

**74–77** Use the Trapezoidal Rule with  $n = 6$  to approximate the integral, and compare the result to the exact value of the integral by determining the absolute value of the error  $E_T$ .

74.  $\int_1^4 x^{3/2} dx$

75.  $\int_1^2 \frac{1}{x^2} dx$

76.  $\int_0^4 \sqrt{x^2 + 1} dx$

77.  $\int_0^{\pi/3} \tan x dx$

**78–81.** Use the error estimate for the Trapezoidal Rule to estimate  $|E_T|$  for  $n = 6$ , and compare the estimate with the actual error you found in Exercises 74–77.

**82–85.** Use Simpson's Rule to approximate the integrals from Exercises 74–77 with  $n = 6$ . Determine the absolute value of the error  $E_S$ .

**86–89.** Use the error estimate for Simpson's Rule to estimate  $|E_S|$  for  $n = 6$ , and compare the estimate with the actual error you found in Exercises 82–85.

**90.** Prove that if  $f(x) = ax + b$  is a linear function on a closed interval  $[a, b]$ , then for any  $n$ ,  $T_n = \int_a^b f(x) dx$ .

**91–96** Identify the type of the improper integral and determine whether it is convergent or divergent. If it is convergent, find its value.

91.  $\int_2^4 \frac{dx}{(x-2)^4}$

92.  $\int_{-\infty}^0 x^2 e^x dx$

93.  $\int_2^6 \frac{dx}{\sqrt{x-2}}$

94.  $\int_0^{\infty} \frac{2}{(x+3)^{2/3}} dx$

95.  $\int_{-\infty}^{\infty} \frac{2e^x}{e^{2x} + 4} dx$

96.  $\int_e^{\infty} \frac{dx}{x \ln x}$

**97–98** Use the Direct Comparison Test to determine whether the integral converges.

97.  $\int_0^{\infty} \frac{dx}{\sqrt{x^3 + 1}}$

98.  $\int_1^{\infty} \frac{\ln x}{\sqrt{x}} dx$

**99.** Use substitution to turn the improper integral  $\int_0^{\pi^2/4} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$  into a proper one and evaluate it.

**100.** Rotate about the  $x$ -axis the region bounded by the graph of  $y = \sqrt{\ln x}/x^2$  and the  $x$ -axis over the interval  $[1, \infty)$ . Use the disk method to determine if the resulting unbounded solid has finite volume. If so, find the volume.

**101.\*** Rotate about the  $x$ -axis the region bounded by the  $x$ -axis and the graph of  $y = e^{-x}$  over the infinite interval  $[0, \infty)$ . Determine if the resulting infinite solid has finite volume or surface area. If so, find their values.

**102.** Prove that the improper integral  $\int_2^{\infty} \frac{dx}{x(\ln x)^a}$  converges if and only if  $a > 1$ .

**103–106** Find the Laplace transform. (See Exercises 88–93 in Section 7.7.)

103.  $L\{te^{at}\}$

104.\*  $L\{t^2 e^{at}\}$

105.\*  $L\{t \sin kt\}$

106.\*  $L\{t \cos kt\}$

## Concept Check

**107–113** Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

**107.** If  $\lim_{x \rightarrow \infty} f(x) = L$  and  $L \neq 0$ , then  $\int_0^{\infty} f(x) dx$  diverges.

**108.** If  $\lim_{x \rightarrow \infty} f(x) = 0$ , then  $\int_0^{\infty} f(x) dx$  converges.

**109.** Let  $f(x)$  be defined on  $[a, \infty)$  and  $S$  be the solid generated by rotating the graph of  $f(x)$  about the  $x$ -axis. If the surface area of  $S$  is infinite, then the volume of  $S$  is also infinite.

**110.** If  $f(x)$  is an odd function, then  $\int_{-\infty}^{\infty} f(x) dx = 0$ .

**111.** If  $f(x)$  has a vertical asymptote at  $x = 0$  and  $a > 0$ , then  $\int_{-a}^a f(x) dx$  diverges.

**112.** Any rational function is integrable on any finite interval that doesn't include a zero of the denominator.

**113.** If an integrand contains the expression  $\sqrt{a^2 \pm x^2}$ , a trigonometric substitution must be used to evaluate the integral.

## Chapter 7

### Technology Exercises

**114.** Use a graphing utility to find the integral from Exercise 33. If the answer appears different from what you obtained by hand, prove that the answers are equivalent.

**115.** Write a program for a computer algebra system or programmable calculator that evaluates the trapezoidal approximation  $T_n$  for a given input function on a specified interval and positive integer  $n$ . Find the smallest  $n$  that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal.

**116.** Use the program you wrote for Exercise 115 for the integral from Exercise 75.

- 117.** Write a program for a computer algebra system or programmable calculator that evaluates the Simpson approximation  $S_n$  for a given integral and an even positive integer  $n$ . Find the smallest  $n$  that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal. Compare this with your answer for Exercise 115. What  $n$  ensures that the answer is correct to at least five decimal places?
- 118.** Use the program you wrote for Exercise 117 for the integral from Exercise 75.

**119–120** Use the programs you wrote for Exercises 115 and 117 to approximate the given nonelementary integral with  $n = 50$ . Which method (the Trapezoidal Rule or Simpson's Rule) do you expect to be more accurate? Use the built-in numerical integration command of your technology to verify your conjecture.

**119.**  $\int_0^1 \sqrt{1+x^4} \, dx$

**120.**  $\int_0^1 e^{e^x} \, dx$