69. The proper dosage d of a certain over-the-counter medicine for children depends on body weight w according to the function $d(w)=\frac{5}{4}w^{3/5}$, where d is measured in milligrams and w in pounds. Use differentials to estimate how accurately (in terms of percentage error) we need to know a 32-pound child's weight if we cannot stray from the proper dosage by more than 6 percent.

70. A manufacturing business found its daily revenue to be $R(x)=150x-\frac{1}{4}x^2$ dollars when x units are produced and sold.

a. Use linearization and marginal revenue to estimate the extra revenue when production is increased from 100 to 102 units.

b. Use the revenue function to calculate the actual revenue increase. Compare your answers.

71. Use the concept of the derivative function to explain why the graph of $y=x^a$, $a>1$ curves upward, while the graph of $y=x^b$, $0<b<1$ curves downward.

Concept Check

72–82 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

72. If both one-sided derivatives of $f(x)$ either exist or are equal to $\pm\infty$ at c, then f is continuous at c.

73. If $p(x)$ is a polynomial of degree n, then all k^{th}-order derivatives of $p(x)$ for $k>n$ are 0.

74. If $y=\pi^n \sin x$, then $y'=n\pi^{n-1}\sin x+\pi^n \cos x$.

75. If $y=1/(x^2-3x+1)$, then $y'=1/(2x-3)$.

76. If $y=\ln(3x+1)$, then $y'=1/(3x+1)$.

77. If $y=x^x$, then $y'=x\cdot x^{x-1}$.

78. Since $(e^x)'=e^x$, therefore $\left(e^{e^x}\right)'=e^{e^x}$.

79. If $f(x)=x$, then $df=dx$.

80. If $f(x)$ is linear, then its linearization at any point is itself.

81. If $x\to 0$, then $\Delta x\to dx$ and $\Delta y\to dy$.

82. If $\Delta x\to 0$, then $\Delta y/\Delta x\to dy/dx$.

Chapter 3
Technology Exercises

83–85 Use a graphing utility to graph the function and identify all points where the function is not differentiable. Explain.

83. $f(x)=|x^2-x|$

84. $f(x)=|x|(x+2)$

85. $f(x)=\sqrt[4]{x^2-1}$

86. Use the differentiation capabilities of a graphing utility to find the derivative of $f(x)=2\cos^2 x-\cos 2x$. Then find the derivative by hand, applying a trigonometric identity before differentiating. Does your answer agree with that of your technology? If not, what do you think is the reason? Can you "force" your graphing utility to represent its answer in a simpler form?

87. Repeat Exercise 86 for the function $f(x)=2\sin(x/2)\cos(x/2)$.

88. Use a graphing utility to graph the functions $y=\ln x$, $y=a^x$, $a>1$ and $y=x^b$, $0<b<1$ for various values of the parameters a and b. By zooming out appropriately, compare their relative growth rates; that is, conjecture "who wins the race toward infinity" in general among these three types of functions. Use the concept of the derivative to support your conjecture.

89. The displacement of a mass attached to a spring is given by the function $h(t)=e^{-t/6}\cos 2t$.

a. Use a graphing utility to graph the function and explain why it is realistic.

b. Use a graphing utility to graph the velocity and acceleration functions together with $h(t)$ on the same screen. What seems to be the position of the mass when velocity is maximum? When is velocity 0? When is acceleration maximum, and when is it 0?