$$I(t) = e^{-(R/L)t}\left[\int \frac{e^{(R/L)t}V}{L}\,dt + C\right]$$

$$= e^{-(R/L)t}\left[\left(\frac{V}{L}\right)\left(\frac{L}{R}\right)e^{(R/L)t} + C\right] = \frac{V}{R} + Ce^{-(R/L)t}$$

The current I begins at 0 when $t = 0$, so it must be the case that $C = -V/R$ and hence the particular solution is as follows.

$$I(t) = \frac{V}{R} - \frac{V}{R}e^{-(R/L)t} = \frac{V}{R}\left[1 - e^{-(R/L)t}\right]$$

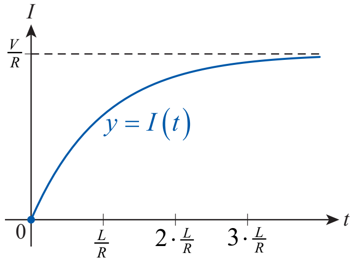

Figure 4

As $t \to \infty$, $\left[1 - e^{-(R/L)t}\right] \to 1$, so the *steady-state* current in the circuit is simply V/R. The inductance in the circuit is most significant immediately after voltage is applied—it is then that the term $1 - e^{-(R/L)t}$ has greatest effect, as shown in Figure 4.

The length of time L/R is referred to as the circuit's *time constant* and is a measure of how its inductance and resistance interact to affect the current. In Exercise 44, you will show that $I(t)$ attains slightly more than 95% of its steady-state value after three time constants.

8.2 Exercises

1–6 Classify the differential equation as linear or nonlinear. (Do not attempt to solve the equation.)

1. $xy' - \frac{e^x}{x+1}y = \sqrt{x} - \frac{2y}{x}$
2. $xy' - \frac{e^x}{x+1}y = \sqrt{y} - \frac{2y}{x}$
3. $\frac{y}{x} = x^2y'$
4. $yy' + (x+1)y = x^2$
5. $\tan x = \frac{2x + y'}{y}$
6. $y' - xy^2 = x$

7–10 Decide if the equation is linear in the dependent variable y. If not, check whether it is linear when x is considered to be the dependent variable.

7. $2\,dy - x\,dx = y\left(1 - 3\sqrt{x}\right)dx$
8. $x\,dy = \cos y\,dy - y\,dx$
9. $y\,dx = (5x + 2y - 4)\,dy$
10. $y\,dy - \left(1 + x^2\right)y\,dx = e^x\,dx$

11–25 Solve the linear differential equation. (**Hint:** In some cases, x has to be the dependent variable in order for the equation to be linear.)

11. $y' + 3\frac{y}{x} = 0$
12. $xy' - 2y = x^3e^x$
13. $xy' + 4y = 1$
14. $y' + 2xy = 4x$
15. $x\frac{dy}{dx} = y + 2x^2 - 3x + 5$
16. $(x-1)y' = 4(x-1)^3 - y$
17. $dx + \frac{8x}{y}dy = y\,dy$
18. $y' - y\tan x = e^{\sin x}$
19. $y' = \sin x + \cos x - y$
20. $yx' - x = y^3e^y$
21. $\left(x^4 + 1\right)y' = x^4 - 4x^3y + 1$
22. $\cot x\,dy + y\,dx = \csc x\,dx$
23. $x(\ln x)y' + y = \ln x$
24. $dx - xy\,dy = y\,dy$
25. $\frac{dx}{dt} + x\tan t = t\tan t + \sec t + 1$

26–33 Solve the given initial value problem.

26. $\frac{dy}{dx}+4y=16x; \quad y(0)=0$

27. $\frac{dy}{dx}+3x^2y=x^2; \quad y(0)=2$

28. $(1+x^2)y'+2xy=1; \quad y(0)=10$

29. $(1+x^2)y'-2xy=1+x^2; \quad y(0)=10$

30. $t\frac{dS}{dt}-S=2t; \quad S(1)=7$

31. $(1+x)y'-xy=\frac{e^x}{1+x}; \quad y(0)=0$

32. $y'\sin x+y\cos x=\frac{x}{\csc x}; \quad y(\pi/2)=-3$

33. $y\,dx=(4\ln y-2x)\,dy; \quad x(1)=4$

34–37 Find a first-order linear differential equation in standard form that has the given general solution. (**Hint:** Identify the integrating factor and "reverse" the solution technique discussed in the text.)

34. $y=\frac{e^x}{x^2}+\frac{1}{x}+Cx^{-2}$

35. $y=\sin x+\frac{C}{x}$

36. $y=\frac{x^3+C}{e^x}$

37. $y=1+\frac{x+C}{\ln x}$

38–41 A first-order differential equation of the form

$$\frac{dy}{dx}+P(x)y=Q(x)y^{\alpha}$$

is called the **Bernoulli equation** (named after Jakob Bernoulli), where α is any real number. You should check that by introducing the new dependent variable $u=y^{1-\alpha}$ $(\alpha \neq 0, \alpha \neq 1)$ and noting that

$$\frac{du}{dx}=(1-\alpha)y^{-\alpha}\frac{dy}{dx},$$

we can turn a Bernoulli equation into the following standard-form linear one.

$$\frac{du}{dx}+(1-\alpha)P(x)u=(1-\alpha)Q(x)$$

In Exercises 38–41, use the above substitution method to solve the given Bernoulli equation.

38. $\frac{dy}{dx}+2y=x\sqrt{y}$

39. $y'+\frac{y}{x}=y^2$

40. $y'+2y=e^xy^2$

41. $y'+\frac{y}{x}=2y^{3/2}$

42. Explain why it is no loss of generality to always let the constant of integration be 0 when determining the integration factor in the solution of a linear differential equation.

43. Find the time when the additive in the tank of Example 4 reaches a maximum.

44. Show that $I(t)$ in Example 5 attains slightly more than 95% of its steady-state value after three time constants.

45. A 50-gallon tank is filled with brine (water nearly saturated with salt; used as a preservative) holding 12 pounds of salt in solution. A salt solution containing 0.5 pounds of salt per gallon is added to the tank at the rate of 1 gallon per minute. The contents of the tank are continuously and thoroughly mixed and drained out at 5 quarts per minute. What is the amount of salt in the tank after an hour? (Compare with Exercise 54 of Section 8.1.)

46. A tank contains 2000 gallons of diesel fuel. A fuel mixture containing a lubricity additive is pumped into the tank through two inlets. The mixture flowing in through the first inlet contains 0.48 oz of additive per gallon and is being pumped in at a rate of 25 gal/min. Meanwhile, the mixture being allowed in by the second inlet at a rate of 10 gal/min contains 10.4 oz of additive per gallon. The mixture in the tank is continuously and thoroughly mixed and drained out at the rate of 20 gal/min. If there should be 16 oz of additive for every 120 gallons of diesel fuel, how long will it take to reach the right mixture? (Compare with Exercise 55 of Section 8.1.)

47. Suppose that V_0 gallons of gasoline contain a_0 pounds of a seasonal additive. A gasoline mixture containing a_1 pounds of additive per gallon is added to the tank at the rate of $r_1 \text{ gal/min}$. The gasoline solution in the tank is continuously and thoroughly mixed and drained out at a rate of $r_2 \text{ gal/min}$. Set up the initial value problem whose solution is $y(t)$, the amount of additive in the mixture at time t.

48. In learning theory, the rate of memorization, or learning, is considered to be proportional to the amount of material yet to be memorized. On the other hand, the amount forgotten is proportional to the amount already learned. If T stands for the total amount of material to be memorized, and $M(t)$ is the amount memorized at time t, find a differential equation satisfied by $M(t)$. What type of differential equation did you obtain?

49. Find $I(t)$ in a simple RL circuit if 5 volts are applied at time $t = 0$, $I(0) = 0$, the inductance is 0.2 henries and the resistance is 10 ohms.

50. Find a formula for $I(t)$ in Exercise 49 if the impressed voltage is $V(t) = 1.2t$.

51. Suppose that 150 volts are impressed on an RC circuit with resistance of 50 ohms and capacitance of 10^3 farads. If $q(0) = 0$, find the charge $q(t)$ on the capacitor. What happens to the charge as $t \to \infty$? (**Hint:** See Exercise 62 of Section 8.1.)

52.* Find the current in the RL circuit of Example 5 if the impressed voltage (also called electromotive force) is $V(t) = V \sin(\omega t)$.

53. Antibiotics are taken by a patient at the rate of d milligrams per day. Assume that the drug is removed by the body from the bloodstream at a rate proportional to the amount present.

a. Find the differential equation satisfied by $A(t)$, the amount of antibiotics present in the bloodstream.

b. Solve for $A(t)$, assuming $A(0) = 0$, and determine what happens as $t \to \infty$.

54.* Find the current induced by a discharging capacitor in an RC circuit if $V(t) = 0$ and $I(0) = I_0$. (**Hint:** Consider the differential equation satisfied by $q(t)$, and start by differentiating both sides of the equation.)

Concept Check

55–58 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

55. A linear differential equation cannot be separable.

56. The equation $y\,dx + 3x\,dy - 2y\,dy = y^4\,dy$ is linear.

57. The only way to solve a linear differential equation is by the use of an integrating factor.

58. The integrating factor for a standard-form linear differential equation is the function $I(x) = e^{\int P(x)\,dx}$.

8.2 Technology Exercises

59–69. Use a computer algebra system to solve the equations in Exercises 15–25. Compare the results with the ones you obtained by hand.

70. A skydiver jumps out of a plane at 2000 meters and deploys his chute after 10 seconds of free fall. The total mass of the diver and his gear is 80 kilograms. Assume that air resistance is proportional to velocity both before and after deploying the chute, with respective constants of proportionality of 8 and 100. Use a computer algebra system to create a model to find how long after the jump the skydiver will land. (Distance is measured in meters, time in seconds.)