56.* A container in a lab contains 14 gallons of pure distilled water. 10% and 25% acid solutions are pumped into the container through two respective inlets. The 10% solution is flowing in at a rate of 0.2 gallons per minute, while the 25% solution is being allowed in by the second inlet at a rate of 0.5 gallons per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the rate of 0.7 gallons per minute. How long does it take to form 14 gallons of 14% solution in this way?

57. A hailstone is melting so that its volume $V(t)$ decreases at a rate proportional to its surface area.

a. Assuming that the hailstone is nearly spherical, find a differential equation satisfied by $V(t)$.

b. If a hailstone of diameter 1 inch loses 20% of its volume in half an hour, predict how long it takes for it to completely melt away. (Consider it melted away when your model predicts less than 1 percent remaining).

58. If a vertical cylindrical tank of radius $\frac{1}{2}$ meters and height 4 meters is initially full of water but is draining through a circular orifice of diameter 2 centimeters that is on the bottom of the tank, what is the water level in the tank 2 minutes later? (**Hint:** See Exercise 59 in Section 8.1.)

59. Find the charge $q(t)$ of the 10^2-farad capacitor in an RC circuit if the impressed voltage on the circuit is $V(t) = t$ and the resistance is 25 ohms. Assume $q(0) = 0$. (**Hint:** See Exercise 62 in Section 8.1.)

60. Suppose that the impressed voltage in a simple RL circuit is $V(t) = 2t,\ I(0) = 0,$ the inductance is 0.1 henries, and the resistance 0.5 ohms. Find the electric current I at time $t = 4$ seconds. (**Hint:** See Example 5 in Section 8.2.)

61. A baking dish is removed from a 210 °C oven and left at 20 °C room temperature. Two and a half minutes later the dish's temperature is 155 °C. Find the bakeware's temperature 10 minutes after it was removed from the oven. (**Hint:** See Example 2 in Section 8.3.)

62. A snapping turtle population grows logistically with a carrying capacity of 200 turtles and constant of proportionality $k = 0.2$ per year.

a. Find the population size $P(t)$ as a function of time if initially 50 turtles are present in the habitat. (**Hint:** See Example 3 in Section 8.3.)

b. How long does it take for the population to reach 100 turtles?

63. Suppose that an object of mass 200 grams stretches a spring by 10 centimeters. If it is pulled upward to a position of 5 centimeters above equilibrium and released with a downward velocity of $1\ \text{m/s}$, find and graph the resulting displacement function, assuming that the surrounding medium offers resistance with a damping constant of $c = 0.5\ \text{kg/s}$. (**Hint:** See Example 5 in Section 8.4.)

Concept Check

64–70 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

64. The equation $(y')^2 + xy' - 3y = 0$ is a first-order differential equation.

65. The equation $y' = -y$ is not linear.

66. If $y_1(x)$ and $y_2(x)$ are solutions of a homogeneous linear differential equation, then so is $3y_1(x) - 2y_2(x)$.

67. Only autonomous equations have slope fields.

68. The logistic equation discussed in this text is autonomous.

69. The equations $y = 2e^{x/2}$ and $y = xe^{x/2}$ are linearly independent solutions of $4y'' - 4y' + y = 0$.

70. A second-order BVP with two boundary conditions always has a solution.

Chapter 8
Technology Exercises

71–72. Use a graphing utility to display the slope fields of the differential equations in Exercises 33 and 34. Compare the graphs to your original sketches.

73. Write a program for a computer algebra system that accepts a spring constant, a damping constant, and the mass of an oscillating object as inputs, and graphs the displacement function as output. Use it to check your answer for Exercise 63.