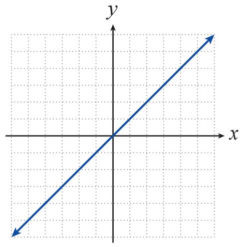
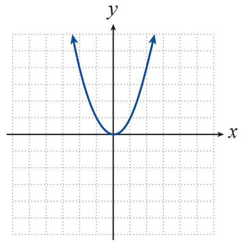
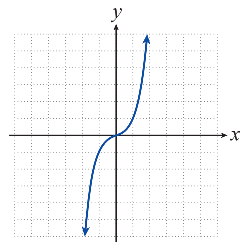
GRAPHS OF BASIC FUNCTIONS 4.4, 7.1, 7.3



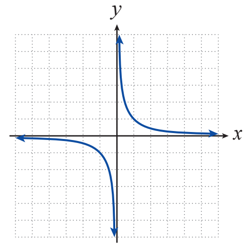
$f(x)=x$



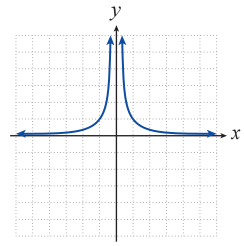
$f(x)=x^2$



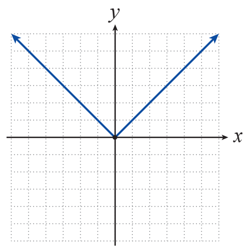
$f(x)=x^3$



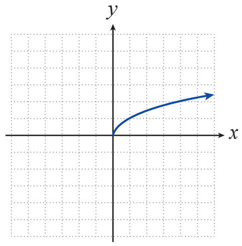
$f(x)=x^{-1}=\dfrac{1}{x}$



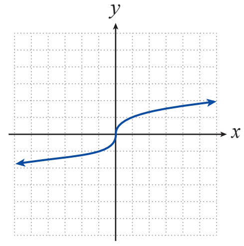
$f(x)=x^{-2}=\dfrac{1}{x^2}$



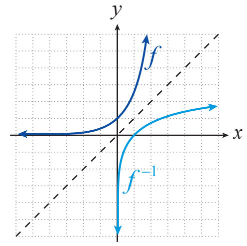
$f(x)=|x|$



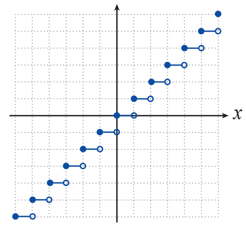
$f(x)=x^{\frac{1}{2}}=\sqrt{x}$



$f(x)=x^{\frac{1}{3}}=\sqrt[3]{x}$

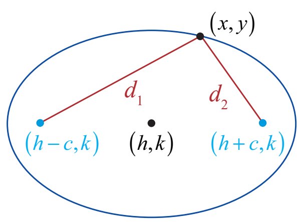


$f(x)=e^x$ and $f^{-1}(x)=\ln x$



$f(x)=[\![x]\!]$

CONIC SECTIONS 8.1, 8.2, 8.3



Ellipse

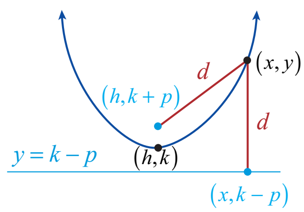
Let $a, b>0$ with $a>b$. The standard form of the equation of an ellipse centered at (h,k) with major axis of length $2a$ and minor axis of length $2b$ is as follows.

- $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$
  (major axis is horizontal)
- $\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$
  (major axis is vertical)

The foci are located on the major axis c units away from the center of the ellipse where $c^2=a^2-b^2$.

Eccentricity: $e=\dfrac{c}{a}=\dfrac{\sqrt{a^2-b^2}}{a}$

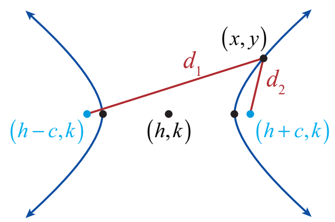
Note: If we let $a=b$, then $e=0$ and the ellipse is a circle.



Parabola

Let p be a nonzero real number. The standard form of the equation of a parabola with vertex at (h,k) is as follows.

- $(x-h)^2=4p(y-k)$
  (vertically oriented)
  Focus: $(h, k+p)$
  Directrix: $y=k-p$
- $(y-k)^2=4p(x-h)$
  (horizontally oriented)
  Focus: $(h+p, k)$
  Directrix: $x=h-p$



Hyperbola

Let $a, b>0$. The standard form of the equation of a hyperbola with center at (h,k) is as follows.

- $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$
  (foci are aligned horizontally)
  Asymptotes: $y-k=\pm\dfrac{b}{a}(x-h)$
- $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$
  (foci are aligned vertically)
  Asymptotes: $y-k=\pm\dfrac{a}{b}(x-h)$

The foci are located c units away from the center, where $c^2=a^2+b^2$.

The vertices are located a units away from the center.

GEOMETRIC FORMULAS

A = area, P = perimeter, C = circumference, SA = surface area, V = volume

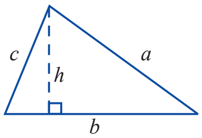
Triangle

$$A = \frac{1}{2}bh$$

Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

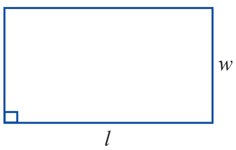
where $s = \dfrac{a+b+c}{2}$



Rectangle

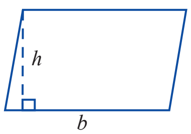
$A = lw$

$P = 2l + 2w$



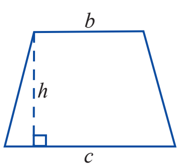
Parallelogram

$A = bh$



Trapezoid

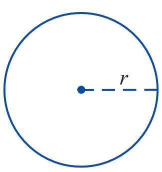
$$A = \frac{1}{2}h(b+c)$$



Circle

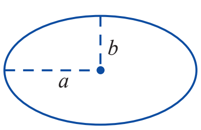
$A = \pi r^2$

$C = 2\pi r$



Ellipse

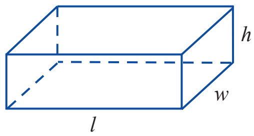
$A = \pi ab$



Rectangular Prism

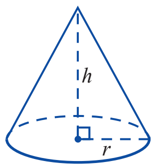
$V = lwh$

$SA = 2lh + 2wh + 2lw$



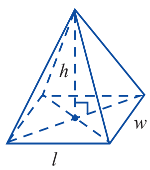
Circular Cone

$$V = \frac{1}{3}\pi r^2 h$$



Rectangular Pyramid

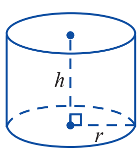
$$V = \frac{1}{3}lwh$$



Right Circular Cylinder

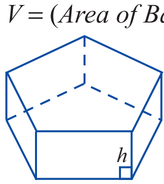
$V = \pi r^2 h$

$SA = 2\pi r^2 + 2\pi rh$



Right Cylinder

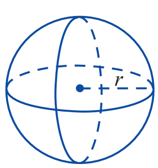
$V = (\text{Area of Base})h$



Sphere

$$V = \frac{4}{3}\pi r^3$$

$SA = 4\pi r^2$



PROPERTIES OF ABSOLUTE VALUE 1.1

For all real numbers a and b:

$|a| \geq 0$ $\qquad$ $|-a| = |a|$

$a \leq |a|$ $\qquad$ $|ab| = |a||b|$

$\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$, $b \neq 0$

$|a+b| \leq |a| + |b|$ (the triangle inequality)

PROPERTIES OF EXPONENTS AND RADICALS 1.3, 1.4

$a^n \cdot a^m = a^{n+m}$ $\qquad$ $(a^n)^m = a^{nm}$ $\qquad$ $(ab)^n = a^n b^n$

$\dfrac{a^n}{a^m} = a^{n-m}$ $\qquad$ $a^{-n} = \dfrac{1}{a^n}$ $\qquad$ $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$ $\qquad$ $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ $\qquad$ $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\qquad$ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

SPECIAL PRODUCT FORMULAS 1.6

$(A-B)(A+B) = A^2 - B^2$

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

FACTORING SPECIAL BINOMIALS 1.6

$A^2 - B^2 = (A-B)(A+B)$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

QUADRATIC FORMULA 2.3

The solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

PYTHAGOREAN THEOREM 3.1

Given a right triangle with legs a and b and hypotenuse c,

$$a^2 + b^2 = c^2.$$

DISTANCE FORMULA 3.1

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MIDPOINT FORMULA 3.1

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

STANDARD FORM OF THE EQUATION OF A CIRCLE 3.2

The standard form of the equation of a circle of radius r and center (h,k) is

$$(x-h)^2 + (y-k)^2 = r^2.$$

SLOPE OF A LINE 3.4

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines $y = c$ have a slope of 0.

Vertical lines $x = c$ have an undefined slope.

FORMS OF LINEAR EQUATIONS 3.4

Standard form: $ax + by = c$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

PARALLEL AND PERPENDICULAR LINES 3.5

Given a line with slope m:

slope of parallel line $= m$

slope of perpendicular line $= -\frac{1}{m}$

TRANSFORMATIONS OF FUNCTIONS 5.1

Horizontal Shifting

The graph of $g(x) = f(x - h)$ has the same shape as the graph of f, but shifted h units to the right if $h > 0$ and shifted h units to the left if $h < 0$.

Vertical Shifting

The graph of $g(x) = f(x) + k$ has the same shape as the graph of f, but shifted upward k units if $k > 0$ and downward k units if $k < 0$.

Reflecting with Respect to Axes

- The graph of the function $g(x) = -f(x)$ is the reflection of the graph of f with respect to the x-axis.
- The graph of the function $g(x) = f(-x)$ is the reflection of the graph of f with respect to the y-axis.

Stretching and Compressing

- The graph of the function $g(x) = af(x)$ is stretched vertically compared to the graph of f by a factor of a if $a > 1$.
- The graph of the function $g(x) = af(x)$ is compressed vertically compared to the graph of f by a factor of a if $0 < a < 1$.
- The graph of the function $g(x) = f(ax)$ is stretched horizontally compared to the graph of f by a factor of $\frac{1}{a}$ if $0 < a < 1$.
- The graph of the function $g(x) = f(ax)$ is compressed horizontally compared to the graph of f by a factor of $\frac{1}{a}$ if $a > 1$.

OPERATIONS WITH FUNCTIONS 5.3

Let f and g be functions.

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ when } g(x) \neq 0$$

$$(f \circ g)(x) = f(g(x))$$

RATIONAL ZERO THEOREM 6.3

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial with integer coefficients with $a_n \neq 0$, then any rational zero of f must be of the form $\frac{p}{q}$, where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

FUNDAMENTAL THEOREM OF ALGEBRA 6.4

If p is a polynomial of degree n, with $n \geq 1$, then p has at least one zero. That is, the equation $p(x) = 0$ has at least one solution. (**Note:** The solution may be a nonreal complex number.)

COMPOUND INTEREST 7.2

An investment of P dollars, compounded n times per year at an annual interest rate of r, has a value after t years of

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

An investment compounded continuously has an accumulated value of $A(t) = Pe^{rt}$.

PROPERTIES OF LOGARITHMS 7.3, 7.4

For $a, x, y > 0$, $a \neq 1$, and any real number r:

$\log_a x = y$ and $x = a^y$ are equivalent

$\log_a 1 = 0$ $\qquad$ $\log_a a = 1$

$\log_a (a^x) = x$ $\qquad$ $a^{\log_a x} = x$

$\log_a (xy) = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a (x^r) = r \log_a x$

CHANGE OF BASE FORMULA 7.4

For $a, b, x > 0$ and $a, b \neq 1$:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

DETERMINANT OF A 2×2 MATRIX 9.3

The determinant of a 2×2 matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by the formula

$$|A| = a_{11}a_{22} - a_{21}a_{12}.$$

DETERMINANT OF AN $n \times n$ MATRIX 9.3

The minor of the element a_{ij} is the determinant of the $(n-1)\times(n-1)$ matrix formed from A by deleting the i^{th} row and the j^{th} column.

The cofactor of the element a_{ij} is $(-1)^{i+j}$ times the minor of a_{ij}. Find the determinant of an $n \times n$ matrix by expanding along a fixed row or column.

- To expand along the i^{th} row, each element of that row is multiplied by its cofactor and the n products are then added.
- To expand along the j^{th} column, each element of that column is multiplied by its cofactor and the n products are then added.

CRAMER'S RULE 9.3

A system of n linear equations in the n variables $x_1, x_2, \ldots, x_n$ can be written in the following form.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \qquad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

The solution of the system is given by the n formulas

$$x_1 = \frac{D_{x_1}}{D}, x_2 = \frac{D_{x_2}}{D}, \ldots, x_n = \frac{D_{x_n}}{D},$$

where D is the determinant of the coefficient matrix and D_{x_i} is the determinant of the same matrix with the i^{th} column of constants replaced by the column of constants $b_1, b_2, \ldots, b_n$.

MATRIX ADDITION 9.4

$A + B =$ the matrix such that $c_{ij} = a_{ij} + b_{ij}$ (c_{ij} is the element in the i^{th} row and j^{th} column of $A + B$).

SCALAR MULTIPLICATION 9.4

$cA =$ the matrix such that the element in the i^{th} row and j^{th} column is equal to ca_{ij}.

MATRIX MULTIPLICATION 9.4

$AB =$ the matrix such that $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$. (The length of each row in A must be the same as the length of each column of B.)

PROPERTIES OF SIGMA NOTATION 10.1

For sequences $\{a_n\}$ and $\{b_n\}$ and a constant c:

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{k} a_i + \sum_{i=k+1}^{n} a_i \text{ (for any } 1 \le k \le n-1)$$

SUMMATION FORMULAS 10.1

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

SEQUENCES AND SERIES

Arithmetic 10.2

Let $\{a_n\}$ be an arithmetic sequence with common difference d.

General term: $a_n = a_1 + (n-1)d$

Partial sum: $S_n = na_1 + d\left(\frac{(n-1)n}{2}\right) = \left(\frac{n}{2}\right)(a_1 + a_n)$

Geometric 10.3

Let $\{a_n\}$ be a geometric sequence with common ratio r.

General term: $a_n = a_1 r^{n-1}$

Partial sum: $S_n = \frac{a_1(1-r^n)}{1-r}$, if $r \ne 0, 1$

Infinite sum: $S = \sum_{n=0}^{\infty} a_1 r^n = \frac{a_1}{1-r}$, if $|r| < 1$

PERMUTATION FORMULA 10.5

$${}_nP_k = \frac{n!}{(n-k)!}$$

COMBINATION FORMULA 10.5

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

BINOMIAL THEOREM 10.5

$$(A+B)^n = \sum_{k=0}^{n} \binom{n}{k} A^k B^{n-k}$$

MULTINOMIAL COEFFICIENTS 10.5

$$\binom{n}{k_1, k_2, \ldots, k_r} = \frac{n!}{k_1! k_2! \cdots k_r!}$$

MULTINOMIAL THEOREM 10.5

$$(A_1 + A_2 + \cdots + A_r)^n = \sum_{k_1 + k_2 + \cdots + k_r = n} \binom{n}{k_1, k_2, \ldots, k_r} A_1^{k_1} A_2^{k_2} \cdots A_r^{k_r}$$

INDEX OF SYMBOLS

Symbol	Meaning
$\mathbb{N}$	set of all natural numbers $\{1,2,3,4,5,\ldots\}$
$\mathbb{Z}$	set of all integers $\{\ldots,-4,-3,-2,-1,0,1,2,3,4,\ldots\}$
$\mathbb{Q}$	set of all rational numbers; that is, the set of all numbers that can be represented as a ratio of integers
$\mathbb{R}$	set of all real numbers
π	the ratio of the circumference to the diameter of a circle
E	calculator notation for scientific notation (for example, aEb means $a\times10^b$)
∞	infinity
i	imaginary unit defined as $\sqrt{-1}$
$\Leftrightarrow$	"is equivalent to"
$\Rightarrow$	"implies"
$\cup$	the union of two sets (that is, the set of all elements found in either set)
$\cap$	the intersection of two sets (that is, the set of all elements found in both sets)
$\in$	"is an element of"
$\varnothing$	empty set or the set containing no elements
$\rightarrow$	"approaches"
$\approx$	"approximately equal to"
Δ	"change in"
$\mathbb{R}^2$	the plane of all real x-values by all real y-values (the Cartesian plane)
$\sum_{i=a}^{n}$	summation notation (or sigma notation) used to express the sum of a sequence from a to n
e	base of the natural logarithm
$n!$	"n factorial" stands for the product of all the integers from 1 to n ($0! = 1$ and $1! = 1$)
$\binom{n}{k}$	"n choose k"; combination of n objects taken k at a time