Chapter 1
Technology Exercises

77–79 Mentally sketch the graph of the given function by identifying the basic shape that has been shifted, reflected, stretched, or compressed. Then use a graphing utility to graph the function and check your reasoning.

77. $f(x)=\ln(x+1)+2$

78. $f(x)=-\dfrac{2}{x-3}+1$

79. $f(x)=\sin \pi x-1$

80. The annual expenditures (in millions of dollars) for a corporation are given in the table below.

Annual Expenditures

Year	2017	2018	2019	2020	2021	2022
Expenditures (in millions)	$16.2	$17.1	$18.8	$19.6	$21.1	$22.9

a. Find the least-squares line of best fit for the data. (Let $x = 0$ correspond to the year 2017.)

b. Estimate the expenditures for 2023.

81–82 Use a graphing utility to approximate the solution(s) of the given equation, rounded to four decimal places. (**Hint:** Zoom in on the *x*-intercepts or points of intersection as appropriate for each equation.)

81. $x^5-x^3-3=0$

82. $x^2+6=2^{x+1}$

83–84 Use a graphing utility to graph the given function, and describe the characteristics of the graph as *c* varies. Use different viewing windows.

83. $u(x)=\dfrac{1-e^{c/x}}{1+e^{c/x}}$

84. $v(x)=\dfrac{x}{c^2}\sqrt[4]{c^4-x^4}$

56. Use the Intermediate Value Theorem to show that the graphs of $f(x)=x^3$ and $g(x)=e^{-x}$ intersect.

57–58 Find the equation of the tangent line to the graph of $f(x)$ at the given point.

57. $f(x)=x^2+x;\quad (1,2)$

58. $f(x)=\sqrt{x};\quad (4,2)$

59–60 Use the definition (also called the limit process) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

59. $f(x)=2x-x^2$

60. $f(x)=\dfrac{3}{x-2}$

61–62 Sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

61. f is continuous at 0, $f(0)=1$, $f'(x)<0$ for $x<0$, $f'(x)>0$ for $x>0$, and $f'(0)$ does not exist

62. $g(1)<0$, $g'(1)>0$, and $g(2)>0$, but $g'(2)<0$

63. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.

64. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.

a. How high does it go and when does it reach the ground?

b. What is the speed of impact?

(**Hint:** Use $h(t)=-5t^2+12t+15$ as the position function, where h is in meters, t in seconds.)

65. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p=D(x)=0.2x+30$ dollars. The total cost as a function of x is given by $C(x)=0.1x^2+15x+247.5$ dollars.

a. Find the profit function $P(x)$.

b. Find any break-even points.

c. Find the marginal profit function.

Concept Check

66–73 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

66. Instantaneous velocity can be interpreted as the slope of a tangent line.

67. If $\lim_{x\to c} f(x)$ doesn't exist, then $f(x)$ has a vertical asymptote at $x=c$.

68. Any rational function has at least one vertical asymptote.

69. If $\lim_{x\to c} f(x)=A$ and $\lim_{x\to c} g(x)=B$, then $\lim_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{A}{B}$.

70. If f is defined on $[a,b]$, L is a real number between $f(a)$ and $f(b)$, and $\lim_{x\to c} f(x)$ exists for all $x\in(a,b)$, then there is a c in the interval (a,b) such that $f(c)=L$.

71. If f is continuous at c, then $f(c)$ is equal to both one-sided limits at c.

72. If both one-sided limits of f exist at c, and if f is defined at c, then f is continuous at c.

73. If $g(x)\le f(x)\le h(x)$ for all x in some open interval containing c, and if $\lim_{x\to c} g(x)=\lim_{x\to c} h(x)=L$, then by the Squeeze Theorem $f(c)=L$ as well.

Chapter 2
Technology Exercises

74. Use a computer algebra system to find approximations for the areas in Exercises 10 and 11 by using $n=100$. (Round your answers to four decimal places.)

75. Use a computer algebra system to find approximations for the arc lengths in Exercises 12 and 13 by using $n=100$. (Round your answers to four decimal places.)

76. Use a graphing utility to verify your answers given for Exercises 14–17.

77. Use a graphing utility to approximate the solutions for Exercises 55 and 56. Round your answers to four decimal places.

78–81 Use a graphing utility to graph the function, and estimate from the graph the value of the given limit.

78. $\lim_{x\to\infty} x^{1/x}$

79. $\lim_{x\to 0}\dfrac{\arcsin x}{x}$

80. $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{2x}$

81. $\lim_{x\to 1}\dfrac{\ln(x^3)}{x-1}$

69. The proper dosage d of a certain over-the-counter medicine for children depends on body weight w according to the function $d(w) = \frac{5}{4}w^{3/5}$, where d is measured in milligrams and w in pounds. Use differentials to estimate how accurately (in terms of percentage error) we need to know a 32-pound child's weight if we cannot stray from the proper dosage by more than 6 percent.

70. A manufacturing business found its daily revenue to be $R(x) = 150x - \frac{1}{4}x^2$ dollars when x units are produced and sold.

a. Use linearization and marginal revenue to estimate the extra revenue when production is increased from 100 to 102 units.

b. Use the revenue function to calculate the actual revenue increase. Compare your answers.

71. Use the concept of the derivative function to explain why the graph of $y = x^a$, $a > 1$ curves upward, while the graph of $y = x^b$, $0 < b < 1$ curves downward.

Concept Check

72–82 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

72. If both one-sided derivatives of $f(x)$ either exist or are equal to $\pm\infty$ at c, then f is continuous at c.

73. If $p(x)$ is a polynomial of degree n, then all k^{th}-order derivatives of $p(x)$ for $k > n$ are 0.

74. If $y = \pi^n \sin x$, then $y' = n\pi^{n-1}\sin x + \pi^n \cos x$.

75. If $y = 1/(x^2 - 3x + 1)$, then $y' = 1/(2x - 3)$.

76. If $y = \ln(3x+1)$, then $y' = 1/(3x+1)$.

77. If $y = x^x$, then $y' = x \cdot x^{x-1}$.

78. Since $\left(e^x\right)' = e^x$, therefore $\left(e^{e^x}\right)' = e^{e^x}$.

79. If $f(x) = x$, then $df = dx$.

80. If $f(x)$ is linear, then its linearization at any point is itself.

81. If $x \to 0$, then $\Delta x \to dx$ and $\Delta y \to dy$.

82. If $\Delta x \to 0$, then $\Delta y/\Delta x \to dy/dx$.

Chapter 3
Technology Exercises

83–85 Use a graphing utility to graph the function and identify all points where the function is not differentiable. Explain.

83. $f(x) = |x^2 - x|$

84. $f(x) = |x|(x+2)$

85. $f(x) = \sqrt[4]{x^2 - 1}$

86. Use the differentiation capabilities of a graphing utility to find the derivative of $f(x) = 2\cos^2 x - \cos 2x$. Then find the derivative by hand, applying a trigonometric identity before differentiating. Does your answer agree with that of your technology? If not, what do you think is the reason? Can you "force" your graphing utility to represent its answer in a simpler form?

87. Repeat Exercise 86 for the function $f(x) = 2\sin(x/2)\cos(x/2)$.

88. Use a graphing utility to graph the functions $y = \ln x$, $y = a^x$, $a > 1$ and $y = x^b$, $0 < b < 1$ for various values of the parameters a and b. By zooming out appropriately, compare their relative growth rates; that is, conjecture "who wins the race toward infinity" in general among these three types of functions. Use the concept of the derivative to support your conjecture.

89. The displacement of a mass attached to a spring is given by the function $h(t) = e^{-t/6}\cos 2t$.

a. Use a graphing utility to graph the function and explain why it is realistic.

b. Use a graphing utility to graph the velocity and acceleration functions together with $h(t)$ on the same screen. What seems to be the position of the mass when velocity is maximum? When is velocity 0? When is acceleration maximum, and when is it 0?

78. A vending machine sells 500 bars of a certain type of candy when the price is \$1.50. It was discovered that 10 fewer customers will buy the candy bar for each 5¢ increase in price. What is the price that will bring maximum revenue from the sales of this type of candy bar?

79. Maximize the surface area of the can in Example 3 of Section 4.6. Explain your findings.

80. Minimize the cost of producing the can in Example 3 of Section 4.6 if the top and bottom are produced using a material that is 50% more expensive than the material used for the side.

81. Nate needs to reach a restaurant that is 600 ft upstream on the other side of a 150 ft wide river. Find the point where he has to reach the other side in order to make the best time if he can swim at 5 ft/s and walk at 9 ft/s. (Ignore the flow of the river.)

82–89 Find the general antiderivative of the given function, and check your answer by differentiation. (If necessary, rewrite the function before antidifferentiation.)

82. $f(x) = 2x^3 - 6x^2 + 3x$

83. $f(x) = 5x^4 - 4.8x^3 + e^2$

84. $f(x) = x(x+2)(2x-3)$

85. $f(x) = 0.4x\sqrt{x} - \dfrac{2}{\sqrt{x}}$

86. $f(x) = \dfrac{x^4 - 4x}{x^2}$

87. $f(x) = 2(x + \sec^2 2x)$

88. $f(x) = 6e^{3x}$

89. $f(x) = \dfrac{3}{4x^2 + 1}$

90–91 Find $f(x)$ that satisfies the specified conditions.

90. $f''(x) = x, \quad f'(1) = 1, \quad f(1) = 0$

91. $f'''(x) = 2, \quad f''(2) = -1, \quad f'(2) = 2, \quad f(2) = 3$

92. A tennis ball is thrown upward from an initial height of 4 feet with an initial velocity of 56 feet per second. How high will it go and for how long is it rising? (Ignore air resistance.)

93. With what initial velocity do we need to throw a golf ball vertically upward in order for it to rise 100 feet high? (Ignore the initial height and air resistance.)

94. A pebble is shot horizontally using a slingshot at 10 meters per second from the top of a building that is 20 meters high. If the terrain around the building is nearly flat, approximately how far from the building will the pebble hit the ground? (Use the approximation $g \approx 10 \text{ m/s}^2$ and ignore air resistance.)

Concept Check

95–101 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

95. A continuous function on a finite interval always attains its maximum and minimum.

96. If $f(x)$ has a relative maximum or minimum at $x = c$, then $f'(c) = 0$.

97. If $f(x)$ has a relative maximum or minimum at $x = c$, then c is a critical point of f.

98. A cubic polynomial has exactly one inflection point.

99. If $f(x)$ is a polynomial, then between two consecutive local extrema there must be an $x = c$ so that $f''(c) = 0$.

100. If $f(x)$ is a polynomial and c is a critical point, then there is a relative maximum or minimum at $x = c$.

101. If $f'''(c) = 0$, then $f'(x)$ has a point of inflection at $x = c$.

Chapter 4 Technology Exercises

102–111. Use a graphing utility to verify the answers you obtained for Exercises 51–60.

112–113. Use a graphing utility to verify the conclusions of Exercises 15 and 16.

71. Consider the function $f(x) = 1/x^2$ defined on some interval $[a,b]$. Partition $[a,b]$ and in each subinterval $[x_{i-1}, x_i]$ choose the sample point $x_i^* = \sqrt{x_{i-1}x_i}$ (the geometric mean of the endpoints). Show that

$$\frac{1}{(x_i^*)^2}\Delta x_i = \frac{1}{x_{i-1}} - \frac{1}{x_i}$$

and use this observation to prove the following formula.

$$\int_a^b \frac{1}{x^2}\,dx = \frac{1}{a} - \frac{1}{b}$$

72. Prove that if the conditions of Part I of the Fundamental Theorem of Calculus are satisfied and $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, where $g(x)$ and $h(x)$ are differentiable, then $F'(x) = f(h(x))h'(x) - f(g(x))g'(x)$. (**Hint:** See Example 3 of Section 5.3.)

73. Prove that if f is a linear function, then its definite integral on an interval $[a,b]$ is the average of its left and right Riemann sums, that is,

$$\int_a^b f(x)\,dx = \frac{L_n + R_n}{2}.$$

What is your expectation regarding the integral and the average above if f is concave up? Concave down?

Concept Check

74–81 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

74. If $n_1 < n_2$, then the Riemann sum R_{n_2} is always a better approximation of the integral than R_{n_1}.

75. If f is piecewise continuous on a closed interval, then the limit of its Riemann sums always exists.

76. When applying the Fundamental Theorem of Calculus, we must choose the antiderivative with $C = 0$.

77. $\int \frac{1}{e^x}\,dx = \ln(e^x) + C = x + C$

78. The definite integral of the velocity function of a moving object on $[t_1, t_2]$ is equal to the total distance traveled by the object from time $t = t_1$ to $t = t_2$.

79. $\int_a^b f(x)\,dx > 0$ if and only if $f(x) > 0$ on $[a,b]$.

80. $\int \sec x\,dx = \sec x \tan x + C$

81. $\int_{-1}^{1} \frac{1}{x^3}\,dx = \left.\frac{-1}{2x^2}\right]_{-1}^{1} = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

Chapter 5 Technology Exercises

82. Use the summation feature of a graphing utility to verify your answers for Exercises 9–15.

83. Write a program for a graphing calculator or computer algebra system that calculates the n^{th} Riemann sum for a given function on a given interval, using subintervals of equal width and sample points of your choice. Use your program to verify your answers for Exercises 16–18.

84. Use a graphing utility to evaluate the limit of Exercise 28. What do you find? (Answer will vary depending on the capabilities of the particular software used.)

84. Suppose a fighter plane fires a missile at 500 mph in the forward direction at a moment when the plane itself is flying at 900 mph. Use Einstein's relativistic formula to find the missile's velocity relative to Earth and compare it with Galileo's prediction of $500 + 900 = 1400$ mph. Approximate the speed of light by 3×10^8 m/s.

85. In this exercise, we are going to up the numbers of Exercise 84 significantly. Suppose a rocket is traveling away from Earth at a speed of $0.7c$ and fires another rocket at $0.5c$. Use Einstein's formula to calculate the velocity of the second rocket relative to Earth.

Concept Check

86–92 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

86. The disk method is based on the idea of integrating slices.

87. When both the disk method and the shell method are applied to calculate the volume of a solid of revolution, the variable of integration is always the same.

88. If the area of the region bounded by $y = f(x)$ and $y = g(x)$ is A, then the volume of the solid obtained by revolving the same region about the x-axis is $V = \pi A^2$.

89. If the area of the region bounded by $y = f(x)$ and $y = g(x)$ is A, then the volume of the solid obtained by revolving the same region about the x-axis never equals πA^2.

90. $\lim_{x\to-\infty} \tanh x = -1$

91. $\cosh 2x = 2\sinh^2 x + 1$

92. The work needed to pump fluid out of a tank through an opening on its top equals the total weight of the fluid multiplied by the distance traveled by its center of mass.

Chapter 6 Technology Exercises

93–96 Use a graphing utility to find (or approximate) the volume of the solid generated by rotating the region bounded by the graphs of the given equations about the indicated axis.

93. $y = \sin(x^2)$, $y = 0$, $x = 0$, $x = \sqrt{\pi}$; about the x-axis

94. $y = \arccos x$, $y = 0$, $x = 0$, $x = 1$; about the x-axis

95. $y = \sinh^{-1} x$, $y = 0$, $x = 4$; about the y-axis

96. $y = x^2 \sin^2 x$, $y = 0$, $x = 0$, $x = \pi$; about the y-axis

97–98 Use a graphing utility to find the arc length of the graph of the equation over the given interval.

97. $y = \dfrac{1}{x^2+1}$; $-1 \le x \le 1$

98. $y = \sin x$; $0 \le x \le \pi$

99–100 Use a graphing utility to find the surface area of the solid generated by revolving the given curve about the indicated axis.

99. $y = \sin x$; $0 \le x \le \pi$; about the x-axis

100. $y = \sqrt{\ln x}$; $1 \le x \le e$; about the y-axis

74–77 Use the Trapezoidal Rule with $n = 6$ to approximate the integral, and compare the result to the exact value of the integral by determining the absolute value of the error E_T.

74. $\int_1^4 x^{3/2}\,dx$

75. $\int_1^2 \frac{1}{x^2}\,dx$

76. $\int_0^4 \sqrt{x^2+1}\,dx$

77. $\int_0^{\pi/3} \tan x\,dx$

78–81. Use the error estimate for the Trapezoidal Rule to estimate $|E_T|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 74–77.

82–85. Use Simpson's Rule to approximate the integrals from Exercises 74–77 with $n = 6$. Determine the absolute value of the error E_S.

86–89. Use the error estimate for Simpson's Rule to estimate $|E_S|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 82–85.

90. Prove that if $f(x) = ax + b$ is a linear function on a closed interval $[a,b]$, then for any n, $T_n = \int_a^b f(x)\,dx$.

91–96 Identify the type of the improper integral and determine whether it is convergent or divergent. If it is convergent, find its value.

91. $\int_2^4 \frac{dx}{(x-2)^4}$

92. $\int_{-\infty}^0 x^2 e^x\,dx$

93. $\int_2^6 \frac{dx}{\sqrt{x-2}}$

94. $\int_0^\infty \frac{2}{(x+3)^{2/3}}\,dx$

95. $\int_{-\infty}^\infty \frac{2e^x}{e^{2x}+4}\,dx$

96. $\int_e^\infty \frac{dx}{x\ln x}$

97–98 Use the Direct Comparison Test to determine whether the integral converges.

97. $\int_0^\infty \frac{dx}{\sqrt{x^3+1}}$

98. $\int_1^\infty \frac{\ln x}{\sqrt{x}}\,dx$

99. Use substitution to turn the improper integral $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ into a proper one and evaluate it.

100. Rotate about the x-axis the region bounded by the graph of $y = \sqrt{\ln x}/x^2$ and the x-axis over the interval $[1,\infty)$. Use the disk method to determine if the resulting unbounded solid has finite volume. If so, find the volume.

101.* Rotate about the x-axis the region bounded by the x-axis and the graph of $y = e^{-x}$ over the infinite interval $[0,\infty)$. Determine if the resulting infinite solid has finite volume or surface area. If so, find their values.

102. Prove that the improper integral $\int_2^\infty \frac{dx}{x(\ln x)^a}$ converges if and only if $a > 1$.

103–106 Find the Laplace transform. (See Exercises 88–93 in Section 7.7.)

103. $L\{te^{at}\}$

104.* $L\{t^2e^{at}\}$

105.* $L\{t\sin kt\}$

106.* $L\{t\cos kt\}$

Concept Check

107–113 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

107. If $\lim_{x\to\infty} f(x) = L$ and $L \neq 0$, then $\int_0^\infty f(x)\,dx$ diverges.

108. If $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

109. Let $f(x)$ be defined on $[a,\infty)$ and S be the solid generated by rotating the graph of $f(x)$ about the x-axis. If the surface area of S is infinite, then the volume of S is also infinite.

110. If $f(x)$ is an odd function, then $\int_{-\infty}^\infty f(x)\,dx = 0$.

111. If $f(x)$ has a vertical asymptote at $x = 0$ and $a > 0$, then $\int_{-a}^a f(x)\,dx$ diverges.

112. Any rational function is integrable on any finite interval that doesn't include a zero of the denominator.

113. If an integrand contains the expression $\sqrt{a^2 \pm x^2}$, a trigonometric substitution must be used to evaluate the integral.

Chapter 7 Technology Exercises

114. Use a graphing utility to find the integral from Exercise 33. If the answer appears different from what you obtained by hand, prove that the answers are equivalent.

115. Write a program for a computer algebra system or programmable calculator that evaluates the trapezoidal approximation T_n for a given input function on a specified interval and positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal.

116. Use the program you wrote for Exercise 115 for the integral from Exercise 75.

117. Write a program for a computer algebra system or programmable calculator that evaluates the Simpson approximation S_n for a given integral and an even positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal. Compare this with your answer for Exercise 115. What n ensures that the answer is correct to at least five decimal places?

118. Use the program you wrote for Exercise 117 for the integral from Exercise 75.

119–120 Use the programs you wrote for Exercises 115 and 117 to approximate the given nonelementary integral with $n = 50$. Which method (the Trapezoidal Rule or Simpson's Rule) do you expect to be more accurate? Use the built-in numerical integration command of your technology to verify your conjecture.

119. $\int_0^1 \sqrt{1+x^4}\,dx$

120. $\int_0^1 e^{e^x}\,dx$

56.* A container in a lab contains 14 gallons of pure distilled water. 10% and 25% acid solutions are pumped into the container through two respective inlets. The 10% solution is flowing in at a rate of 0.2 gallons per minute, while the 25% solution is being allowed in by the second inlet at a rate of 0.5 gallons per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the rate of 0.7 gallons per minute. How long does it take to form 14 gallons of 14% solution in this way?

57. A hailstone is melting so that its volume $V(t)$ decreases at a rate proportional to its surface area.

a. Assuming that the hailstone is nearly spherical, find a differential equation satisfied by $V(t)$.

b. If a hailstone of diameter 1 inch loses 20% of its volume in half an hour, predict how long it takes for it to completely melt away. (Consider it melted away when your model predicts less than 1 percent remaining).

58. If a vertical cylindrical tank of radius $\frac{1}{2}$ meters and height 4 meters is initially full of water but is draining through a circular orifice of diameter 2 centimeters that is on the bottom of the tank, what is the water level in the tank 2 minutes later? (**Hint:** See Exercise 59 in Section 8.1.)

59. Find the charge $q(t)$ of the 10^2-farad capacitor in an RC circuit if the impressed voltage on the circuit is $V(t) = t$ and the resistance is 25 ohms. Assume $q(0) = 0$. (**Hint:** See Exercise 62 in Section 8.1.)

60. Suppose that the impressed voltage in a simple RL circuit is $V(t) = 2t$, $I(0) = 0$, the inductance is 0.1 henries, and the resistance 0.5 ohms. Find the electric current I at time $t = 4$ seconds. (**Hint:** See Example 5 in Section 8.2.)

61. A baking dish is removed from a 210 °C oven and left at 20 °C room temperature. Two and a half minutes later the dish's temperature is 155 °C. Find the bakeware's temperature 10 minutes after it was removed from the oven. (**Hint:** See Example 2 in Section 8.3.)

62. A snapping turtle population grows logistically with a carrying capacity of 200 turtles and constant of proportionality $k = 0.2$ per year.

a. Find the population size $P(t)$ as a function of time if initially 50 turtles are present in the habitat. (**Hint:** See Example 3 in Section 8.3.)

b. How long does it take for the population to reach 100 turtles?

63. Suppose that an object of mass 200 grams stretches a spring by 10 centimeters. If it is pulled upward to a position of 5 centimeters above equilibrium and released with a downward velocity of 1 m/s, find and graph the resulting displacement function, assuming that the surrounding medium offers resistance with a damping constant of $c = 0.5 \text{ kg/s}$. (**Hint:** See Example 5 in Section 8.4.)

Concept Check

64–70 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

64. The equation $(y')^2 + xy' - 3y = 0$ is a first-order differential equation.

65. The equation $y' = -y$ is not linear.

66. If $y_1(x)$ and $y_2(x)$ are solutions of a homogeneous linear differential equation, then so is $3y_1(x) - 2y_2(x)$.

67. Only autonomous equations have slope fields.

68. The logistic equation discussed in this text is autonomous.

69. The equations $y = 2e^{x/2}$ and $y = xe^{x/2}$ are linearly independent solutions of $4y'' - 4y' + y = 0$.

70. A second-order BVP with two boundary conditions always has a solution.

Chapter 8 Technology Exercises

71–72. Use a graphing utility to display the slope fields of the differential equations in Exercises 33 and 34. Compare the graphs to your original sketches.

73. Write a program for a computer algebra system that accepts a spring constant, a damping constant, and the mass of an oscillating object as inputs, and graphs the displacement function as output. Use it to check your answer for Exercise 63.

Chapter 9
Technology Exercises

91. Find the equation of the graph of $r = 1 - 2\cos\theta$ after a clockwise rotation by $\pi/4$ radians. Name the resulting curve and use a graphing utility to sketch it. (See Exercise 73 in Section 9.3.)

92–93 Use a graphing utility to sketch the given curve for various values of the parameter(s) and explore the effects on the shape of your graph.

92. $r = \theta\cos k\theta$

93. $x = \pm a\cos^{2/n} t, \quad y = \pm b\sin^{2/n} t, \quad a, b, n > 0$
(Lamé curves)

94–95 Use a graphing utility to sketch the curve and then find all horizontal and vertical tangent lines. Confirm your results by paper and pencil calculations.

94. $x = t^3 - t, \quad y = t^2 + 1, \quad -2 \le t \le 2$

95. $r = 2\sin 2\theta, \quad 0 \le \theta \le \pi/2$

96–97 Use a graphing utility to sketch the region enclosed by the given curve and find its area.

96. $x = t\sin t, \quad y = \sin 2t, \quad 0 \le t \le \pi$

97. Inner loop of $r = 2 - 3\cos\theta$

98–99 Use a graphing utility to approximate the arc length of the curve with the given parametrization.

98. $x = \sin 2t, \quad y = \sin t, \quad 0 \le t \le \pi$

99. $r = \cos(2\sin\theta), \quad 0 \le \theta \le 2\pi$

100–101 Use a graphing utility to approximate the surface area of the solid obtained by rotating the given curve about the indicated axis.

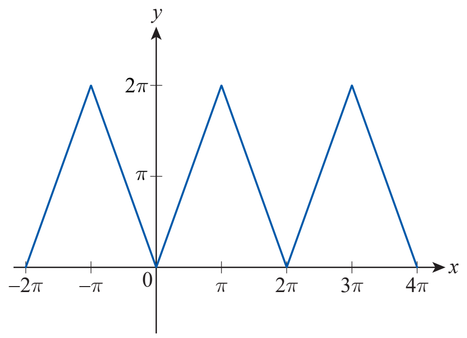
100. $\left(t^3 - 3t, t^2 - 2\right), \quad 0 \le t \le \sqrt{3};$ about the y-axis

101. $r = 3\sin 2\theta, \quad \pi/4 \le \theta \le \pi/2;$ about the polar axis

110.* Use Taylor's formula to provide a proof of the Second Derivative Test as follows. Assuming that $f'(c)=0$, use Taylor's formula to conclude that $f(x)=f(c)+\frac{1}{2}f''(a)(x-c)^2$, for some a between x and c. Then examine the signs of $f(x)-f(c)$ and $f''(a)$. Next, assuming $f'(c)=f''(c)=0$ and $f'''(c)\neq 0$, argue that $f(c)$ is neither a relative maximum nor a minimum. (Assume initially that f is continuosly differentiable through at least the third order; then think about whether you can relax this condition.)

111. Find a second solution to Exercise 71 using long division.

112. Find the Fourier series expansion of the 2π-periodic extension of the function graphed below.



Concept Check

113–124 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

113. If a_n is monotonically decreasing, then $\lim_{n\to\infty} a_n = -\infty$.

114. If $\{a_n\}$ is convergent, then $\{a_n/n\}$ is a null sequence.

115. If $\{a_n/n\}$ is a null sequence, then $\{a_n\}$ is convergent.

116. If $\{a_n\}$ is convergent, then $\{a_{n+1}-a_n\}$ is a null sequence.

117. If $\{a_n\}$ is monotonically decreasing to zero, then $\sum_{n=1}^{\infty}(-1)^n a_n$ is absolutely convergent.

118. If $\{a_n\}$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

119. If $\sum_{n=1}^{\infty}|a_n|$ is divergent, then either $\sum_{n=1}^{\infty} a_n$ or $\sum_{n=1}^{\infty}(-a_n)$ is divergent.

120. If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty}|a_n|$ is divergent.

121. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

122. If the power series $\sum_{n=1}^{\infty} a_n x^n$ diverges at $x=c$, then it diverges at $x=-c$.

123. All power series converge at infinitely many x-values.

124. There is a power series whose convergence set is empty.

Chapter 10 Technology Exercises

125–127 Use a graphing utility to solve the problem.

125. We already know that the harmonic series diverges to infinity, and that it does so at a very slow pace. In this exercise, we will examine this series a bit further.

a. Find out how many terms are needed for the partial sum of the harmonic series to exceed 12.

b. What is the sum of the first 2 million terms? (Compare with Example 4 of Section 10.3. Notice that this calculation takes a bit of time even for today's powerful technology!)

126. The simple series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$ was shown by Gregory and Leibniz to converge to $\pi/4$ (hence its name, the *Gregory series*). However, it converges rather slowly. Find out how many terms of this series are necessary to approximate π accurate to two decimal places.

127. **a.** Graph $y=\sin x$ and its 11^{th}-order Maclaurin polynomial on the same screen, over the interval $[-4\pi,4\pi]$. Visually estimate the subinterval over which you find the approximation acceptable.

b. Repeat part a. with the 21^{st}-order Maclaurin polynomial.