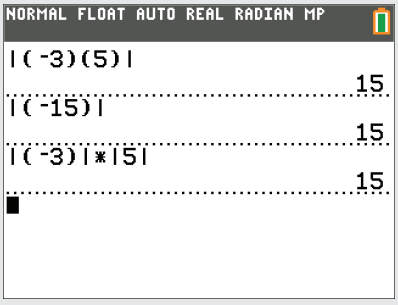
TECHNOLOGY



Example 7: Using Absolute Value Properties

a. $\left|(-3)(5)\right| = \left|-15\right| = 15 = \left|-3\right|\left|5\right|$

b. $1 = \left|-3+4\right| \le \left|-3\right| + \left|4\right| = 7$

c. $7 = \left|-3-4\right| \le \left|-3\right| + \left|-4\right| = 7$

d. $\left|\frac{-3}{7}\right| = \frac{\left|-3\right|}{\left|7\right|} = \frac{3}{7}$

TOPIC 6: Working with Repeating Decimals

By definition, any rational number can be written as a ratio of two integers. A rational number such as 9.35 can be written as the sum $9 + \frac{35}{100}$, which can in turn be written as the ratio $\frac{935}{100}$. But a rational number that appears as a repeating decimal requires a little more work in order to be rewritten as a ratio of two integers. Example 8 illustrates a procedure for doing so.

Example 8: Working with Repeating Digits

As a sum, the decimal number $1.87\overline{35}$ stands for $1 + \frac{87}{100} + 0.00\overline{35}$. The first two terms pose no problem, but we will have to work to write the third term as a ratio of integers.

Let $x = 0.00\overline{35}$. Then $100x = 0.\overline{35}$, so $100x = 0.35 + 0.00\overline{35}$ or $100x = 0.35 + x$. Thus, $99x = 0.35$ or $99x = \frac{35}{100}$. Solving for x, we have $x = \frac{35}{9900}$. (We have just solved a linear equation; we will study these in detail later.) Altogether, we have

$$1.87\overline{35} = 1 + \frac{87}{100} + \frac{35}{9900} = \frac{18{,}548}{9900} = \frac{4637}{2475}.$$

1.1 EXERCISES

PRACTICE

Which elements of the following sets are **a.** natural numbers, **b.** whole numbers, **c.** integers, **d.** rational numbers, **e.** irrational numbers, **f.** real numbers, **g.** undefined? See Example 1.

1. $\left\{19,\ -4.3,\ -\sqrt{3},\ \frac{15}{0},\ \frac{0}{15},\ 2^5,\ -33\right\}$

2. $\left\{5\sqrt{7},\ 4\pi,\ \sqrt{16},\ 3.\overline{3},\ -1,\ \frac{22}{7},\ \left|-8\right|\right\}$

3. $\left\{5.41, \left|-16\right|, \frac{12}{3}, 0, \sqrt{4}, 2.\overline{145}, \frac{1}{4}\right\}$

4. $\left\{2\sqrt{25}, -4, 0.125, \left|32\right|, 2.1563, 6, \sqrt[3]{8}\right\}$

Plot the real numbers in the following sets on a number line. Choose the unit length appropriately for each set. See Example 2.

5. $\{-4.5, -1, 2.5\}$

6. $\{5.1, 5.2, 5.8\}$

7. $\{-24, 2, 15\}$

8. $\left\{0, \frac{1}{2}, \frac{5}{6}\right\}$

Select all of the symbols from the set $\{<, \leq, >, \geq\}$ that can be placed in the blank to make each statement true. See Example 3.

9. $12 \_\_\_ 14$

10. $-3.4 \_\_\_ -3.5$

11. $-102 \_\_\_ 9$

12. $3 \_\_\_ 3$

13. $-50 \_\_\_ -45$

14. $-\frac{1}{4} \_\_\_ -\frac{1}{3}$

15. $0.0087 \_\_\_ -42.9$

16. $\frac{2}{16} \_\_\_ 0.125$

17. $-7 \_\_\_ -9$

18. $-8 \_\_\_ 2$

Write each statement as an inequality, using the appropriate inequality symbol. See Example 3.

19. "$2a + b$ is strictly greater than c"

20. "2 is less than or equal to x"

21. "9 is greater than or equal to 7"

22. "7 is less than or equal to 9"

23. "$x + 5$ is strictly less than 3"

24. "$2c$ is no more than $3d$"

25. "9 is no less than 8"

26. "$6 + x$ is greater than or equal to $4x$"

Describe each of the following sets using set-builder notation. There may be more than one correct way to do this. See Example 4.

27. $\{-6, -3, 0, 3, 6, 9\}$

28. $\{5, 6, 7, \ldots, 105\}$

29. $\{2, 3, 5, 7, 11, 13, 17, \ldots\}$

30. $\{1, 2, 4, 8, 16, 32, \ldots\}$

31. $\left\{\ldots, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\right\}$

32. $\{0, 1, 2, 3, 4, 5, \ldots\}$

Write each set as an interval using interval notation. See Example 5.

33. $x < 15$

34. $-9 \leq x \leq 6$

35. $2.5 < x \leq 3.7$

36. $\{x \mid -3 \leq x < 19\}$

37. $\{x \mid x < 4\}$

38. The positive real numbers

39. $\left\{x \middle| -\frac{1}{2} < x < \frac{2}{5}\right\}$

40. $\{x \mid 1 \leq x \leq 2\}$

41. The nonnegative real numbers

Graph the following intervals.

42. $[5,14)$ **43.** $[-9,-1]$ **44.** $(0,2)$

45. $(-3,18]$ **46.** $(-\infty,7]$ **47.** $(25,\infty)$

Evaluate the absolute value expressions. See Examples 6 and 7.

48. $-|-11|$ **49.** $|3-7|$ **50.** $-|4-9|$

51. $\left|\sqrt{3}-\sqrt{5}\right|$ **52.** $\sqrt{|-4|}$ **53.** $-|-4-|-11||$

54. $\left|-\sqrt{2}\right|$ **55.** $\dfrac{|-x|}{|x|}\ (x \neq 0)$ **56.** $|(-7)(-5)|$

57. $-\left|\sqrt{16}-5\right|$ **58.** $\left|2-\sqrt{7}\right|$ **59.** $-\left|-\sqrt{|-9|}-|-9|\right|$

Find the distance on the real number line between each pair of numbers given. See Example 6.

60. $a=8,\ b=3$ **61.** $a=6,\ b=14$ **62.** $a=5,\ b=5$

63. $a=4,\ b=-2$ **64.** $a=-7,\ b=7$ **65.** $a=-12,\ b=-1$

Write the following rational numbers as ratios of integers. See Example 8.

66. $2.\overline{3}$ **67.** $-5.0\overline{82}$ **68.** $0.\overline{41836}$

69. $0.\overline{9}$ **70.** $-1.\overline{01}$ **71.** $7.\overline{421}$

Write the following rational numbers in decimal form.

72. $-\dfrac{11}{3}$ **73.** $\dfrac{13}{12}$ **74.** $\dfrac{6}{7}$

APPLICATIONS

75. Jess, Stan, Nina, and Michele are in a marathon. Twenty-five minutes after beginning, Jess has run 3.4 miles, Stan has run 4 miles, Nina has run 2.25 miles, and Michele has walked 1.6 miles. Using 0 as the beginning point, plot each competitor's location on a real number line using an appropriate interval.

76. Freddie, Sarah, Elizabeth, JR, and Aubrey are trying to line up by height for a photo shoot. JR is the tallest and Elizabeth is the shortest. Freddie is taller than Sarah, and Sarah is taller than Aubrey. Express their line-up using appropriate inequality symbols.

77. Sue boards an eastbound train in Center Station at the same time Joy boards a westbound train in Center Station. After riding the Straight Line for 20 minutes, Sue's train has traveled 13 miles east, while Joy's train (also on the Straight Line) has traveled 7 miles west. Find the distance between the two trains at this time. (Assume the Straight Line is true to its name and that the tracks lie literally along a straight line.)

78. The admission prices at the local zoo are as follows.

Admission Prices	
Children under 2	free
Children under 12	$3
Adults	$7
Seniors (65 and up)	$5

Express the age range for each of these prices in set-builder notation and interval notation.

79. A particular fudge recipe calls for at least 3 but no more than 4 cups of sugar and at least $\frac{1}{2}$ but no more than $\frac{2}{3}$ of a cup of walnuts. Express the amount of sugar and nuts needed in both set-builder and interval notation.

WRITING & THINKING

80. Can a natural number be irrational? Explain.

81. Are all whole numbers also integers? Are all integers also whole numbers? Explain your answers.

82. In your own words, define absolute value.

83. Write a short paragraph explaining the similarities and differences between $>$ and $\geq$.

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Select all of the symbols from the set $\{<, \leq, >, \geq\}$ that can be placed in the blank to make each statement true. Use a graphing utility to check your answers.

84. $-2.9 \_\_\_ -3.1$

85. $2.1 \_\_\_ -5.5$

86. $100 \_\_\_ -4$

87. $0.001 \_\_\_ -99.8$

88. $\frac{1}{3} \_\_\_ \frac{1}{4}$

89. $-\frac{1}{5} \_\_\_ -\frac{3}{4}$