $\sqrt{27xy^2} - 4\sqrt{3xy^2}$

53. $\sqrt{7x} - \sqrt[3]{7x}$

54. $|x|\sqrt{8xy^2z^3} - |yz|\sqrt{18x^3z}$

55. $-x^2\sqrt[3]{54x} + 3\sqrt[3]{2x^7}$

56. $\sqrt[5]{32x^{13}} + 3x\sqrt[5]{x^8}$

57. $\sqrt[3]{-16z^4} + 6z\sqrt[3]{2z}$

58. $\sqrt[3]{7y} - \sqrt[4]{7y}$

59. $-x^2\sqrt[3]{16x} + 2\sqrt[3]{2x^7}$

Simplify the following expressions, writing your answer using the same notation as the original expression. See Example 7.

60. $\sqrt[3]{\sqrt[4]{x^{36}}}$

61. $\left(3x^2-4\right)^{\frac{1}{3}}\left(3x^2-4\right)^{\frac{5}{3}}$

62. $32^{\frac{-3}{5}}$

63. $81^{\frac{3}{4}}$

64. $\dfrac{(x-z)^y}{(x-z)^4}$

65. $\sqrt[7]{n^9}\cdot\sqrt[7]{n^5}$

66. $(-8)^{\frac{2}{3}}$

67. $\dfrac{x^{\frac{1}{5}}y^{\frac{-2}{3}}}{x^{\frac{-3}{5}}y}$

68. $(1024)^{-\frac{2}{5}}$

69. $(625)^{-\frac{3}{4}}$

70. $\sqrt[8]{49a^2}$

71. $\sqrt[3]{\sqrt[5]{y^{25}}}$

72. $\dfrac{(a-b)^{-\frac{2}{3}}}{(a-b)^{-2}}$

73. $\left(ax^2+by\right)^{\frac{3}{4}}\left(by+ax^2\right)^{-\frac{2}{3}}$

74. $\dfrac{\sqrt[3]{a^2}}{\sqrt[3]{a^5}}$

Convert the following expressions from radical notation to exponential notation, or vice versa. Simplify each expression in the process, if possible.

75. $\sqrt[4]{a^3}\cdot\sqrt[3]{a^9}$

76. $256^{-\frac{3}{4}}$

77. $\sqrt[12]{x^3}$

78. $\left(9y^2\right)^{\frac{3}{2}}\left(y^6\right)^{\frac{5}{3}}$

79. $\sqrt[6]{\dfrac{2}{72}}$

80. $\left(36n^4\right)^{\frac{5}{6}}$

Simplify the following expressions. See Example 8.

81. $\sqrt{5}\cdot\sqrt[4]{5}$

82. $\sqrt[4]{25}$

83. $\sqrt[16]{y^4}$

84. $\sqrt[4]{36}$

85. $\sqrt[3]{x^7}\cdot\sqrt[9]{x^6}$

86. $\sqrt[5]{y^{16}}\cdot\sqrt[25]{y^{20}}$

87. $\sqrt[4]{7}\cdot\sqrt[16]{7}$

88. $\sqrt{y^4}\cdot\sqrt[6]{y^3}$

Apply the definition of rational exponents to demonstrate the following properties.

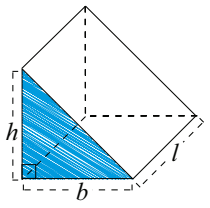
89. $\sqrt[n]{ab} = \sqrt[n]{a}\cdot\sqrt[n]{b}$

90. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

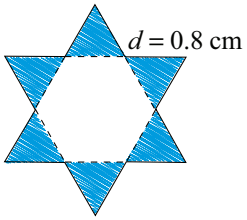
91. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

APPLICATIONS

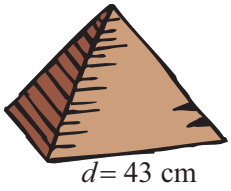
92. The prism shown below is a right triangular cylinder, where the base is a right triangle. Find the surface area of the prism in terms of b, h, and l.



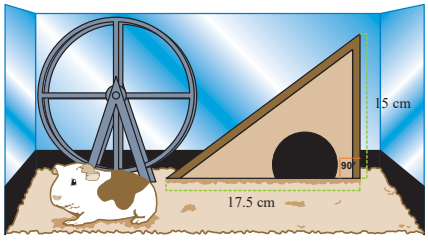
93. A jeweler decides to construct a pendant for a necklace by simply attaching equilateral triangles to each edge of a regular hexagon. The edge length of one of the points of the resulting star is $d = 0.8$ cm. Find the formula for the area of the star in terms of d and then evaluate for $d = 0.8$ cm (rounding to three decimal places). Remember that the area of an equilateral triangle of side length d is $A = \frac{d^2\sqrt{3}}{4}$. See Example 9.



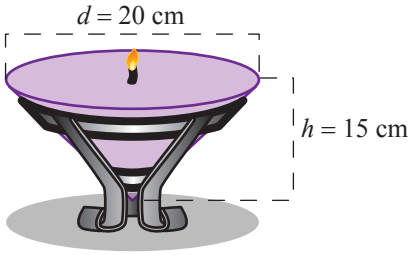
94. The pyramids in Egypt each consist of a square base and four triangular sides. For a class project, Karim constructs a model pyramid with equilateral triangles as sides. The side length is $d = 43$ cm. Find the total surface area of the pyramid (rounding to the nearest square centimeter). See Example 9.



95. Ilyana has made a home for her pet guinea pig (Ralph) in the shape of a right triangular cylinder. Before she can put the new home in Ralph's cage, she must paint it with a nontoxic outer coat. If the front of the home has a base of 17.5 cm and a height of 15 cm and the length of the home is 25 cm, what is the surface area of Ralph's home, rounded to the nearest square centimeter? The small bottle of nontoxic coating will cover up to 1500 cm^2. Will the small bottle contain enough nontoxic coating to cover Ralph's home?



96. Conic Candles plans to design a new candle. The designers have determined that the diameter across the top should be 20 cm and the height of the candle should be 15 cm. Find the volume of the candle, which is in the shape of a circular cone. Round to the nearest cubic centimeter.



97. Einstein's Theory of Special Relativity tells us that $E = mc^2$, where E is energy (in joules, J), m is mass (in kilograms, kg), and c is the speed of light (in meters per second, m/s). This equation may also be written as $\sqrt{\frac{E}{m}} = c$. Assume you know $E = 418{,}400$ J and $m = 4.655 \times 10^{-12}$ kg. Use this information to estimate the speed of light.

WRITING & THINKING

98. Explain, in your own words, why the square root of a negative number is not a real number.

99. Explain, in your own words, why exponents and roots are evaluated at the same time in the order of operations.