CHAPTER 9 REVIEW EXERCISES

Section 9.1

Use any convenient method to solve the following systems of equations. If a system is dependent, express the solution set in terms of one or more of the variables, as appropriate.

1. $\begin{cases} 3x - y + z = 2 \\ -x + y - 2z = -4 \\ -6x + 2y - 2z = -7 \end{cases}$

2. $\begin{cases} 2x - y = 13 \\ 5x - 2y - z = 25 \\ 7x - 6z = -2 \end{cases}$

3. $\begin{cases} x + y - z = 1 \\ 3x - 4y - 5z = -1 \\ 6x - 3y + z = 20 \end{cases}$

4. $\begin{cases} 6x - 5y = 17 \\ -4x + 9y = -17 \end{cases}$

5. $\begin{cases} 3x - y = 2 \\ -6x + 2y = 5 \end{cases}$

6. $\begin{cases} 3x - 2y = -10 \\ x + 2y = 2 \end{cases}$

7. $\begin{cases} \frac{x}{3} + y - 1 = 0 \\ x + 3y = 3 \end{cases}$

8. $\begin{cases} \frac{x}{3} - \frac{y+1}{2} = 1 \\ \frac{x}{2} - \frac{y}{4} = \frac{3}{4} \end{cases}$

9. $\begin{cases} x + y = 5 \\ 2x - y = 4 \\ 5x + y = 17 \end{cases}$

10. $\begin{cases} 2x + 3y - 4z = -7 \\ x - y + 4z = 6 \\ x + y + z = 2 \end{cases}$

11. $\begin{cases} 3x - 2y + z = 10 \\ x + y + z = 30 \\ 2x - y - z = -6 \end{cases}$

12. $\begin{cases} 3x - 2y - 2z = -8 \\ x - y - z = -5 \\ x + y + z = -3 \end{cases}$

13. Find the equation of a parabola $y = ax^2 + bx + c$, passing through the points $(1,0)$, $(-4,9)$, and $(-1,2)$.

Section 9.2

14. Let $A = \begin{bmatrix} 2 & -8 & 9 \\ 7 & 3 & 0 \\ 11 & 6 & 1 \end{bmatrix}$. Determine the following, if possible:

a. The order of A **b.** The value of a_{12} **c.** The value of a_{21}

15. Let $B = \begin{bmatrix} 13 & 8 & 20 & 5 \end{bmatrix}$. Determine the following, if possible:

a. The order of B **b.** The value of b_{12} **c.** The value of b_{31}

Construct the augmented matrix that corresponds to each of the following systems of equations. (Answers may appear in slightly different, but equivalent, forms.)

16. $\begin{cases} 2x+(y-z)=3 \\ 2(y-x)+y-2=z \\ 3x-\dfrac{3-z}{2}=4y \end{cases}$

17. $\begin{cases} z-4x=5y \\ 14z+7(x+3y)=21 \\ 8x-y=-2(x-3z) \end{cases}$

Construct the system of equations that corresponds to each of the following matrices.

18. $\left[\begin{array}{cc|c} 8 & -2 & 2 \\ -1 & 5 & 3 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} 8 & 0 & 7 & 5 \\ 0 & -3 & 4 & 16 \\ 16 & -2 & 1 & 2 \end{array}\right]$

20. $\left[\begin{array}{ccc|c} 3 & -7 & 6 & 9 \\ -11 & 0 & 3 & -14 \\ 0 & 0 & 8 & 2 \end{array}\right]$

Fill in the blanks by performing the indicated elementary row operations.

21. $\left[\begin{array}{cc|c} 3 & 1 & -2 \\ 1 & 2 & 3 \end{array}\right] \xrightarrow{-3R_2+R_1} \underline{?}$

22. $\left[\begin{array}{cc|c} 2 & 3 & 5 \\ -4 & -1 & 2 \end{array}\right] \xrightarrow{2R_1+R_2} \underline{?}$

23. $\left[\begin{array}{cc|c} 1 & -4 & -4 \\ 3 & -1 & 3 \end{array}\right] \xrightarrow{-2R_1+R_2} \underline{?}$

24. $\left[\begin{array}{ccc|c} -1 & 0 & 2 & -6 \\ 1 & -3 & 4 & 1 \\ -2 & -1 & -3 & 0 \end{array}\right] \xrightarrow[-R_1+R_3]{2R_2} \underline{?}$

Use Gaussian elimination and back-substitution to solve the following systems of equations.

25. $\begin{cases} 3x-y=7 \\ x-4y=6 \end{cases}$

26. $\begin{cases} \dfrac{x}{5}-\dfrac{y}{3}=2 \\ -6x+5y=20 \end{cases}$

Use Gauss-Jordan elimination to solve the following systems of equations.

27. $\begin{cases} 5x-4y=35 \\ 25x-18y=165 \end{cases}$

28. $\begin{cases} x-3y-4z=-5 \\ -x+7y+8z=17 \\ 2x-10y-12z=-10 \end{cases}$

Section 9.3

Evaluate the following determinants.

29. $\begin{vmatrix} x^3 & -x^2 \\ x^2 & x \end{vmatrix}$

30. $\begin{vmatrix} -1 & 3 & 1 \\ 1 & -4 & 0 \\ 0 & 2 & 3 \end{vmatrix}$

31. $\begin{vmatrix} -2 & -1 & -3 & 0 \\ 3 & 3 & 1 & 5 \\ 4 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{vmatrix}$

32. $\begin{vmatrix} x^4 & x & x & 2x \\ 0 & x & x^3 & x \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x^2 \end{vmatrix}$

Use the matrix $A = \begin{bmatrix} 0 & -3 & 1 \\ 2 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$ to evaluate the minor and cofactor of the following elements.

33. a_{12}

34. a_{31}

Use Cramer's Rule to solve each system of equations.

35. $\begin{cases} x + 6y = 2 \\ 3x - y = -13 \end{cases}$

36. $\begin{cases} x + 2y - 3z = -3 \\ -5x - y + 4z = -5 \\ 3x + y + z = 6 \end{cases}$

37. $\begin{cases} -4x + 2y = 3 \\ 2x - y = 4 \end{cases}$

38. $\begin{cases} x - 2y = 0 \\ x + y + z = 6 \\ 3x - y - 4z = 10 \end{cases}$

Section 9.4

Given $A = \begin{bmatrix} 2 & -8 & 3 \\ -1 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 & -6 \\ 8 & -3 & -7 \end{bmatrix}$, and $D = \begin{bmatrix} 0 & 4 \\ -3 & 11 \\ 7 & 1 \end{bmatrix}$, determine the following, if possible.

39. BA

40. B^2

41. $CD + C$

42. BD

43. $3A + C$

44. $AD + B$

Determine values of the variables that will make the following equations true, if possible.

45. $\begin{bmatrix} w & 5x \\ 2y & z \end{bmatrix} - 3\begin{bmatrix} w & x \\ 2 & -z \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ y-3 & -16 \end{bmatrix}$

46. $\begin{bmatrix} 4x & 2y^2 & z \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \\ -2 \end{bmatrix}$

47. $2\begin{bmatrix} x \\ -3y \end{bmatrix} - \begin{bmatrix} y \\ 2x \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \end{bmatrix}$

48. $\begin{bmatrix} 3x \\ 5y \end{bmatrix} - \begin{bmatrix} y \\ -2x \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

Evaluate the following matrix products, if possible.

49. $\begin{bmatrix} 7 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 2 & 1 \\ -3 & -3 \end{bmatrix}$

50. $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\begin{bmatrix} -3 & 2 & 3 \end{bmatrix}$

Section 9.5

Write each of the following systems of equations as a single matrix equation.

51. $\begin{cases} x_1 - x_2 + 2x_3 = -4 \\ 2x_1 - 3x_2 - x_3 = 1 \\ -3x_1 + 6x_3 = 5 \end{cases}$

52. $\begin{cases} 3x - y + z = 4 \\ 2x - 5z = 1 \\ 4x + 3y - 6 = 0 \end{cases}$

Find the inverse of each of the following matrices, if possible.

53. $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$

54. $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$

55. $\begin{bmatrix} 4 & 12 \\ 3 & 9 \end{bmatrix}$

56. $\begin{bmatrix} -1 & 2 & 3 \\ 1 & -1 & 4 \\ 2 & 0 & -2 \end{bmatrix}$

For each pair of matrices, determine if either matrix is the inverse of the other.

57. $\begin{bmatrix} 3 & 12 \\ 2 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ \frac{2}{3} & 3 \end{bmatrix}$

58. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}, \begin{bmatrix} -2 & -1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

59. $\begin{bmatrix} -2 & 4 & -3 \\ 0 & 6 & -3 \\ 0 & 8 & -3 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{4}{3} & 1 \end{bmatrix}$

60. $\begin{bmatrix} 5 & -3 & 7 \\ 6 & 0 & 2 \\ -9 & 1 & 0 \end{bmatrix}, \begin{bmatrix} -9 & 2 & 1 \\ 7 & 0 & -3 \\ 1 & 8 & 2 \end{bmatrix}$

Solve the following systems by the inverse matrix method, if possible. If the inverse matrix method doesn't apply, use any other method to determine if the system is inconsistent or dependent.

61. $\begin{cases} 5x + 9y = 2 \\ -2x - 3y = -1 \end{cases}$

62. $\begin{cases} 2y + 3z = 3 \\ -2x = 0 \\ 8x + 4y + 5z = -1 \end{cases}$

Solve the following set of systems by the inverse matrix method.

63. $\begin{cases} 2x - z = 3 \\ x + 4y + 2z = -1 \\ x + y = 5 \end{cases}$ $\begin{cases} 2x - z = 0 \\ x + 4y + 2z = 2 \\ x + y = 1 \end{cases}$ $\begin{cases} 2x - z = -1 \\ x + 4y + 2z = 1 \\ x + y = 2 \end{cases}$

Section 9.6

Graph the solution set of each of the following systems of inequalities.

64. $\begin{cases} 7x - 2y \ge 8 \\ y < 5 \end{cases}$

65. $\begin{cases} y - x > 0 \\ x < 2 \end{cases}$

Construct the constraints and graph the feasible regions for the following situations.

66. Each bag of nuts contains peanuts and cashews. The total number of nuts in the bag cannot exceed 60. There must be at least 20 peanuts and 10 cashews per bag. There can be no more than 40 peanuts or 40 cashews per bag. What is the region of constraint for the number of nuts per bag?

67. You wish to study at least 15 hours (over a 4-day span) for your upcoming statistics and biology tests. You need to study a minimum of 6 hours for each test. The maximum you wish to study for statistics is 10 hours and for biology is 8 hours. What is the region of constraint for the numbers of hours you should study for each test?

Find the minimum and maximum values of the given functions, subject to the given constraints.

68. Objective Function:

$f(x,y)=6x+10y$

Constraints:

$$\begin{cases} x\geq 0,\ y\geq 0 \\ 2x+5y\leq 10 \end{cases}$$

69. Objective Function:

$f(x,y)=5x+2y$

Constraints:

$$\begin{cases} x\geq 0,\ y\geq 0 \\ x+y\leq 10 \\ x+2y\geq 10 \\ 2x+y\geq 10 \end{cases}$$

70. Objective Function:

$f(x,y)=5x+4y$

Constraints:

$$\begin{cases} x\geq 0,\ y\geq 0 \\ 2x+3y\leq 12 \\ 3x+y\leq 12 \\ x+y\geq 2 \end{cases}$$

71. Objective Function:

$f(x,y)=70x+82y$

Constraints:

$$\begin{cases} x\geq 0,\ y\geq 0 \\ x\leq 10,\ y\leq 20 \\ x+y\geq 5 \\ x+2y\leq 18 \end{cases}$$

72. Krueger's Pottery manufactures two kinds of hand-painted pottery: a vase and a pitcher. There are three processes to create the pottery: throwing (forming the pottery on the potter's wheel), baking, and painting. No more than 90 hours are available per day for throwing, only 120 hours are available per day for baking, and no more than 60 hours per day are available for painting. The vase requires 3 hours for throwing, 6 hours for baking, and 2 hours for painting. The pitcher requires 3 hours for throwing, 4 hours for baking, and 3 hours for painting. The profit for each vase is $25 and the profit for each pitcher is $30. How many of each piece of pottery should be produced a day to maximize profit? What would the maximum profit be if Krueger's produced this amount?

73. Pranas produces bionic arms and legs for those that are missing a limb. Pranas can produce at least 20, but no more than 60 arms in a week due to the lab limitations. They can produce at least 15, but no more than 40 legs in a week. To keep their research grant, the company must produce at least 50 limbs per week. It costs $450 to produce the bionic arm and $550 to produce the bionic leg. How many of each should be produced per week to minimize the cost? What would the minimum cost be if Pranas produced this amount?

Section 9.7

Use graphing to approximate the real solution(s) of the following systems, and then verify that your answers are correct.

74. $\begin{cases} (x-2)^2 + y = 2 \\ x - y = 2 \end{cases}$

75. $\begin{cases} x^2 + y^2 = 25 \\ -x - y = 5 \end{cases}$

Solve the following systems of nonlinear equations. Be sure to check for nonreal solutions.

76. $\begin{cases} x^2 + 2y^2 = 1 \\ x^2 = y \end{cases}$

77. $\begin{cases} x^2 + y^2 = 25 \\ 2x^2 - y^2 = 23 \end{cases}$

78. $\begin{cases} y = (x-1)^2 \\ y + 8 = (x+1)^2 \end{cases}$

Draw the graph and determine whether the ordered pairs are solutions to the system of inequalities.

79. $\begin{cases} y^2 \le 9 - x^2 \\ y < |x| \\ y > -|x| \end{cases}$ **a.** $(2,5)$ **b.** $(7,8)$ **c.** $(5,0)$ **d.** $(3,4)$

Graph the following systems of inequalities.

80. $\begin{cases} y \le \sin x \\ y > -\sin x \end{cases}$

81. $\begin{cases} y \le \sqrt{x+1} \\ y > x^2 - 1 \end{cases}$

82. $\begin{cases} x^2 y \le 1 \\ 2y \le x^2 + 2 \\ y < 16x^2 \end{cases}$

83. The product of two positive integers is 144, and their sum is 25. What are the integers?

84. Stephen and Scott were driving the same 72-mile route, and they departed at the same time. After 30 minutes, Stephen was 6 miles ahead of Scott. If it took Scott one more hour than Stephen to reach their destination, how fast were they each driving?