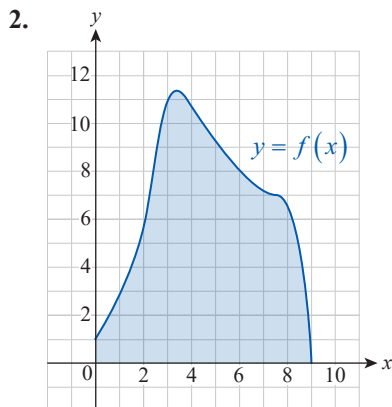
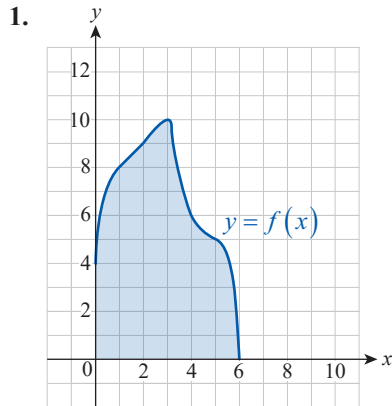


7.6 Exercises

1–2 The function $f(x)$ is given by its graph. Use the Trapezoidal Rule and Simpson's Rule, respectively, to approximate the shaded area $\int_a^b f(x) dx$ by **a.** T_6 and **b.** S_6 .



3–17 Use the Trapezoidal Rule and Simpson's Rule with $n = 8$ to approximate the integral. Then find the exact value and compare your answers.

3. $\int_0^8 x^4 dx$

4. $\int_1^5 \frac{1}{x} dx$

5. $\int_1^5 \frac{1}{x^2} dx$

6. $\int_0^4 \sqrt{x} dx$

7. $\int_0^4 x^3 dx$

8. $\int_0^{2\pi} |\sin x| dx$

9. $\int_{-2}^6 \sqrt[3]{x+2} dx$

10. $\int_0^2 e^x dx$

11. $\int_1^5 \ln x dx$

12. $\int_{-2}^6 \left(4 - \frac{1}{2}x\right) dx$

13. $\int_{-4}^4 (16 - x^2) dx$

14. $\int_{-4}^4 \sqrt{16 - x^2} dx$

15. $\int_0^4 x\sqrt{x^2 + 2} dx$

16. $\int_0^{16} \frac{1}{\sqrt{x+1}} dx$

17. $\int_0^8 \frac{x}{\sqrt{x^2 + 1}} dx$

18–20 Use **a.** the Trapezoidal Rule and **b.** Simpson's Rule to approximate the definite integral for the indicated value of n .

18. $\int_0^4 \sqrt[4]{x} dx; n = 4$

19. $\int_{-\pi/2}^{\pi/2} \cos x dx; n = 6$

20. $\int_0^5 \sqrt{x^4 + 4} dx; n = 10$

21–23 Some texts discuss the *Midpoint Rule* as a numerical integration method. The idea is simply forming a Riemann sum by choosing the midpoint of each subinterval as the sample point.

Use the Midpoint Rule with $n = 8$ to approximate the integral and compare your answers to those in Exercises 3–5.

21. $\int_0^8 x^4 dx$

22. $\int_1^5 \frac{1}{x} dx$

23. $\int_1^5 \frac{1}{x^2} dx$

24–29 Use the formula discussed in the text to find an error estimate for the Trapezoidal Rule when it is used to approximate the integral of $f(x)$ over the given interval. Use $n = 10$.

24. $f(x) = x^2, [0, 2]$

25. $f(x) = \cos 2x, [\pi/2, 3\pi/2]$

26. $f(x) = \sqrt{x}, [1, 4]$

27. $f(x) = x^4, [0, 1]$

28. $f(x) = \frac{1}{\sqrt{x+1}}, [0, 3]$

29. $f(x) = \frac{1}{x}, [1, 11]$

30–35. Repeat Exercises 24–29, this time providing an error estimate for Simpson's Rule with $n = 10$. Compare your error bounds to those you gave for the Trapezoidal Rule.

36–41 Determine a number of subintervals that will guarantee an approximation of the given definite integral to within an error of 0.001 using the Trapezoidal Rule. (Answers will vary.)

36. $\int_0^1 \frac{x^4}{2} dx$

37. $\int_0^{\pi} \sin^2 x dx$

38. $\int_1^2 \frac{2-x^2}{x^2} dx$

39. $\int_0^1 e^{x^2} dx$

40. $\int_1^3 x \ln x dx$

41. $\int_0^{\pi/2} x \sin x dx$

42–47. For the integrals in Exercises 36–41, find a number of subintervals that will guarantee an approximation of the same accuracy (within an error of 0.001), this time using Simpson's Rule. Compare your answers to those you gave for the Trapezoidal Rule.

48. Use Simpson's Rule with $n = 6$ to approximate $\ln 2 = \int_1^2 \frac{1}{x} dx$, and find the percentage error of your approximation (the actual error divided by the true value expressed as a percentage).

49. Repeat Exercise 48 to approximate $\pi = \int_0^1 \frac{4}{x^2 + 1} dx$.

50. Use Simpson's Rule with $n = 24$ to approximate the area of the region bounded by the graphs of $y = \sqrt{1+x^4}$, $x = -6$, $x = 6$, and the x -axis. (Notice that this problem leads to a nonelementary integral.)

51. Use the shell method along with the Trapezoidal Rule to find the approximation T_7 for the volume of the solid obtained by revolving the region between the graph of $y = e^x/x^2$ ($1 \leq x \leq 2$) and the x -axis about the y -axis.

52. Combine the disk method with Simpson's Rule to find the approximation S_6 for the volume of the solid obtained by revolving the region between the graph of $y = e^{x^2}$ ($0 \leq x \leq 1$) and the x -axis about the x -axis.

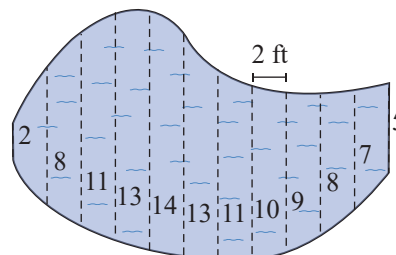
53. The following table summarizes acceleration data for a Ford Mustang Boss 302 Laguna Seca. Use Simpson's Rule to estimate the total distance traveled by "the Boss" during its timed 0–120 mph run. (**Hint:** Sketching a graph similar to the one in Example 2 is useful. Be sure to identify which area you can approximate and how it yields the answer to the problem.)

Acceleration Times

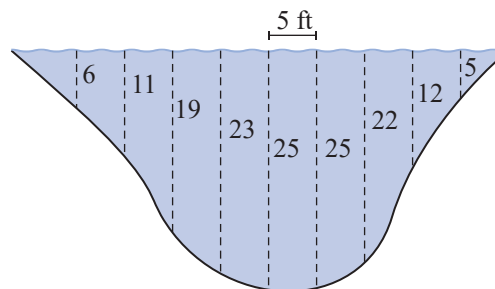
Miles per Hour	Seconds
0–120	13.0
0–110	10.9
0–100	9.1
0–90	7.6
0–80	6.3
0–70	5.2
0–60	4.1
0–50	3.3
0–40	2.4
0–30	1.7
0–20	1.1
0–10	0.4

Source: *Road & Track*

54. Use the Trapezoidal Rule to estimate the amount of water needed to raise the water level by two inches in a pool with the shape shown in the figure. At 2-foot intervals, the distances across the pool (in feet) are as indicated in the diagram.



55. The figure shows the vertical cross-section of the Lazee river where the Dinkatown ferry docks. The depth of the river is indicated at 5-foot intervals in the diagram. If the river flows at 5 ft/s, use Simpson's Rule to estimate the amount of water passing by the dock every second.



56. Prove that Simpson's Rule actually gives the exact answer for definite integrals of all polynomials of degree 3 or less.

7.6 Technology Exercises

57–60 Write a short program for a computer algebra system to approximate the given integral using the Trapezoidal Rule with $n = 100$. Then ask the software to evaluate the integral directly and compare your approximation with the exact value.

57. $\int_0^2 \sqrt{x} \sqrt{x+2} dx$ 58. $\int_0^6 \frac{dx}{3+x^2}$

59. $\int_0^{20} \frac{dx}{1+x^3}$ 60. $\int_0^{10} \sqrt{1+\sqrt{1+\sqrt{x}}} dx$

61–64. Modify the program you wrote for Exercises 57–60 to find S_{100} for the integrals. Compare your answers with the exact values you found in Exercises 57–60.

65–70 Use the program you wrote for Exercises 61–64 to find the Simpson approximation S_{50} for the given nonelementary integral. To get a feel for how close your answer is to the exact value of the integral, use the graph of the fourth derivative of the integrand, and identify an error bound as small as you can by using the theorem from this section. (Answers may vary slightly. **Hint:** You may check your error bound by using the command specific to your CAS to obtain the “exact value” of the integral. Note that the CAS uses numerical algorithms to obtain that answer and, thus, it isn’t exact either. It is, however, more accurate than what we can obtain by Simpson’s Rule with $n = 50$.)

65. $\int_0^1 e^{x^2} dx$

66. $\int_1^3 \frac{e^x}{x} dx$

67. $\int_0^\pi \cos x^2 dx$

68. $\int_1^\pi \frac{\sin x}{x} dx$

69. $\int_2^4 \ln(\ln x) dx$

70. $\int_e^5 \frac{1}{\ln x} dx$

71–73 With the help of a computer algebra system and Simpson’s Rule, approximate the solution of the given integral equation. (Use $n = 100$).

71. $\int_0^x e^{\sqrt{t}} dt = 3$

72. $\int_0^x \tan t^2 dt = 1$

73. $\int_0^x \sqrt{1+t^4} dt = 4$