CHAPTER 1 REVIEW EXERCISES

Section 1.1

Which elements of the following set are **a.** natural numbers, **b.** whole numbers, **c.** integers, **d.** rational numbers, **e.** irrational numbers, **f.** real numbers?

1. $\left\{\frac{3}{7},\ -\sqrt{4},\ 2^3,\ 5.3,\ |-2.1|,\ \sqrt{17},\ 0\right\}$

Describe the following set using set-builder notation. There may be more than one correct way to do this.

2. $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10},\ldots\right\}$

Write each set as an interval using interval notation.

3. $4 \le x < 17$

4. $\{x \mid -8 \le x \le -1\}$

Evaluate the absolute value expressions.

5. $-|-4-3|$

6. $-|11-2|$

7. $\left|\sqrt{9}-7\right|$

8. $\left|\sqrt{5}-\sqrt{11}\right|$

9. $-\frac{|x|}{|-x|}$

Write the following rational numbers as ratios of integers.

10. $7.\overline{6}$

11. $-2.0\overline{42}$

12. Liz, Monica, Peter, James, and Melissa are comparing their ages. Liz is older than Peter and Melissa is the youngest. James is the oldest and Peter is older than Monica. Order them from youngest to oldest.

Section 1.2

Identify the components of the algebraic expressions, as indicated.

13. Identify the terms in the expression $\frac{x^2}{2y}+12.1x-\sqrt{y+5}$.

14. Identify the coefficients in the expression $\frac{x^2}{2y}+12.1x-\sqrt{y+5}$.

Evaluate the following algebraic expressions for the given values of the variables.

15. $7y^2-\frac{1}{3}\pi xy+8x^3$ for $x=-2$ and $y=2$

16. $x^2z^3+5\sqrt{3x-2y}$ for $x=2, y=1,$ and $z=-1$

17. $\left|-3x+x^2y\right|-\frac{xy}{2}$ for $x=-3$ and $y=4$

18. $3\sqrt{\frac{xy}{3}}-2y^2$ for $x=2$ and $y=6$

Identify the property that justifies each of the following statements. If one of the cancellation properties is being used to transform an equation, identify the quantity that is being added to both sides or the quantity by which both sides are being multiplied.

19. $-4+x=x-4$

20. $12a^2=8b \Leftrightarrow 3a^2=2b$

21. $(x-3)(z-2)=0 \Rightarrow x-3=0 \text{ or } z-2=0$

Simplify the following set expressions.

22. $(-4,8)\cup[5,13]$

23. $(-4,8)\cap[5,13]$

Section 1.3

Use the properties of exponents to simplify each of the following expressions, writing your answer with only positive exponents.

24. $\left(2^3a^{-2}b^4\right)^{-1}c^{-3}$

25. $\dfrac{-4t^0\left(s^2t^{-2}\right)^{-3}}{2^3st^{-3}}$

26. $\left[\left(3y^{-2}z\right)^{-1}\right]^{-3}$

27. $\dfrac{3^2x^{-4}\left(y^2z\right)^{-2}}{\left(2z^{-3}\right)^{-1}y^{-6}}$

Convert each number from scientific notation to standard notation, or vice versa, as indicated.

28. -3.005×10^{-4}; convert to standard

29. 69,520,000; convert to scientific

Evaluate each expression, using the properties of exponents. Use a calculator only to check your final answer.

30. $\left(3.46\times10^8\right)\left(1.2\times10^4\right)$

31. $\dfrac{2.4\times10^{-12}}{1.2\times10^{-4}}$

32. Sam is making a piñata in the shape of a sphere and needs to know how much candy to buy to fill it. If the radius of the piñata is 10 inches, what is the volume of the piñata?

Section 1.4

Evaluate the following radical expressions.

33. $\sqrt{3^2+4^2}$

34. $\dfrac{\sqrt[3]{\sqrt{15}}}{\sqrt{\sqrt[3]{5}}}$

Simplify the following radical expressions. Rationalize all denominators and use only positive exponents.

35. $\sqrt{25x^{20}}$

36. $\sqrt{16x^2}$

37. $\sqrt[3]{-64x^{-9}y^3}$

38. $\dfrac{\sqrt{3a^3}}{\sqrt{12a}}$

39. $\sqrt[3]{\dfrac{8x^2}{3y^{-4}}}$

40. $\sqrt[4]{\dfrac{a^9b^{-4}}{81}}$

41. $\dfrac{4}{\sqrt{2}-\sqrt{6}}$

42. $\dfrac{3}{\sqrt{x}+\sqrt{2}}$

Simplify the following expressions.

43. $\sqrt{18x^3y} - \sqrt[3]{16x^4y}$

44. $\left(2\sqrt{3} - 5\sqrt{2}\right)^2$

Convert the following expressions from radical notation to exponential notation, or vice versa. Simplify each expression in the process, if possible.

45. $\sqrt{x^{-5}} \cdot \sqrt[4]{x^3}$

46. $\left(49x^4\right)^{\frac{1}{2}}\left(16x^{12}\right)^{\frac{3}{4}}$

Section 1.5

Add or subtract the polynomials, as indicated.

47. $\left(-4m^2 - 5m^3 + 4\right) + \left(m^4 + 7m^2 - 2\right)$

48. $\left(2xy + 3x\right) - \left(8x^2y - 6xy + 3x - y\right)$

Multiply the polynomials, as indicated.

49. $\left(x^2 + y\right)\left(3x - 4y^3\right)$

50. $\left(a + 5b\right)\left(5a - 7ab + 2b\right)$

Section 1.6

Factor each of the following polynomials.

51. $x^2 - x - 12$

52. $2x^2 + x - 15$

53. $6a^2 - 7a - 5$

54. $4a^2 - 9b^4$

55. $36x^6 - y^2$

56. $nx + 3mx - 2ny - 6my$

57. $2x^2 + 6x - 5xy - 15y$

58. $8x^3y^2 + 4x^3y - 12xy^2$

Factor the following algebraic expressions.

59. $\left(3x - 2y\right)^{\frac{4}{3}} - \left(3x - 2y\right)^{\frac{2}{3}}$

60. $8x^{-2} + 5x^{-1}$

Section 1.7

Simplify the following rational expressions, indicating which real values of the variable must be excluded.

61. $\dfrac{x^3 + 6x^2 + 9x}{x^3 - 9x}$

62. $\dfrac{x^2 - 9}{x^3 - 27}$

Perform the indicated operations on the rational expressions and simplify your answer.

63. $\dfrac{1}{x} - \dfrac{3}{x+2} - \dfrac{6}{x^2 + 2x}$

64. $\dfrac{a^3 - 8}{a^2 - 4} \div \dfrac{a^3 + 2a^2 + 4a}{a^3 + 2a^2} \cdot \dfrac{1}{a^2 + a}$

Simplify the complex rational expressions.

65. $\dfrac{\dfrac{1}{2a} - \dfrac{1}{2b}}{\dfrac{2}{a} + \dfrac{2}{b}}$

66. $\dfrac{\dfrac{x}{3} - \dfrac{3}{x}}{-\dfrac{3}{x} + 1}$

67. $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{x^{-1} - y^{-1}}$

Section 1.8

Evaluate the following square root expressions.

68. $-\sqrt{-8x}$

69. $i^3\sqrt{-9}$

Simplify the following expressions.

70. $(7-2i)+(9i-5)$

71. $(5-3i)-(-12i)$

72. $(3-i)(6i^2-4)$

73. $\dfrac{17}{4-i}$

74. $\dfrac{2i}{3-i}$

75. $\dfrac{3+4i}{3-4i}$

76. $\left(\sqrt{-3}\right)\left(\sqrt{-16}\right)$

77. $\left(8-\sqrt{-2}\right)^2$

78. $\dfrac{2i\sqrt{-27}}{\sqrt{-16}}$

CHAPTER 2 REVIEW EXERCISES

Section 2.1

Solve the following linear equations.

1. $2y-(1-y)=y+2(y-1)$

2. $\frac{x}{2}-\frac{1}{3}=x-\frac{1}{3}-\frac{x}{2}$

3. $-0.2x-0.5=-0.4x+0.75$

4. $-2(x-5)+1=3+(7x-2)$

Solve the following absolute value equations.

5. $|2x-7|=1$

6. $|2y-5|-1=|3-y|$

7. $|7z+5|+3=8$

8. $|w-5|=|3w+1|$

Solve the following absolute value equations geometrically and algebraically.

9. $|-2x+1|=7$

10. $|x+4|-|x-1|=0$

Solve each of the following equations for the indicated variable.

11. Area of a trapezoid: $A=\frac{1}{2}h(b+c)$; solve for c

12. Volume of a rectangular pyramid: $V=\frac{1}{3}lwh$; solve for l

13. Temperature conversions: $F=\frac{9}{5}C+32$; solve for C

14. Two trains leave the station at the same time in opposite directions. One travels at an average rate of 90 miles per hour, and the other at an average rate of 95 miles per hour. How far apart are the two trains after an hour and twenty minutes? Round your answer to one decimal place.

15. Two firefighters, Jake and Rose, each have $5000 to invest. Jake invests his money in a money market account with an annual return of 3.25%, while Rose invests hers in a CD paying 4.95% annually. How much more money does Rose have than Jake after 1 year?

Section 2.2

Solve the following linear inequalities. Describe each solution set using interval notation and by graphing.

16. $-8x+3\geq -9x+10$

17. $4(2x-5)<-3(-3x+8)$

18. $\frac{-2(x-1)}{3}\leq\frac{-2x}{4}$

19. $3.1(2x-1)>7.2-4.1x$

20. $-5<3m+1<13$

21. $-14<-2(3+y)\leq 8$

22. $2<\frac{x+1}{4}\leq 7$

23. $-5|3+t|>-10$

24. $3+|2x-1|<1$

25. $-2|x-1|+|3x-3|\geq 7$

26. $6+\frac{x}{5}\le\frac{4}{5}$ or $5+2x\ge x-2$

27. $\frac{8x-5}{9}\le 3$ or $2(3x-16)\ge 4(x-3)$

28. $2.9x+1.8<3(1.3x+6)$ and $7x<5x+34$

Section 2.3

Solve the following quadratic equations.

29. $5x^2-13x-6=0$

30. $x^2=7$

31. $2(x-2)^2=-18$

32. $15x^2+3x+2=-8x$

33. $x^2-8x+14=0$

34. $3x^2-x+3=-7x$

35. $x^2=6x-16$

36. $-2x-7=-4x^2$

37. $2x^2+3x-10=10$

38. $x^2-7x-2=-12$

39. $1.7z^2-3.8z-2=0$

40. $2x^2+7x=x^2+2x-6$

Section 2.4

Solve the following quadratic-like equations.

41. $(x^2+2)^2-7(x^2+2)+12=0$

42. $y^{\frac{2}{3}}+y^{\frac{1}{3}}-6=0$

43. $(t+2)^2-2(t+2)=24$

44. $x^4-13x^2+36=0$

Solve the following equations by factoring.

45. $x^3-4x^2-2x+8=0$

46. $2x^3+2x=5x^2$

47. $x^3-x^2+4x-4=0$

48. $x^4+7x^2-18=0$

49. $x^{\frac{7}{2}}-3x^{\frac{5}{2}}-4x^{\frac{3}{2}}=0$

50. $x^{\frac{7}{3}}+7x^{\frac{4}{3}}-8x^{\frac{1}{3}}=0$

51. $(x-2)^{\frac{3}{4}}+2(x-2)^{\frac{7}{4}}=0$

52. $(x-1)^{-\frac{1}{2}}+4(x-1)^{\frac{1}{2}}=0$

Use the connection between solutions of polynomial equations and polynomial factoring to answer the following questions.

53. Find b and c so the equation $x^3+bx^2+cx=0$ has solutions of -2, 0, and 4.

54. Given that the equation $x^2-6x+m-1=0$ has only one root, find m.

55. If the sum of the roots of the equation $x^2+mx-6=0$ is 5, then what is m?

Section 2.5

Solve the following rational equations.

56. $\frac{1}{x+2}+\frac{1}{x-3}-\frac{x}{x-3}=0$

57. $\frac{1}{x-2}-\frac{x}{x+2}=\frac{2}{x^2-4}$

58. $\frac{y}{y-1}+\frac{1}{y-4}=\frac{y^2}{y^2-5y+4}$

59. $\frac{2}{x+1}-\frac{x}{x-3}=\frac{3x-21}{x^2-2x-3}$

60. Jim cleans a house in 6 hours. John cleans the same house in 8 hours. How long does it take for them to clean the house together?

Section 2.6

Solve the following equations.

61. $\sqrt{-4-x}-4=x$

62. $\sqrt{5x-1}=4+\sqrt{x+3}$

63. $\sqrt{2x^2+8x+1}-x-3=0$

64. $\sqrt{10x^2-14x+16}+1=3x$

65. $x+2=\left(-x^2+11x+19\right)^{\frac{1}{2}}$

66. $\left(2x^2+14x\right)^{\frac{1}{4}}=\left(-x^2-8\right)^{\frac{1}{4}}$

67. $(2x-5)^{\frac{1}{6}}=(x-2)^{\frac{1}{6}}$

68. $\left(x^2+x-16\right)^{\frac{1}{3}}=2(x-1)^{\frac{1}{3}}$

69. The formula for the volume of a cone with radius r and height h is $V=\frac{1}{3}\pi r^2 h$. Solve the equation for r.

CHAPTER 3 REVIEW EXERCISES

Section 3.1

Plot the following sets of points in the Cartesian plane.

1. $\{(7,3),(-2,4),(3,0),(-1,-6)\}$

2. $\{(4,-4),(-6,3),(-3,-1),(-4,2)\}$

3. $\{(2,1),(-4,5),(3,-7),(2,3)\}$

Identify the quadrant in which each point lies, if possible. If a point lies on an axis, specify which part (positive or negative) of which axis (x or y).

4. $(0,0)$

5. $(1,0)$

6. $(3,-2)$

For each of the following equations, determine the value of the missing entries in the accompanying table of ordered pairs. Then plot the ordered pairs and sketch your guess of the complete graph of the equation.

7. $3x-2y=6$

x	y
?	0
0	?
−1	?
?	−2
−2	?

8. $3x=y^2-4$

x	y
0	?
?	0
?	$-\sqrt{7}$
−1	?
?	3

Determine **a.** the distance between the following pairs of points, and **b.** the midpoint of the line segment joining each pair of points.

9. $(2,-6)$ and $(3,-7)$

10. $(-4,-3)$ and $(4,-9)$

11. $(-3,6)$ and $(-7,0)$

12. $(5,-1)$ and $(-4,3)$

13. Given $A(-4,2)$, $B(x,y)$, and $C(1,-1)$, find $x+y$ if C is the midpoint of the line segment $\overline{AB}$.

Find the perimeter of the triangle whose vertices are the specified points in the plane.

14. $(-3,2)$, $(-3,0)$, and $(-6,-3)$

15. $(8,-3)$, $(2,-3)$, and $(2,5)$

16. Use the distance formula to prove that the triangle with vertices at the points $(-2,2),(0,3)$, and $(4,-5)$ is a right triangle and determine the area of the triangle.

Section 3.2

Find the standard form of the equation for each circle described below.

17. Radius 4; center $\left(\sqrt{5},-\sqrt{2}\right)$

18. Endpoints of a diameter are $(1,-3)$ and $(-5,3)$.

19. Center at $(2,-1)$; passes through $(4,3)$

20. Endpoints of a diameter are $(1,2)$ and $(-5,8)$.

21. What is the radius and center of the circle $(x+3)^2+(y-1)^2=8$?

22. Given that point $(a,4)$ is on the circle $x^2+y^2=25$, find a.

Sketch a graph of the circle defined by the given equation. Then state the radius and center of the circle.

23. $(x+5)^2+(y-2)^2=16$

24. $x^2+(y-3)^2=10$

25. $(x-1)^2+(y+4)^2=9$

26. $x^2+y^2+6x-10y=-5$

Section 3.3

Determine if the following equations are linear.

27. $3x+y(4-2x)=8$

28. $y-3(y-x)=8x$

29. $9x^2-(3x+1)^2=y-3$

30. $8x-3y=4(x-1)+y$

31. $2x(3y-1)=7$

32. $3x^2+2=(x+2)^2-1$

Find the *x*- and *y*-intercepts of the given equations, if possible, and then sketch their graphs.

33. $4y-12=8x$

34. $3(2y+1)=5y-4x+3$

35. $2x+y-2=2(3+x)$

36. $3y-4x=-2(3x-y)$

37. $2x+3y=18$

38. $4x+y=12+y$

Section 3.4

Determine the slope of the line passing through the specified points.

39. $(-2,5)$ and $(-3,-7)$

40. $(3,6)$ and $(7,-10)$

41. $(3,5)$ and $(3,-7)$

Use the slope-intercept form to graph the equations.

42. $6x-3y=9$ **43.** $2y+5x+9=0$ **44.** $15y-5x=0$

Find the equation, in standard form, of the line passing through the given point with the given slope.

45. point $(4,-1)$; slope of 1 **46.** point $(-2,3)$; slope of $\frac{3}{2}$

Find the equation, in slope-intercept form, of the line with the given *y*-intercept and slope.

47. *y*-intercept $(0,-2)$; slope of $\frac{5}{9}$ **48.** *y*-intercept $(0,9)$; slope of $-\frac{7}{3}$

Find the equation, in standard form, of the line passing through the specified points.

49. $(5,7)$ and $(3,-2)$ **50.** $\left(\frac{3}{2},1\right)$ and $\left(-3,\frac{5}{2}\right)$

51. A sales person receives a monthly salary of \$2800 plus a commission of 8% of sales. Write a linear equation for the sales person's monthly wage W, in terms of monthly sales, s.

Section 3.5

Determine if the two lines are perpendicular, parallel, or neither.

52. $x-4y=3$ and $4x-y=2$

53. $3x+y=2$ and $x-3y=25$

54. $\frac{3x-y}{3}=x+2$ and $\frac{y}{3}+x=9$

Find the equation, in slope-intercept form, for the line parallel to the given line and passing through the indicated point.

55. $y-3x=10$; $(-2,4)$ **56.** $3(y+1)=\frac{x-3}{2}$; $(-6,3)$

57. $y=2x+1$; $(1,-1)$ **58.** $3y-2=-5(2x-1)$; $(2,-5)$

Find the equation, in slope-intercept form, for the line perpendicular to the given line and passing through the indicated point.

59. $y=\frac{3}{4}x-1$; $(6,-2)$ **60.** $2(y-3)=\frac{2x+3}{3}$; $(-5,-4)$

61. $y=8$; $(7,1)$ **62.** $5x+7y-2=10$; $\left(\frac{2}{7},-1\right)$

Each set of four ordered pairs defines the vertices, in counterclockwise order, of a quadrilateral. Determine if the quadrilateral is a rectangle.

63. $\{(-2,1),(-1,-1),(3,1),(2,3)\}$ **64.** $\{(-2,2),(-3,-1),(2,-3),(2,1)\}$

Section 3.6

Solve the following linear inequalities by graphing their solution sets.

65. $x - 2y < 4$ **66.** $y < 3x + 2$ **67.** $\frac{4x + y}{3} \geq 2$

Graph the solution sets that satisfy the following inequalities.

68. $7x - 2y \geq 8$ and $y < 5$ **69.** $x - 4y \geq 6$ or $y > -2$

70. $y - x > 0$ and $x < 2$

Graph the solution sets that satisfy the following linear absolute value inequalities.

71. $|2x + 5| < 3$ **72.** $|2x - 1| < 5$

73. $|x - y| < 3$ **74.** $-5 + |x - 3| > -1$

75. $|2x + 1| < 3$ or $|y + 3| \geq 4$ **76.** $|x| > 4$ and $\left|\frac{2y - 1}{3}\right| < 3$

77. A candle store makes a \$3 profit for every novelty candle sold and a \$4 profit for every accompanying candle holder sold. Write a linear inequality describing the number of each type of item that needs to be sold in order to make a total profit of at least \$1500.

CHAPTER 4 REVIEW EXERCISES

Section 4.1

For each of the following relations, determine the domain and range and determine whether the relation is a function.

1. $R = \{(-2,9),(-3,-3),(-2,2),(-2,-9)\}$

2. $f = \{(-3,0),(-1,4),(0,3),(3,3),(4,-1)\}$

3. $R = \{(x,2) | x \in \mathbb{R}\}$

4. $S = \{(x,4x) | x \in \mathbb{Z}\}$

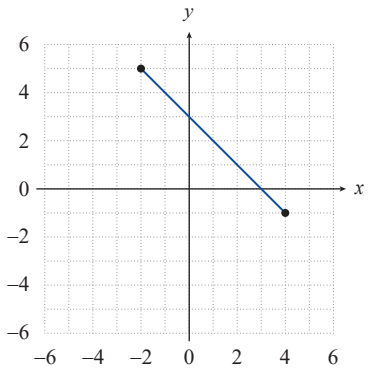
5. $3x - 4y = 17$

6. $x = y^2 - 6$

7. $x = \sqrt{y-4}$

8. $y = -5$

9.



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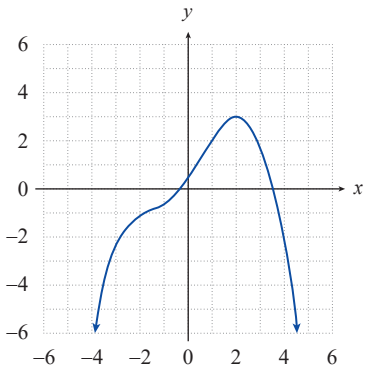
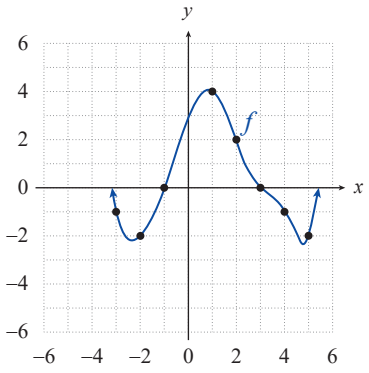


Rewrite each of the following relations as a function of x. Then evaluate the function at $x = -2$.

11. $\dfrac{y+4}{\sqrt{x+11}} - 3y = 3(1-y)$

12. $x^2 - 4x + 3y = x + 2y$

Use the graph below of the function f to answer each of the following questions.



13. What is the value of $f(1)$?

14. What is the value of $f(3)$?

15. For what integer value(s) of x is $f(x) = 0$?

16. For what integer values(s) of x is $f(x) = -2$?

Given $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x^2}$, evaluate the following.

17. $f(x+h)$

18. $\dfrac{f(x+h)-f(x)}{x+h}$

19. $g(x+h)$

20. $\dfrac{g(x+h)-g(x)}{h}$

Identify the domain, the codomain, and the range of each of the following functions.

21. $g:\mathbb{N}\to\mathbb{R}$ by $g(x)=\dfrac{3x}{4}$

22. $h:\mathbb{R}\to\mathbb{R}$ by $h(x)=5x+1$

Determine the implied domain of each of the following functions.

23. $f(x)=\sqrt{|-x-3|}$

24. $f(x)=\dfrac{x}{1-x}$

Section 4.2

Graph the following linear functions.

25. $f(x)=7x-2$

26. $g(x)=\dfrac{2x-6}{3}$

27. $p(x)=-2x-3$

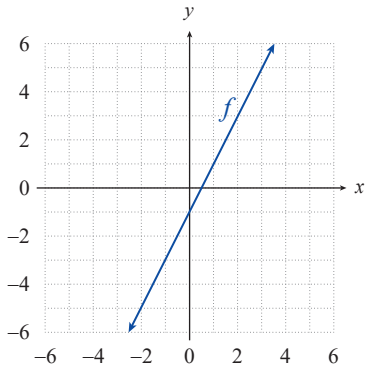
28. $h(x)=-5+\dfrac{x}{4}$

29. $k(x)=2(1-2x)+x$

30. $f(x)=\dfrac{4x+3}{2}-2x$

Find a formula for the linear function depicted in each of the following graphs.

31.



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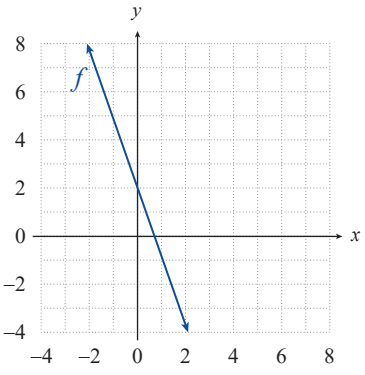


For the given points, **a.** use linear regression to find and graph the line of best fit along with the points, and **b.** find the Pearson correlation coefficient *r*.

33. $(1,2),(2,2),(3,4),(4,6),(5,5)$

Section 4.3

Graph the following quadratic functions, accurately locating the vertices and *x*-intercepts (if any).

34. $f(x)=(x-1)^2-1$

35. $g(x)=-(x+3)^2-2$

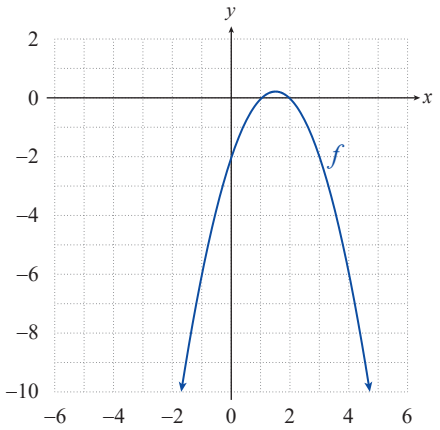
36. $p(x)=x^2-2$

37. $k(x)=-x^2+4x$

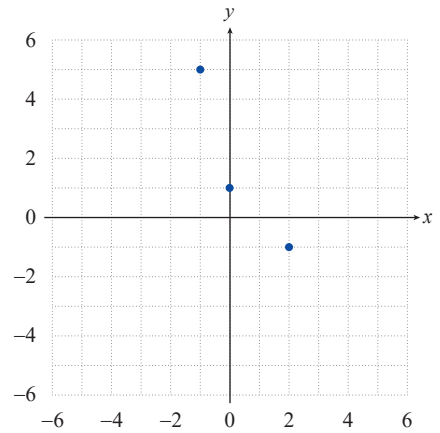
38. $h(x)=x^2+2x-3$

39. $f(x)=-x^2+5$

40. For the parabolic graph, **a.** find a formula for the corresponding quadratic function, and **b.** use the formula to determine the coordinates of the parabola's vertex.



41. Given the points graphed in the following figure, **a.** find the quadratic function that best fits the points, and **b.** use your result to determine the coordinates of the vertex of the best-fitting parabola.



42. The total revenue for McDaniel's Storage Plus is given as the function

$$R(x) = -0.4x^2 + 100x - 5250,$$

where x is the number of storage units rented. What number of units rented produces the maximum revenue?

Section 4.4

Sketch the graphs of the following functions. Pay particular attention to intercepts, if any, and locate these accurately.

43. $f(x) = -4|x|$

44. $g(x) = 3\sqrt{x}$

45. $r(x) = \dfrac{1}{x^2}$

46. $p(x) = -2x^4$

47. $q(x) = -\dfrac{1}{x^3}$

48. $k(x) = \dfrac{\sqrt[3]{x}}{2}$

49. $f(x) = 4x^3$

50. $f(x) = -\dfrac{2}{x^2}$

51. $f(x) = \left[\!\left[\dfrac{2x}{3}\right]\!\right]$

52. $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \dfrac{1}{x} & \text{if } x \geq 1 \end{cases}$

53. $g(x) = \begin{cases} (x+1)^2 - 1 & \text{if } x \leq 0 \\ \sqrt[3]{x} & \text{if } x > 0 \end{cases}$

54. $h(x) = \begin{cases} -|x| & \text{if } x < 3 \\ (x-4)^2 + 1 & \text{if } x \geq 3 \end{cases}$

55. $f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\ \dfrac{1}{x^2} & \text{if } x > -2 \end{cases}$

56. $q(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ x^4 & \text{if } x \geq 1 \end{cases}$

57. $g(x) = \begin{cases} 2|x| & \text{if } x < 2 \\ \sqrt{x} & \text{if } x \geq 2 \end{cases}$

Section 4.5

Find the mathematical model for each of the following verbal statements.

58. The quantity V varies directly as the product of r squared and h.

59. The value of y varies directly as the cube of a and inversely as the square root of b.

Solve the following variation problems.

60. Suppose that y varies directly as the square of x and that $y = 567$ when $x = 9$. What is y when $x = 4$?

61. Suppose that y is inversely proportional to the square root of x and that $y = 45$ when $x = 64$. What is y when $x = 25$?

62. A video store manager observes that the number of videos rented seems to vary inversely as the price of a rental. If the store's customers rent 1050 videos per month when the price per rental is \$3.49, how many videos per month does he expect to rent if he lowers the price to \$2.99?

Section 4.6

63. Determine the approximate distance between Earth, which has a mass of approximately 6.4×10^{24} kg, and an object that has a mass of 6.42×10^{22} kg, if the gravitational force between them equals approximately 4.95×10^{21} N. Remember, $F=\dfrac{km_1m_2}{d^2}$ and the universal gravitational constant equals $6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

64. **a.** Find a model for the volume of a cylindrical can in terms of the can's radius r, given that the surface area of the rectangle used to make the cylindrical portion of the can is constrained to be 100 in^2.

b. Find the height of such a can if the volume is to be 150 in^3.

65. Robert is planning to build a fenced rectangular garden by the side of a road, with fencing that costs \$8/ft for the length along the road and fencing that costs \$5/ft for the other three sides. He wants the garden to have an area of 1200 ft^2.

a. Find a model for the total cost of the fencing.

b. Estimate the size of the garden that will minimize the total cost of the fencing and estimate that minimum cost.

66. Carlotta throws a baseball straight up as hard as she can. It reaches a maximum height of 30 meters, and the table below shows its height in quarter-second intervals from that point on.

a. Graph the heights (either by hand or with a graphing utility) and estimate the time the ball hits the ground.

b. Find the linear function of best fit that models the height of the ball, and graph the function along with the given heights. By the linear model, what is the extrapolated time when the ball hits the ground? What is the calculated linear-model height of the ball at time $t=0$?

c. Find the quadratic function of best fit that models the height of the ball, and graph the function along with the given heights. By the quadratic model, what is the extrapolated time when the ball hits the ground? What is the calculated quadratic-model height of the ball at time $t=0$?

Time t (in seconds)	Height (in meters)
0	30
0.25	29.4
0.5	27.6
0.75	24.5
1.0	20.2
1.25	14.7
1.5	8.0

CHAPTER 5 REVIEW EXERCISES

Section 5.1

Sketch the graphs of the following functions by first identifying the more basic functions that have been shifted, reflected, stretched, or compressed. Then determine the domain and range of each function.

1. $f(x)=(x-1)^3+2$

2. $G(x)=4|x+3|$

3. $m(x)=\dfrac{1}{(x+2)^2}$

4. $g(x)=-\sqrt[3]{x}+4$

5. $r(x)=\dfrac{1}{x-2}-3$

6. $f(x)=\sqrt{x-1}+3$

7. $g(x)=\sqrt{\dfrac{x}{2}}+1$

8. $f(x)=-\sqrt{-4x}$

Write a formula for each of the functions described below.

9. Use the function $g(x)=x^2$. Move the function 1 unit to the right and 2 units down.

10. Use the function $g(x)=|x|$. Move the functions 3 units to the right and reflect across the x-axis.

11. Use the function $g(x)=\sqrt{x}$. Reflect the function across the x-axis and move it 4 units up.

Section 5.2

Determine if each of the following relations is a function. If so, determine whether it is even, odd, or neither. Also determine if it has y-axis symmetry, x-axis symmetry, origin symmetry, or none of the above.

12. $y=|2x-1|$

13. $y=\dfrac{1}{x^2}+1$

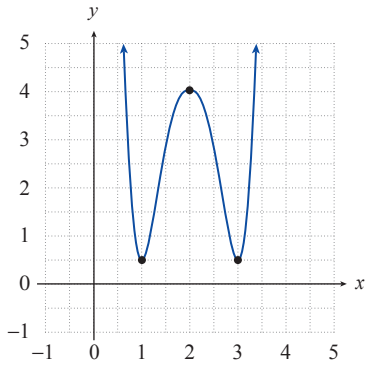
14. $x=-5|y|$

For each of the following functions, find the open intervals of monotonicity where the function is increasing, decreasing, or constant.

15. $f(x)=(x-2)^4-6$

16. $R(x)=\begin{cases}(x+2)^2 & \text{if } x<-1\\ -x & \text{if } x\geq -1\end{cases}$

17. Given the following graph of a function determine, **a.** the locations and types of the local extrema, and **b.** the values of the local extrema.



For each given function and interval, determine the average rate of change of the function over the interval.

18. $f(x)=x^2;\ [3,4]$

19. $f(x)=\frac{1}{x};\ [1,3]$

20. $f(x)=\sqrt{x};\ [1,4]$

21. $f(x)=x^2-x^3;\ [-1,2]$

Section 5.3

In each of the following exercises, use the information given to determine **a.** $(f+g)(2)$, **b.** $(f-g)(2)$, **c.** $(fg)(2)$, and **d.** $\left(\frac{f}{g}\right)(2)$.

22. $f(x)=-x^2+x$ and $g(x)=\frac{1}{x}$

23. $f(2)=4$ and $g(2)=-1$

24. $f(x)=\sqrt{2x}$ and $g(x)=x+3$

25. $f=\{(0,4),(2,8)\}$ and $g=\{(-2,2),(0,3),(2,-10)\}$

In each of the following exercises, find **a.** the formula and domain for $f+g$ and **b.** the formula and domain for $\frac{f}{g}$.

26. $f(x)=x^2$ and $g(x)=\sqrt{x}$

27. $f(x)=\frac{1}{x-2}$ and $g(x)=\sqrt[3]{x}$

28. $f(x)=3x$ and $g(x)=(x-1)^2$

29. $f(x)=x^2-4$ and $g(x)=\sqrt[3]{x}-1$

In each of the following exercises, use the information given to determine $(f\circ g)(3)$.

30. $f(x)=-x+1$ and $g(x)=-x-1$

31. $f(x)=\frac{x^{-1}}{18}-3$ and $g(x)=\frac{x-4}{x^3}$

32. $f(-3)=4$ and $g(3)=-3$

33. $f(x)=\frac{x}{3}$ and $g(x)=-\sqrt{x+1}$

In each of the following exercises, find **a.** the formula and domain for $f\circ g$ and **b.** the formula and domain for $g\circ f$.

34. $f(x)=4x-1$ and $g(x)=x^3+2$

35. $f(x)=\frac{1}{\sqrt{x-4}}$ and $g(x)=x+2$

36. $f(x)=2x^2+1$ and $g(x)=x-4$

37. $f(x)=3x$ and $g(x)=\sqrt{x-3}$

Write each of the following functions as a composition of two functions. Answers may vary.

38. $f(x)=\frac{3}{3x^2+1}$

39. $f(x)=\frac{\sqrt{x+2}}{x^2+4x+4}$

In each of the following exercises, use the information given to find $g(x)$.

40. $f(x)=6x-1$ and $(f\circ g)(x)=x+3$

41. $f(x)=\sqrt{x}+3$ and $(g\circ f)(x)=\frac{2}{\sqrt{x}+3}+1$

Section 5.4

Graph the inverse of each of the following relations, and state its domain and range.

42. $R=\{(3,4),(-1,5),(0,2),(-6,-1)\}$

43. $y=3x+1$

44. $y=\frac{\sqrt{x}}{2}$

Find a formula for the inverse of each of the following functions.

45. $r(x)=\dfrac{2}{7x-1}$ **46.** $g(x)=\dfrac{4x-3}{x}$ **47.** $f(x)=x^{\frac{1}{5}}-6$

48. $p(x)=2\sqrt{x-1}+3$ **49.** $f(x)=\dfrac{2x-3}{x+1}$ **50.** $f(x)=\sqrt[3]{x+2}-1$

51. $f(x)=8x+3$ **52.** $f(x)=(x-1)^2-3,\ x\geq 1$

Verify that $f\left(f^{-1}(x)\right)=x$ and that $f^{-1}\left(f(x)\right)=x$.

53. $f(x)=\dfrac{6x-7}{2-x}$ and $f^{-1}(x)=\dfrac{2x+7}{6+x}$

CHAPTER 6 REVIEW EXERCISES

Section 6.1

Verify that the given values of x solve the corresponding polynomial equations.

1. $4x^3 - 5x^2 = -3x + 18;\ x = 2$

2. $x^2 - 6x = -13;\ x = 3 + 2i$

3. $x^3 + x = 6x^2 - 164;\ x = 5 - 4i$

4. $x^3 + (1+4i)x = (7-2i)x^2 - 2i + 36;\ x = -2i$

Solve the following polynomial equations by factoring and/or using the quadratic formula, making sure to identify all the solutions.

5. $x^4 - 7x^2 + 10 = 0$

6. $x^5 - x^3 - 2x = 0$

7. $x^4 + 4 = 4x^2$

8. $6x^2 + 8x = -x^3$

9. $x^4 + x^3 = x^2$

10. $x^2 + 4x + 7 = 0$

For each of the following polynomial functions, determine the behavior of its graph as $x \to \pm\infty$ and identify the x- and y-intercepts. Use this information to then sketch the graph of each polynomial.

11. $f(x) = (x+2)(x-1)(x-3)$

12. $f(x) = (x-2)^2(x+1)^2$

13. $g(x) = x^2 - 5x + 4$

14. $h(x) = -x^3 - 7x^2 - 10x$

Solve the following polynomial inequalities.

15. $2x^2 + 15 \le 11x$

16. $(x-3)^2(x+1)^2 > 0$

17. $(x-4)(x+2)(x^2-1) \le 0$

18. $x^3 - 2x^2 - 8x \ge 0$

19. $x^2(x-2)(1-x) < 0$

20. $-3x^2 + 7x - 2 > 0$

21. A manufacturer has determined that the revenue from the sale of x video games is given by $r(x) = -x^2 + 12x$. The cost of producing x video games is $C(x) = 120 - 22x$. Given that profit is revenue minus cost, what value(s) for x will give the company a nonnegative profit?

Section 6.2

Use polynomial long division to rewrite each of the following fractions in the form $q(x) + \dfrac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

22. $\dfrac{8x^4 - 6x^3 + 2x^2 + 3x + 4}{2x^2 - 1}$

23. $\dfrac{11x^2 + 2x - 5}{x - 3}$

24. $\dfrac{x^4 - 3x^2 + x - 8}{x^2 + 3x + 2}$

25. $\dfrac{2x^5 - 4x^3 - x^2 + x - 2}{x^2 - x}$

26. $\dfrac{2x^3 + ix^2 - 12x - 4 + i}{2x + i}$

Use synthetic division to determine if the given value for c is a zero of the corresponding polynomial. If not, determine $p(c)$.

27. $p(x) = 6x^5 - 23x^4 - 95x^3 + 70x^2 + 204x - 72;\ \ c = 1$

28. $p(x) = 48x^4 + 10x^3 - 51x^2 - 10x + 3;\ \ c = \frac{1}{6}$

29. $p(x) = 18x^5 - 87x^4 + 110x^3 - 28x^2 - 16x + 3;\ \ c = \frac{2}{3}$

Use synthetic division to rewrite each of the following fractions in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

30. $\dfrac{x^4 - 2x^3 - x^2 + x - 21}{x - 3}$

31. $\dfrac{-x^4 - x^3 - x^2 + 2x + 69}{x + 3}$

32. $\dfrac{x^5 + 2x^4 + 3x^3 + 6x^2 - 5x + 13}{x + 2}$

33. $\dfrac{-x^4 + 8x^3 - 6x^2 - 4x + 2}{x - 1}$

34. $\dfrac{x^4 + (4 - 2i)x^3 - (1 + 8i)x^2 + (3 + 2i)x - 6i}{x - 2i}$

Construct a polynomial function with the stated properties.

35. Second-degree, zeros of -2 and 6, and goes to ∞ as $x \to \infty$.

36. Fourth-degree and a single x-intercept of -4 and y-intercept $(0, 128)$.

37. Third-degree, zeros of ± 2 and 3 and passing through the point $(4, 24)$.

Section 6.3

List all of the potential rational zeros of the following polynomials. Then use polynomial division and the quadratic formula, if necessary, to identify the actual zeros.

38. $f(x) = x^4 + 3x^3 - 3x^2 - 11x - 6$

39. $g(x) = 2x^3 - 11x^2 + 18x - 9$

40. $h(x) = 2x^3 + 2x^2 - 9x + 9$

41. $p(x) = x^4 + 8x^3 + 22x^2 + 24x + 9$

Using the Rational Zero Theorem or your answers to the preceding problems, solve the following polynomial equations.

42. $2x^4 - 6x^2 = -6x^3 + 22x + 12$

43. $2x^3 - 9x^2 + 18x = 9 + 2x^2$

44. $2x^3 + 9 = 9x - 2x^2$

45. $x^4 - x^5 = -x^5 - 8x^3 - 22x^2 - 24x - 9$

Use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of each of the following polynomials.

46. $f(x) = 2x^4 - 3x^3 - x^2 + 3x + 10$

47. $g(x) = x^6 - 4x^5 - 2x^4 + x^3 - 6x^2 - 11x + 6$

Use synthetic division to identify integer upper and lower bounds of the real zeros of the following polynomials.

48. $f(x) = 2x^3 - 11x^2 + 3x + 36$

49. $g(x) = 4x^3 - 16x^2 - 79x - 35$

Using your answers to the preceding problems, polynomial division, and the quadratic formula, if necessary, find all of the zeros of the following polynomials.

50. $f(x) = 2x^3 - 11x^2 + 3x + 36$

51. $g(x) = 4x^3 - 16x^2 - 79x - 35$

Use the Intermediate Value Theorem to show that each of the following polynomials has a real zero between the indicated values.

52. $f(x) = 2x^4 - 6x^3 + x - 5;\ -2 \text{ and } 0$

53. $f(x) = -x^3 + 3x^2 + x - 3;\ 2 \text{ and } 4$

Find all of the real zeros of the following functions.

54. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

55. $g(x) = x^3 - 4x^2 + 9x - 36$

56. $f(x) = x^3 + 6x^2 + 11x + 6$

57. $f(x) = x^3 - 7x^2 + 13x - 3$

Solve the following equations.

58. $x^4 - 2x^3 + 10x^2 = 9(2x - 1)$

59. $2x^3 = 7x^2 - 4x - 4$

60. $-8 = 3x^3 + 4x^2 + 6x$

Section 6.4

Throughout these exercises, a graphing utility may be helpful in identifying zeros and in checking your graphs.

Sketch the graph of each factored polynomial.

61. $f(x) = (x+4)^2(x-1)$

62. $g(x) = x(x-3)(x+4)^3$

Factor each of the following polynomials completely, and then sketch the graph of each one.

63. $f(x) = x^3 - 3x^2 + x - 3$

64. $f(x) = x^5 - x^4 - 2x^3 - x^2 + x + 2$

Solve each polynomial equation.

65. $3x^5 + x^4 + 5x^3 = x^2 + 28x + 20$

66. $8x^5 + 12x^4 - 18x^3 - 35x^2 = 18x + 3$

67. $x^5 + 3x^4 + 3x^3 + 9x^2 = 4(x+3)$

Factor each of the following polynomials completely, making use of the given zero.

68. $f(x)=14x^4-109x^3+296x^2-321x+70;\ 2+i$ is a zero

69. $f(x)=x^4-5x^3+19x^2-125x-150;\ -5i$ is a zero

70. $f(x)=2x^4+3x^3-7x^2+8x+6;\ 1+i$ is a zero

71. $f(x)=4x^3+10x^2-x+15;\ -3$ is a zero

Construct polynomial functions with the stated properties.

72. Fourth-degree, only real coefficients, $\frac{1}{2}$ and $1+2i$ are two of the zeros, y-intercept is -30, leading coefficient is 2.

73. Fifth-degree, only real coefficients, -1 is a zero of multiplicity 3, $\sqrt{6}$ is a zero, y-intercept is -6, leading coefficient is 1.

74. Fifth-degree, only real coefficients, 1 is a zero of multiplicity 3, $\sqrt{3}$ is a zero, y-intercept is 3, leading coefficient is 1.

Section 6.5

Find equations for the vertical asymptotes, if any, for each of the following rational functions.

75. $f(x)=\dfrac{4}{2x-5}$

76. $f(x)=\dfrac{x^2-3x+2}{x-1}$

77. $f(x)=\dfrac{x^2-1}{x-x^2}$

78. $f(x)=\dfrac{x^2-x-6}{x^2-6x+9}$

Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions.

79. $f(x)=\dfrac{2x^3+5x^2-1}{x^2-2x}$

80. $f(x)=\dfrac{x^2-x+8}{3x^2-7}$

81. $f(x)=\dfrac{x^2-9}{x+3}$

82. $f(x)=\dfrac{x^2+2x-3}{(x+1)^3}$

Sketch the graphs of the following rational functions.

83. $f(x)=\dfrac{2x}{x+1}$

84. $f(x)=\dfrac{4x^2}{x^2+3x}$

85. $f(x)=\dfrac{x^2+2}{x+2}$

86. $f(x)=\dfrac{x+1}{x^2-4}$

Solve the following rational inequalities.

87. $\dfrac{7}{x+3}\geq\dfrac{2x}{x+3}$

88. $\dfrac{x}{x^2-5x+6}\leq\dfrac{3}{x^2-5x+6}$

89. $\dfrac{x-4}{x+3}<\dfrac{x}{x-2}$

90. $\dfrac{x-2}{x+3}<2$

CHAPTER 7 REVIEW EXERCISES

Section 7.1

Sketch the graphs of the following functions. State their domain and range.

1. $f(x) = \left(\frac{1}{2}\right)^{x-1} + 3$

2. $r(x) = 2^{-x+4} - 2$

3. $h(x) = 3^x$

4. $f(x) = 1 - 2^{-x}$

5. $p(x) = \left(\frac{1}{4}\right)^x$

6. $s(x) = (0.2)^{x-2}$

7. $g(x) = 4 - 2^x$

8. $m(x) = \frac{1}{2^x} - 3$

9. $f(x) = \frac{1}{2^{4-x}}$

10. $r(x) = \left(\frac{9}{2}\right)^{3-x}$

Solve the following exponential equations.

11. $3^x = 243$

12. $2^{-x} = 16$

13. $0.5^x = 0.25$

14. $3^{3x-5} = 81$

15. $\left(\frac{2}{5}\right)^{-4x} = \left(\frac{25}{4}\right)^{x-1}$

16. $10{,}000^x = 10^{-2x-12}$

17. $9^{x-1} = 27^{-x+2}$

18. $\left(\frac{1}{3}\right)^{x-1} = 81^{\frac{1}{2}}$

19. $5^{3x-6} = 1$

Section 7.2

20. Melissa has recently inherited $15,000 that she wants to deposit into a savings account for 10 years. She has determined that her two best bets are an account that compounds annually at a rate of 3.95% and an account that compounds continuously at an annual rate of 3.85%. Which account would pay Melissa more interest?

21. Bill has come upon a 37-gram sample of iodine-131. He isolates the sample and waits for 2 weeks. After this time period, only 11 grams of iodine-131 remain. What is the half-life of this isotope?

22. Katherine is working in a lab testing bacteria populations. Starting out with a population of 870 bacteria, she notices that the population doubles every 22 minutes. Find **a.** the equation for the population P in terms of time t in minutes, and **b.** the time it would take for the population to reach 7500 bacteria.

23. The number of fruit flies in an experimental population after t hours is given by $Q(t) = 20e^{0.03t}$, $t \geq 0$.

a. How large is the population of fruit flies after 72 hours?
b. Find the initial number of fruit flies in the population.

24. In 1986, a nuclear reactor accident occurred in Chernobyl in what was then the Soviet Union. The explosion spread radioactive chemicals over hundreds of square miles and the government evacuated the city and surrounding areas. To see why the city is now uninhabited, consider the model $P = 10e^{-0.00002845t}$.

This model represents the amount of plutonium that remains (from an initial amount of 10 pounds) after t years. Sketch the graph of this function over the interval from $t = 0$ to $t = 100{,}000$. How much of the 10 pounds will remain after 100,000 years?

Section 7.3

Write the following equations in logarithmic terms.

25. $3^x = 8$ **26.** $(3a)^{\sqrt{2}} = 10$ **27.** $4^{3a} = 4096$

Write the following logarithmic equations as exponential equations.

28. $\log_4 64 = x$ **29.** $\log_3\left(\frac{1}{27}\right) = -3$ **30.** $\log_8(2A) = 3$

Sketch the graphs of the following functions. State their domain and range.

31. $f(x) = -\log_3(-x)$ **32.** $f(x) = \log_{\frac{1}{2}} x$ **33.** $m(x) = \log_{\frac{1}{2}}(x+2)$

Evaluate the following logarithmic expressions without the use of a calculator.

34. $\log_{27} 9^{\log 1000}$ **35.** $\log_{\frac{1}{3}} 9$ **36.** $\log_4\left(\frac{1}{64}\right)$

37. $\log_{\frac{1}{2}} 8$ **38.** $\log_{\sqrt{3}}\left(\frac{1}{3}\right)$ **39.** $\ln\left(\sqrt[3]{e^2}\right)$

Use the elementary properties of logarithms to solve the following equations.

40. $\log_6 6^{\log_5 x} = 3$ **41.** $\log_9 x^{\frac{1}{2}} = \frac{3}{4}$

42. $\log_x\left(\log_{\frac{1}{2}}\left(\frac{1}{16}\right)\right) = 2$ **43.** $\log_4(2x-1) = 2$

Solve the following logarithmic equations, using a calculator if necessary to evaluate the logarithms. Express your answer either as a fraction or a decimal rounded to two decimal places.

44. $\ln(4x) = 3.2$ **45.** $\ln(x-7) = 5$ **46.** $\log_7(4x-3) = 4$

Section 7.4

Use the properties of logarithms to expand the following expressions as much as possible. Simplify any numerical expressions that can be evaluated without a calculator.

47. $\log\sqrt{\dfrac{x^3}{4\pi^5}}$

48. $\ln\left(\dfrac{\sqrt{a^5}mn^2}{e^5}\right)$

49. $\log_3\left(27a^3\right)$

50. $\ln\left(\ln\left(e^{2ex}\right)\right)$

Use the properties of logarithms to condense the following expressions as much as possible, writing each answer as a single term with a coefficient of 1.

51. $\dfrac{1}{3}\left(\log_2\left(a^5\right)-\log_2\left(bc^3\right)\right)$

52. $\ln 4-\ln\left(x^2\right)-7\ln y$

53. $\log_2\left(x^2-9\right)-\log_2\left(x+3\right)$

54. $2\log a+3\log b-\dfrac{1}{2}\log c-\log d$

55. $\log_3\left(x-2\right)+\log_3 x-\log_3\left(x^2+4\right)$

Use the properties of logarithms to write each of the following as a single term that does not contain a logarithm.

56. $6^{3\log_6 x}$

57. $5^{\log_5 x-2\log_5 y}$

Evaluate the following logarithmic expressions.

58. $\log_3 17$

59. $\log_{1.4} 8$

60. $4\log_{\frac{1}{2}} 3$

Without using a calculator, evaluate the following expressions.

61. $\ln\left(\dfrac{1}{e^2}\right)+\ln e^2$

62. $\log_4\left(64^2\right)$

63. On the Richter scale, the magnitude R on a earthquake of intensity I is given by $R=\log\dfrac{I}{I_0}$, where $I_0=1$ is the minimum intensity used for comparison. Find the intensity per unit of area for the following values of R.

a. $R=8.4$ **b.** $R=6.85$ **c.** $R=9.1$

Section 7.5

Solve the following exponential and logarithmic equations. When appropriate, write the answer as both an exact expression and as a decimal approximation. Round your answer to two decimal places if necessary.

64. $e^{8-5x}=16$

65. $10^{\frac{6}{x}}=321$

66. $7^{\frac{x}{3}-4}=19$

67. $e^{4x}=5^{3x+1}$

68. $24=3e^{x+2}$

69. $3^{2x-1}=2^{2-x}$

70. $\ln(x+1)+\ln(x-1)=\ln(x+5)$

71. $\log_2(x+3)+\log_2(x+4)=\log_2(3x+8)$

72. $\log_5(8x-3)=3$

73. $\log_7(4x)-\log_7 6=2$

74. $\ln(5x+8)=\ln(40-3x)$

Using the properties of logarithmic functions, simplify the following functions as much as possible. Write each function as a single term with a coefficient of 1, if possible.

75. $f(x)=0.75\ln x^4$

76. $f(x)=6\log\sqrt{2x}$

77. $f(x)=4\log x^3-\log x^2$

78. $f(x)=0.5\ln(9x^6)$

79. $f(x)=2\log 7^{\log_9 3}$

80. $f(x)=2\ln 3^{\log_4 8}$

81. Rick puts \$6500 in a high interest money market account at 4.36% annual interest compounded monthly. Assuming he makes no deposits or withdrawals, how long will it take for his investment to grow to \$7000?

82. Sodium-24 has a half-life of approximately 15 hours. How long would it take for 350 grams of sodium-24 to decay to 12 grams?

CHAPTER 8 REVIEW EXERCISES

Section 8.1

Find the center, foci, and vertices of the ellipse that each equation describes.

1. $(x-3)^2+4(y+1)^2=16$

2. $9x^2+4y^2+18x-16y+9=0$

Sketch the graphs of the following ellipses and determine the coordinates of the foci.

3. $\dfrac{(x+1)^2}{16}+\dfrac{(y-2)^2}{9}=1$

4. $x^2+9y^2-6x+18y=-9$

5. $3x^2+y^2=27$

6. $25x^2+4y^2-200x+300=0$

In each of the following exercises, an ellipse is described by either a picture or by the properties it possesses. Find the equation, in standard form, for each ellipse.

7. Center at $(-1,4)$, major axis is vertical and of length 8, foci $\sqrt{7}$ units from the center.

8. Foci at $(1,2)$ and $(7,2)$, $e=\dfrac{1}{2}$.

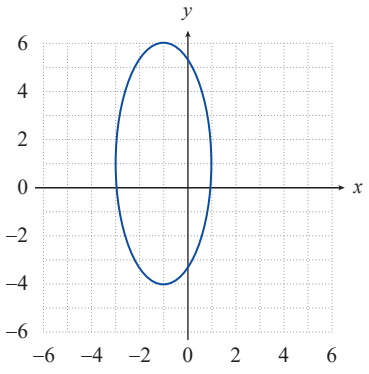
9. Vertices at $\left(\dfrac{7}{2},-1\right)$ and $\left(\dfrac{1}{2},-1\right)$, $e=0$.

10. Vertices at $(1,-8)$ and $(1,2)$, minor axis of length 6.

11. Foci at $(0,0)$ and $(4,0)$, major axis of length 8.

12. Center at $(0,4)$, $a=2c$, and vertices at $(-4,4)$ and $(4,4)$.

13.



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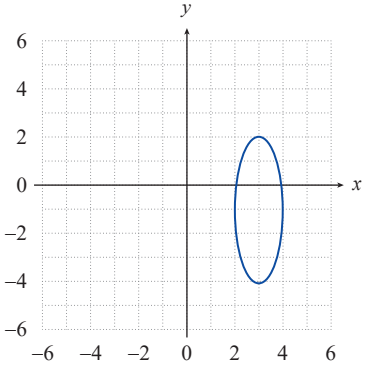


For exercises 15 and 16, use the fact that the area A of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is $A=\pi\cdot a\cdot b$ and $a+b=30$.

15. Write the area of the ellipse as a function of a.

16. Find the equation of an ellipse with an area of 200π square inches.

Section 8.2

Graph the following parabolas and determine the focus and directrix of each.

17. $(y+1)^2 = -12(x+3)$

18. $y^2 - 8y + 2x + 14 = 0$

19. $y^2 + 2y = 4x - 1$

20. $x + \frac{1}{4}y^2 = 0$

21. $2y + 4x^2 = 8$

22. $y^2 - 4y + 2x + 24 = 0$

Find the equation, in standard form, for the parabola with the given properties or with the given graph.

23. Vertex at $(-2,3)$, directrix is the line $y = 2$.

24. Vertex at $(5,-3)$, focus at $(5,1)$.

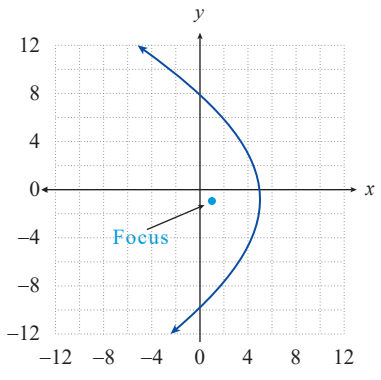
25. Focus at $(3,-1)$, directrix is the line $x = 2$.

26. Focus at $(1,-2)$, directrix is the x-axis.

27. Vertex at $(2,-1)$, directrix is the line $x = -2$.

28. Symmetric with respect to the x-axis, focus at $(-3,0)$, and $p = 4$.

29.



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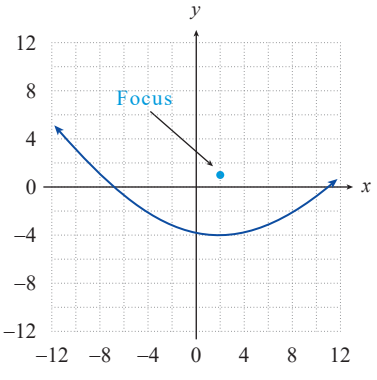


31. A motorcycle headlight is made by placing a strong light bulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 5y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

Section 8.3

Sketch the graphs of the following hyperbolas, using asymptotes as guides. Determine the coordinates of the foci in each case.

32. $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

33. $9x^2 - 4y^2 + 54x - 8y + 41 = 0$

34. $x^2 - y^2 = 1$

35. $\frac{y^2}{25} - \frac{x^2}{144} = 1$

Find the center, foci, and vertices of the hyperbola that each equation describes.

36. $(x+1)^2 - 4(y-2)^2 = 36$

37. $x^2 - 9y^2 + 36y - 72 = 0$

38. $y^2 - 4x^2 - 2y - 32x = 67$

39. $\dfrac{(y-3)^2}{4} - \dfrac{(x-3)^2}{49} = 1$

Find the equation, in standard form, for the hyperbola with the given properties or with the given graph.

40. Vertices at $(4,-1)$ and $(-2,-1)$ and foci at $(5,-1)$ and $(-3,-1)$.

41. Asymptotes of $y = \pm\dfrac{5}{2}(x+1) - 2$ and vertices at $(-3,-2)$ and $(1,-2)$.

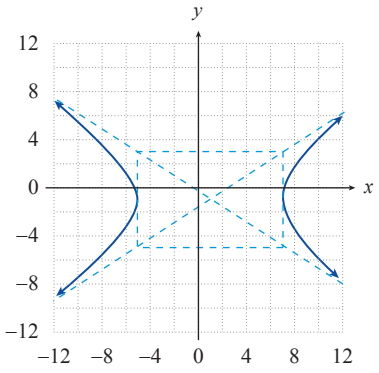
42. Foci at $(-1,-2)$ and $(-1,8)$ and asymptotes of $y = \pm\left(\dfrac{3}{4}x + \dfrac{3}{4}\right) + 3$.

43. Asymptotes of $y = \pm(3x-6) + 2$ and vertices at $(2,-1)$ and $(2,5)$.

44. Vertices at $(\pm 3, 0)$ and foci at $(\pm 5, 0)$.

45. Foci at $\left(-1, 7 \pm \sqrt{13}\right)$ and asymptotes of $y = \pm\dfrac{2}{3}(x+1) + 7$.

46.



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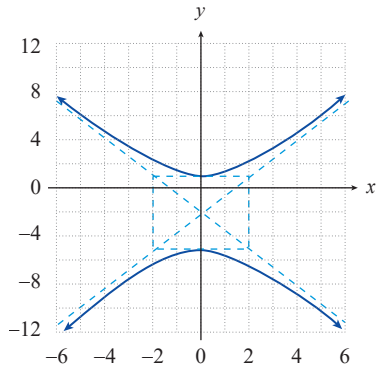

CHAPTER 9 REVIEW EXERCISES

Section 9.1

Use any convenient method to solve the following systems of equations. If a system is dependent, express the solution set in terms of one or more of the variables, as appropriate.

1. $\begin{cases} 3x-y+z=2 \\ -x+y-2z=-4 \\ -6x+2y-2z=-7 \end{cases}$

2. $\begin{cases} 2x-y=13 \\ 5x-2y-z=25 \\ 7x-6z=-2 \end{cases}$

3. $\begin{cases} x+y-z=1 \\ 3x-4y-5z=-1 \\ 6x-3y+z=20 \end{cases}$

4. $\begin{cases} 6x-5y=17 \\ -4x+9y=-17 \end{cases}$

5. $\begin{cases} 3x-y=2 \\ -6x+2y=5 \end{cases}$

6. $\begin{cases} 3x-2y=-10 \\ x+2y=2 \end{cases}$

7. $\begin{cases} \frac{x}{3}+y-1=0 \\ x+3y=3 \end{cases}$

8. $\begin{cases} \frac{x}{3}-\frac{y+1}{2}=1 \\ \frac{x}{2}-\frac{y}{4}=\frac{3}{4} \end{cases}$

9. $\begin{cases} x+y=5 \\ 2x-y=4 \\ 5x+y=17 \end{cases}$

10. $\begin{cases} 2x+3y-4z=-7 \\ x-y+4z=6 \\ x+y+z=2 \end{cases}$

11. $\begin{cases} 3x-2y+z=10 \\ x+y+z=30 \\ 2x-y-z=-6 \end{cases}$

12. $\begin{cases} 3x-2y-2z=-8 \\ x-y-z=-5 \\ x+y+z=-3 \end{cases}$

13. Find the equation of a parabola $y=ax^2+bx+c$, passing through the points $(1,0)$, $(-4,9)$, and $(-1,2)$.

Section 9.2

14. Let $A=\begin{bmatrix} 2 & -8 & 9 \\ 7 & 3 & 0 \\ 11 & 6 & 1 \end{bmatrix}$. Determine the following, if possible:

a. The order of A **b.** The value of a_{12} **c.** The value of a_{21}

15. Let $B=\begin{bmatrix} 13 & 8 & 20 & 5 \end{bmatrix}$. Determine the following, if possible:

a. The order of B **b.** The value of b_{12} **c.** The value of b_{31}

Construct the augmented matrix that corresponds to each of the following systems of equations. (Answers may appear in slightly different, but equivalent, forms.)

16. $\begin{cases} 2x+(y-z)=3 \\ 2(y-x)+y-2=z \\ 3x-\dfrac{3-z}{2}=4y \end{cases}$

17. $\begin{cases} z-4x=5y \\ 14z+7(x+3y)=21 \\ 8x-y=-2(x-3z) \end{cases}$

Construct the system of equations that corresponds to each of the following matrices.

18. $\left[\begin{array}{cc|c} 8 & -2 & 2 \\ -1 & 5 & 3 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} 8 & 0 & 7 & 5 \\ 0 & -3 & 4 & 16 \\ 16 & -2 & 1 & 2 \end{array}\right]$

20. $\left[\begin{array}{ccc|c} 3 & -7 & 6 & 9 \\ -11 & 0 & 3 & -14 \\ 0 & 0 & 8 & 2 \end{array}\right]$

Fill in the blanks by performing the indicated elementary row operations.

21. $\left[\begin{array}{cc|c} 3 & 1 & -2 \\ 1 & 2 & 3 \end{array}\right] \xrightarrow{-3R_2+R_1} \underline{?}$

22. $\left[\begin{array}{cc|c} 2 & 3 & 5 \\ -4 & -1 & 2 \end{array}\right] \xrightarrow{2R_1+R_2} \underline{?}$

23. $\left[\begin{array}{cc|c} 1 & -4 & -4 \\ 3 & -1 & 3 \end{array}\right] \xrightarrow{-2R_1+R_2} \underline{?}$

24. $\left[\begin{array}{ccc|c} -1 & 0 & 2 & -6 \\ 1 & -3 & 4 & 1 \\ -2 & -1 & -3 & 0 \end{array}\right] \xrightarrow[-R_1+R_3]{2R_2} \underline{?}$

Use Gaussian elimination and back-substitution to solve the following systems of equations.

25. $\begin{cases} 3x-y=7 \\ x-4y=6 \end{cases}$

26. $\begin{cases} \dfrac{x}{5}-\dfrac{y}{3}=2 \\ -6x+5y=20 \end{cases}$

Use Gauss-Jordan elimination to solve the following systems of equations.

27. $\begin{cases} 5x-4y=35 \\ 25x-18y=165 \end{cases}$

28. $\begin{cases} x-3y-4z=-5 \\ -x+7y+8z=17 \\ 2x-10y-12z=-10 \end{cases}$

Section 9.3

Evaluate the following determinants.

29. $\begin{vmatrix} x^3 & -x^2 \\ x^2 & x \end{vmatrix}$

30. $\begin{vmatrix} -1 & 3 & 1 \\ 1 & -4 & 0 \\ 0 & 2 & 3 \end{vmatrix}$

31. $\begin{vmatrix} -2 & -1 & -3 & 0 \\ 3 & 3 & 1 & 5 \\ 4 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{vmatrix}$

32. $\begin{vmatrix} x^4 & x & x & 2x \\ 0 & x & x^3 & x \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x^2 \end{vmatrix}$

Use the matrix $A=\begin{bmatrix} 0 & -3 & 1 \\ 2 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$ to evaluate the minor and cofactor of the following elements.

33. a_{12}

34. a_{31}

Use Cramer's Rule to solve each system of equations.

35. $\begin{cases} x+6y=2 \\ 3x-y=-13 \end{cases}$

36. $\begin{cases} x+2y-3z=-3 \\ -5x-y+4z=-5 \\ 3x+y+z=6 \end{cases}$

37. $\begin{cases} -4x+2y=3 \\ 2x-y=4 \end{cases}$

38. $\begin{cases} x-2y=0 \\ x+y+z=6 \\ 3x-y-4z=10 \end{cases}$

Section 9.4

Given $A=\begin{bmatrix} 2 & -8 & 3 \\ -1 & 0 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$, $C=\begin{bmatrix} 3 & 1 & -6 \\ 8 & -3 & -7 \end{bmatrix}$, and $D=\begin{bmatrix} 0 & 4 \\ -3 & 11 \\ 7 & 1 \end{bmatrix}$, determine the following, if possible.

39. BA

40. B^2

41. $CD+C$

42. BD

43. $3A+C$

44. $AD+B$

Determine values of the variables that will make the following equations true, if possible.

45. $\begin{bmatrix} w & 5x \\ 2y & z \end{bmatrix}-3\begin{bmatrix} w & x \\ 2 & -z \end{bmatrix}=\begin{bmatrix} 4 & 2 \\ y-3 & -16 \end{bmatrix}$

46. $\begin{bmatrix} 4x & 2y^2 & z \end{bmatrix}=\begin{bmatrix} 12 \\ 18 \\ -2 \end{bmatrix}$

47. $2\begin{bmatrix} x \\ -3y \end{bmatrix}-\begin{bmatrix} y \\ 2x \end{bmatrix}=\begin{bmatrix} 7 \\ 14 \end{bmatrix}$

48. $\begin{bmatrix} 3x \\ 5y \end{bmatrix}-\begin{bmatrix} y \\ -2x \end{bmatrix}=\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

Evaluate the following matrix products, if possible.

49. $\begin{bmatrix} 7 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 2 & 1 \\ -3 & -3 \end{bmatrix}$

50. $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\begin{bmatrix} -3 & 2 & 3 \end{bmatrix}$

Section 9.5

Write each of the following systems of equations as a single matrix equation.

51. $\begin{cases} x_1-x_2+2x_3=-4 \\ 2x_1-3x_2-x_3=1 \\ -3x_1+6x_3=5 \end{cases}$

52. $\begin{cases} 3x-y+z=4 \\ 2x-5z=1 \\ 4x+3y-6=0 \end{cases}$

Find the inverse of each of the following matrices, if possible.

53. $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$

54. $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$

55. $\begin{bmatrix} 4 & 12 \\ 3 & 9 \end{bmatrix}$

56. $\begin{bmatrix} -1 & 2 & 3 \\ 1 & -1 & 4 \\ 2 & 0 & -2 \end{bmatrix}$

For each pair of matrices, determine if either matrix is the inverse of the other.

57. $\begin{bmatrix} 3 & 12 \\ 2 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ \frac{2}{3} & 3 \end{bmatrix}$

58. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}, \begin{bmatrix} -2 & -1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

59. $\begin{bmatrix} -2 & 4 & -3 \\ 0 & 6 & -3 \\ 0 & 8 & -3 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{4}{3} & 1 \end{bmatrix}$

60. $\begin{bmatrix} 5 & -3 & 7 \\ 6 & 0 & 2 \\ -9 & 1 & 0 \end{bmatrix}, \begin{bmatrix} -9 & 2 & 1 \\ 7 & 0 & -3 \\ 1 & 8 & 2 \end{bmatrix}$

Solve the following systems by the inverse matrix method, if possible. If the inverse matrix method doesn't apply, use any other method to determine if the system is inconsistent or dependent.

61. $\begin{cases} 5x+9y=2 \\ -2x-3y=-1 \end{cases}$

62. $\begin{cases} 2y+3z=3 \\ -2x=0 \\ 8x+4y+5z=-1 \end{cases}$

Solve the following set of systems by the inverse matrix method.

63. $\begin{cases} 2x-z=3 \\ x+4y+2z=-1 \\ x+y=5 \end{cases}$ $\begin{cases} 2x-z=0 \\ x+4y+2z=2 \\ x+y=1 \end{cases}$ $\begin{cases} 2x-z=-1 \\ x+4y+2z=1 \\ x+y=2 \end{cases}$

Section 9.6

Graph the solution set of each of the following systems of inequalities.

64. $\begin{cases} 7x-2y \geq 8 \\ y<5 \end{cases}$

65. $\begin{cases} y-x>0 \\ x<2 \end{cases}$

Construct the constraints and graph the feasible regions for the following situations.

66. Each bag of nuts contains peanuts and cashews. The total number of nuts in the bag cannot exceed 60. There must be at least 20 peanuts and 10 cashews per bag. There can be no more than 40 peanuts or 40 cashews per bag. What is the region of constraint for the number of nuts per bag?

67. You wish to study at least 15 hours (over a 4-day span) for your upcoming statistics and biology tests. You need to study a minimum of 6 hours for each test. The maximum you wish to study for statistics is 10 hours and for biology is 8 hours. What is the region of constraint for the numbers of hours you should study for each test?

Find the minimum and maximum values of the given functions, subject to the given constraints.

68. Objective Function:
$f(x,y)=6x+10y$
Constraints:
$$\begin{cases} x\geq 0,\ y\geq 0 \\ 2x+5y\leq 10 \end{cases}$$

69. Objective Function:
$f(x,y)=5x+2y$
Constraints:
$$\begin{cases} x\geq 0,\ y\geq 0 \\ x+y\leq 10 \\ x+2y\geq 10 \\ 2x+y\geq 10 \end{cases}$$

70. Objective Function:
$f(x,y)=5x+4y$
Constraints:
$$\begin{cases} x\geq 0,\ y\geq 0 \\ 2x+3y\leq 12 \\ 3x+y\leq 12 \\ x+y\geq 2 \end{cases}$$

71. Objective Function:
$f(x,y)=70x+82y$
Constraints:
$$\begin{cases} x\geq 0,\ y\geq 0 \\ x\leq 10,\ y\leq 20 \\ x+y\geq 5 \\ x+2y\leq 18 \end{cases}$$

72. Krueger's Pottery manufactures two kinds of hand-painted pottery: a vase and a pitcher. There are three processes to create the pottery: throwing (forming the pottery on the potter's wheel), baking, and painting. No more than 90 hours are available per day for throwing, only 120 hours are available per day for baking, and no more than 60 hours per day are available for painting. The vase requires 3 hours for throwing, 6 hours for baking, and 2 hours for painting. The pitcher requires 3 hours for throwing, 4 hours for baking, and 3 hours for painting. The profit for each vase is $25 and the profit for each pitcher is $30. How many of each piece of pottery should be produced a day to maximize profit? What would the maximum profit be if Krueger's produced this amount?

73. Pranas produces bionic arms and legs for those that are missing a limb. Pranas can produce at least 20, but no more than 60 arms in a week due to the lab limitations. They can produce at least 15, but no more than 40 legs in a week. To keep their research grant, the company must produce at least 50 limbs per week. It costs $450 to produce the bionic arm and $550 to produce the bionic leg. How many of each should be produced per week to minimize the cost? What would the minimum cost be if Pranas produced this amount?

Section 9.7

Use graphing to approximate the real solution(s) of the following systems, and then verify that your answers are correct.

74. $\begin{cases} (x-2)^2 + y = 2 \\ x - y = 2 \end{cases}$

75. $\begin{cases} x^2 + y^2 = 25 \\ -x - y = 5 \end{cases}$

Solve the following systems of nonlinear equations. Be sure to check for nonreal solutions.

76. $\begin{cases} x^2 + 2y^2 = 1 \\ x^2 = y \end{cases}$

77. $\begin{cases} x^2 + y^2 = 25 \\ 2x^2 - y^2 = 23 \end{cases}$

78. $\begin{cases} y = (x-1)^2 \\ y + 8 = (x+1)^2 \end{cases}$

Draw the graph and determine whether the ordered pairs are solutions to the system of inequalities.

79. $\begin{cases} y^2 \le 9 - x^2 \\ y < |x| \\ y > -|x| \end{cases}$

a. $(2,5)$ **b.** $(7,8)$ **c.** $(5,0)$ **d.** $(3,4)$

Graph the following systems of inequalities.

80. $\begin{cases} y \le \sin x \\ y > -\sin x \end{cases}$

81. $\begin{cases} y \le \sqrt{x+1} \\ y > x^2 - 1 \end{cases}$

82. $\begin{cases} x^2 y \le 1 \\ 2y \le x^2 + 2 \\ y < 16x^2 \end{cases}$

83. The product of two positive integers is 144, and their sum is 25. What are the integers?

84. Stephen and Scott were driving the same 72-mile route, and they departed at the same time. After 30 minutes, Stephen was 6 miles ahead of Scott. If it took Scott one more hour than Stephen to reach their destination, how fast were they each driving?

CHAPTER 10 REVIEW EXERCISES

Section 10.1

Determine the first five terms of each sequence whose n^{th} term is defined as follows.

1. $a_n = (-3)^n$

2. $a_n = (-1)^n \sqrt[3]{n}$

3. $a_1 = -3, a_{n-1} = a_n + 1$ for $n \geq 2$

4. $a_n = \dfrac{n!}{n^n}$

Find a possible formula for the general n^{th} term of each sequence. Answers may vary.

5. $-7, -1, 5, 11, 17, \ldots$

6. $\dfrac{1}{2}, \dfrac{3}{4}, \dfrac{9}{8}, \dfrac{27}{16}, \dfrac{81}{32}, \ldots$

7. $0, 3, 8, 15, 24, 35, \ldots$

8. $\dfrac{3}{2}, \dfrac{5}{3}, \dfrac{7}{4}, \dfrac{9}{5}, \dfrac{11}{6}, \ldots$

9. $-2, -4, -12, -48, -240, \ldots$

10. $2, 6, 12, 20, 30, \ldots$

Translate each expanded sum that follows into summation notation, and vice versa. Then evaluate the sum.

11. $\displaystyle\sum_{i=3}^{8} (-2i + 3)$

12. $\displaystyle\sum_{i=2}^{7} (-2)^{i-1}$

13. $8 + 27 + 64 + \cdots + 343$

14. $\displaystyle\sum_{i=1}^{6} (2i - 3)$

15. $\displaystyle\sum_{i=1}^{5} -4(2^i)$

16. $8 + 18 + 32 + \cdots + 200$

Find a formula for the n^{th} partial sum S_n of each series. If the series is finite, determine the sum. If the series is infinite, determine if it converges or diverges, and if it converges, determine the sum.

17. $\displaystyle\sum_{i=1}^{80} \left(\frac{1}{i+1} - \frac{1}{i+2} \right)$

18. $\displaystyle\sum_{i=1}^{\infty} \left(\frac{1}{i+1} - \frac{1}{i+2} \right)$

19. $\displaystyle\sum_{i=1}^{\infty} (3^i - 3^{i+1})$

Determine the first five terms of each generalized Fibonacci sequence.

20. $a_1 = -2,\ a_2 = 5,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

21. $a_1 = -10,\ a_2 = -12,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

Section 10.2

Find the explicit formula for the general n^{th} term of each arithmetic sequence.

22. $5, 2, -1, -4, -7, \ldots$

23. $a_2 = 14$ and $a_4 = 19$

24. $a_7 = -43$ and $d = -9$

25. $a_1 = 2, a_4 = 11$

26. $a_9 = \dfrac{13}{2}, d = \dfrac{3}{4}$

27. $-5, 4, 13, 22, 31, \ldots$

Use the given information about each arithmetic sequence to answer the question.

28. Given that $a_1 = -2$ and $a_4 = -20$, what is a_{25}?

29. Given that $a_3 = 17$ and $a_7 = 29$, what is a_{89}?

30. In the sequence $8, 19, 30, \ldots$, which term is 668?

31. In the sequence $6, 1, -4, \ldots$, which term is -169?

32. In the sequence $\frac{8}{3}, \frac{10}{3}, 4, \ldots$, which term is $\frac{56}{3}$?

Find the value of the partial sum of each arithmetic sequence.

33. $\sum_{i=1}^{97} (2i - 7)$

34. $\sum_{i=1}^{60} (-4i + 3)$

35. Sylvia suspects that she has an ant infestation in her apartment. The first day she noticed them, she saw 10 ants in her kitchen. Each day she notices 4 more ants than on the previous day. If she doesn't call an exterminator, how many ants would she see on the fifteenth day?

Section 10.3

Find the explicit formula for the general n^{th} term of each geometric sequence.

36. $2, 8, 32, 128, 512, \ldots$

37. $3, \frac{3}{5}, \frac{3}{25}, \frac{3}{125}, \frac{3}{625}, \ldots$

38. $18, -6, 2, -\frac{2}{3}, \frac{2}{9}, \ldots$

39. $a_1 = 6$ and $a_4 = 384$

40. $a_2 = 20$ and $a_6 = 320$

41. $a_1 = 8$ and $a_4 = \frac{1}{8}$

Given the two terms of a geometric sequence, find the common ratio and first five terms of the sequence.

42. $a_1 = 4$ and $a_4 = 108$

43. $a_4 = \frac{5}{3}$ and $a_6 = \frac{20}{27}$

Use the given information about each geometric sequence to answer the question.

44. Given that $a_2 = \frac{3}{5}$ and $a_4 = \frac{1}{15}$, what is the common ratio r?

45. Given that $a_1 = 3$ and $a_4 = -24$, what is the common ratio r?

46. Given that $a_5 = -16$ and $a_6 = -4$, what is a_{11}?

Each of the following sums is a partial sum of a geometric sequence. Use this fact to evaluate the sums.

47. $\sum_{i=3}^{9} 3\left(\frac{1}{2}\right)^i$

48. $5 + 10 + \cdots + 20{,}480$

Determine if each of the following infinite geometric series converges. If so, find the sum.

49. $\sum_{i=0}^{\infty} -3\left(\frac{3}{4}\right)^i$

50. $\sum_{i=1}^{\infty} \left(-\frac{5}{4}\right)^i$

51. $\sum_{i=1}^{\infty} \frac{2}{5}\left(\frac{5}{7}\right)^i$

Section 10.4

Use the Principle of Mathematical Induction to prove the following statements.

52. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

53. $5+8+11+\cdots+(3n+2)=\frac{n(3n+7)}{2}$

54. $1\cdot 3+2\cdot 4+3\cdot 5+\cdots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$

55. For all natural numbers n, 11^n-7^n is divisible by 4.

56. For all natural numbers n, 7^n-1 is divisible by 3.

Section 10.5

Use the Multiplication Principle of Counting and the permutation and combination formulas to answer the following questions.

57. A license plate must contain 4 numerical digits followed by 3 letters. If the first digit cannot be 0 or 1, how many different license plates can be created?

58. How many different 7-digit phone numbers do not contain the digits 6 or 7?

59. In how many different orders can the letters in the word "aardvark" be arranged?

60. In how many different ways can first place, second place, and third place be awarded in a 10-person shot put competition?

61. At a meeting of 21 people, a president, vice president, secretary, treasurer, and recruitment officer are to be chosen. How many different ways can these positions be filled?

62. A college admissions committee selects 4 out of 12 scholarship finalists to receive merit-based financial aid. How many different sets of 4 recipients can be chosen?

Use the Binomial and Multinomial Theorems to expand each of the following expressions.

63. $(1-2y)^5$

64. $(x+2)^7$

65. $(5x^2-2y)^5$

66. $(x+2y+z)^3$

Section 10.6

Apply the formulas for the probabilities of intersection and union to the following sets and determine **a.** $P(E\cap F)$ and **b.** $P(E\cup F)$. Let $n(S)$ equal the size of the sample space.

67. $n(S)=9,\ E=\{3,5,7\},\ F=\{1,2,3,4\}$

68. $n(S)=6,\ E=\{A,B\},\ F=\{X,Y,Z\}$

69. $n(S)=7,\ E=\{\alpha,13\},\ F=\{\alpha,\beta,13,14\}$

70. $n(S)=8,\ E=\{a,4,m,7\},\ F=\{m,3,s\}$

71. $n(S)=10,\ E=\{3,4,X,Y,5,Z\},\ F=\{5,6,7\}$

72. A card is drawn from a standard 52-card deck. Find the probability of drawing
- **a.** a seven or a club.
- **b.** a face card but not a red queen.
- **c.** a black three or a spade.

73. What is the probability of being dealt a five-card hand (from a standard 52-card deck) that contains only face cards?

74. There is a 10% chance of rain each individual day for an entire week. What is the probability that it will rain at least once during this seven- day period?