Because the *y*-intercept is -10, the coefficient a must be chosen so that $f(0) = -10$. To make this calculation simpler, begin my multiplying out $\big(x-\big(2+i\big)\big)\big(x-\big(2-i\big)\big)$.

$$\begin{aligned}\big(x-\big(2+i\big)\big)\big(x-\big(2-i\big)\big) &= \big(x-2-i\big)\big(x-2+i\big)\\ &= x^2-2x+ix-2x+4-2i-ix+2i-i^2\\ &= x^2-2x+\cancel{ix}-2x+4-\cancel{2i}-\cancel{ix}+\cancel{2i}-(-1)\\ &= x^2-2x-2x+4+1\\ &= x^2-4x+5\end{aligned}$$

Now, substitute $f(0) = -10$ and solve for a.

$$\begin{aligned}f(x) &= a\big(x^2-4x+5\big)\big(x+2\big)\\ f(0) &= a\Big(\big(0\big)^2-4\big(0\big)+5\Big)\big(\big(0\big)+2\big)\\ -10 &= a\big(5\big)\big(2\big)\\ -10 &= 10a\\ a &= -1\end{aligned}$$

Thus, the simplified polynomial function is as follows.

$$\begin{aligned}f(x) &= \big(-1\big)\big(x^2-4x+5\big)\big(x+2\big)\\ &= -\big(x^3+2x^2-4x^2-8x+5x+10\big)\\ &= -x^3+2x^2+3x-10\end{aligned}$$

4.R.9 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Regardless of the value of the exponent, the only possible values for any power of i are i and $-i$.

2. The product $\sqrt{a}\cdot\sqrt{b}$ can be rewritten as $\sqrt{ab}$ as long as a and b are real numbers.

3. When i is squared, the product is 1.

4. The conjugate of $4 - 5i$ is $4 + 5i$.

Practice

Perform the indicated operations and write each result in standard form.

5. $-4i(6 - 7i)$

6. $(2 + 7i)(6 + i)$

7. $\dfrac{5}{4i}$

8. $\dfrac{6 + i}{3 - 4i}$

Simplify the following powers of i and write each result in standard form. Assume k is a positive integer.

9. i^{13}

10. i^{-3}

Find the indicated products and simplify.

Writing & Thinking

11. Explain why the product of every complex number and its conjugate is a nonnegative real number.

12. What condition is necessary for the conjugate of a complex number, $a + bi$, to be equal to the reciprocal of this number?