

Looking Ahead

Your review of rational expressions and how to simplify them will help you with identifying the vertical asymptotes, if they exist, of a given rational function.

Example Preview

Find equations for the vertical asymptotes, if any, for the following rational function.

$$f(x) = \frac{-6x^2 + 4x + 2}{-2x + 2}$$

Solution

First, we need to reduce the function by removing any common factors in the numerator and denominator.

$$\begin{aligned} f(x) &= \frac{-6x^2 + 4x + 2}{-2x + 2} \\ &= \frac{\cancel{(-2x + 2)}(3x + 1)}{\cancel{(-2x + 2)}} \\ &= 3x + 1 \end{aligned}$$

Now, the vertical asymptotes of f are at the zeros of the denominator.

Since no denominator remains after factoring the common terms, there are no vertical asymptotes for f . (Note that the domain of f excludes $x = 1$. This means the graph has a “hole” at this x -value instead of a vertical asymptote.)

5.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A simplified rational expression cannot have any common factors other than 1 and -1 in both the numerator and denominator.
2. The difference between a rational number and a rational expression is that a rational expression generally has polynomials in the numerator and/or denominator.

3. While a rational number cannot have a zero denominator, a rational expression can have a zero denominator.

4. If a denominator is $x + 5$, it is defined for all values except 5.

Practice

Reduce each expression to lowest terms. State any restrictions on the variable(s).

5. $\frac{9x^2y^3}{12xy^4}$

6. $\frac{2x-8}{16-4x}$

7. $\frac{xy-3y+2x-6}{y^2-4}$

8. $\frac{x^2+10x+24}{2x^2+x-28}$

9. $\frac{x}{x^2-3x}$

10. Evaluate $\frac{3y-4}{y^2+25}$ for $y=3$

Applications

Solve.

- 11. Event Planning:** The cost of renting a party room with tables, chairs, and simple decorations is \$200 plus \$15 per person attending.
- Write a rational expression that represents the total price per person for renting the party room, where x is the number of people attending.
 - What is the price per person to rent the party room if 10 people are attending?
 - Determine which values of the variable will make the rational expression from Part **a.** undefined.
 - Considering the context of the given problem, are there any additional restrictions on the variable? If so, explain why these restrictions are in place.

- 12. Rectangles:** The area of a rectangle (in square feet) is represented by the polynomial function $A(x) = 4x^2 - 4x - 15$. If the length of the rectangle is $(2x + 3)$ feet, find a representation for the width.

$$A(x) = 4x^2 - 4x - 15$$

$$2x + 3$$

Writing & Thinking

13. a. Define the term rational expression.

b. Give an example of a rational expression that is undefined for $x = -2$ and $x = 3$ and has a value of 0 for $x = 1$. Explain how you determined this expression.

c. Give an example of a rational expression that is undefined for $x = -5$ and never has a value of 0. Explain how you determined this expression.