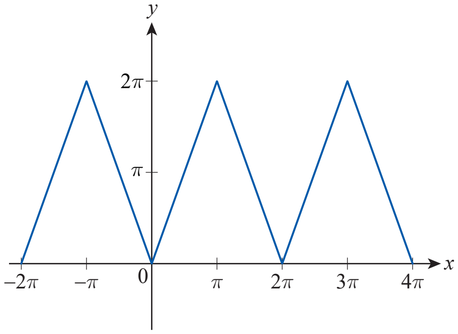
110.* Use Taylor's formula to provide a proof of the Second Derivative Test as follows. Assuming that $f'(c)=0$, use Taylor's formula to conclude that $f(x)=f(c)+\frac{1}{2}f''(a)(x-c)^2$, for some a between x and c. Then examine the signs of $f(x)-f(c)$ and $f''(a)$. Next, assuming $f'(c)=f''(c)=0$ and $f'''(c)\neq 0$, argue that $f(c)$ is neither a relative maximum nor a minimum. (Assume initially that f is continuosly differentiable through at least the third order; then think about whether you can relax this condition.)

111. Find a second solution to Exercise 71 using long division.

112. Find the Fourier series expansion of the 2π-periodic extension of the function graphed below.



Concept Check

113–124 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

113. If a_n is monotonically decreasing, then $\lim_{n\to\infty} a_n = -\infty$.

114. If $\{a_n\}$ is convergent, then $\{a_n/n\}$ is a null sequence.

115. If $\{a_n/n\}$ is a null sequence, then $\{a_n\}$ is convergent.

116. If $\{a_n\}$ is convergent, then $\{a_{n+1}-a_n\}$ is a null sequence.

117. If $\{a_n\}$ is monotonically decreasing to zero, then $\sum_{n=1}^{\infty}(-1)^n a_n$ is absolutely convergent.

118. If $\{a_n\}$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

119. If $\sum_{n=1}^{\infty}|a_n|$ is divergent, then either $\sum_{n=1}^{\infty} a_n$ or $\sum_{n=1}^{\infty}(-a_n)$ is divergent.

120. If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty}|a_n|$ is divergent.

121. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

122. If the power series $\sum_{n=1}^{\infty} a_n x^n$ diverges at $x=c$, then it diverges at $x=-c$.

123. All power series converge at infinitely many x-values.

124. There is a power series whose convergence set is empty.

Chapter 10
Technology Exercises

125–127 Use a graphing utility to solve the problem.

125. We already know that the harmonic series diverges to infinity, and that it does so at a very slow pace. In this exercise, we will examine this series a bit further.

a. Find out how many terms are needed for the partial sum of the harmonic series to exceed 12.

b. What is the sum of the first 2 million terms? (Compare with Example 4 of Section 10.3. Notice that this calculation takes a bit of time even for today's powerful technology!)

126. The simple series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$ was shown by Gregory and Leibniz to converge to $\pi/4$ (hence its name, the *Gregory series*). However, it converges rather slowly. Find out how many terms of this series are necessary to approximate π accurate to two decimal places.

127. a. Graph $y=\sin x$ and its 11^{th}-order Maclaurin polynomial on the same screen, over the interval $[-4\pi, 4\pi]$. Visually estimate the subinterval over which you find the approximation acceptable.

b. Repeat part a. with the 21^{st}-order Maclaurin polynomial.