71. Consider the function $f(x) = 1/x^2$ defined on some interval $[a,b]$. Partition $[a,b]$ and in each subinterval $[x_{i-1}, x_i]$ choose the sample point $x_i^* = \sqrt{x_{i-1}x_i}$ (the geometric mean of the endpoints). Show that

$$\frac{1}{(x_i^*)^2}\Delta x_i = \frac{1}{x_{i-1}} - \frac{1}{x_i}$$

and use this observation to prove the following formula.

$$\int_a^b \frac{1}{x^2}\,dx = \frac{1}{a} - \frac{1}{b}$$

72. Prove that if the conditions of Part I of the Fundamental Theorem of Calculus are satisfied and $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, where $g(x)$ and $h(x)$ are differentiable, then $F'(x) = f(h(x))h'(x) - f(g(x))g'(x)$. (**Hint:** See Example 3 of Section 5.3.)

73. Prove that if f is a linear function, then its definite integral on an interval $[a,b]$ is the average of its left and right Riemann sums, that is,

$$\int_a^b f(x)\,dx = \frac{L_n + R_n}{2}.$$

What is your expectation regarding the integral and the average above if f is concave up? Concave down?

Concept Check

74–81 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

74. If $n_1 < n_2$, then the Riemann sum R_{n_2} is always a better approximation of the integral than R_{n_1}.

75. If f is piecewise continuous on a closed interval, then the limit of its Riemann sums always exists.

76. When applying the Fundamental Theorem of Calculus, we must choose the antiderivative with $C = 0$.

77. $\int \frac{1}{e^x}\,dx = \ln(e^x) + C = x + C$

78. The definite integral of the velocity function of a moving object on $[t_1, t_2]$ is equal to the total distance traveled by the object from time $t = t_1$ to $t = t_2$.

79. $\int_a^b f(x)\,dx > 0$ if and only if $f(x) > 0$ on $[a,b]$.

80. $\int \sec x\,dx = \sec x \tan x + C$

81. $\int_{-1}^{1} \frac{1}{x^3}\,dx = \left.\frac{-1}{2x^2}\right]_{-1}^{1} = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

Chapter 5
Technology Exercises

82. Use the summation feature of a graphing utility to verify your answers for Exercises 9–15.

83. Write a program for a graphing calculator or computer algebra system that calculates the n^{th} Riemann sum for a given function on a given interval, using subintervals of equal width and sample points of your choice. Use your program to verify your answers for Exercises 16–18.

84. Use a graphing utility to evaluate the limit of Exercise 28. What do you find? (Answer will vary depending on the capabilities of the particular software used.)