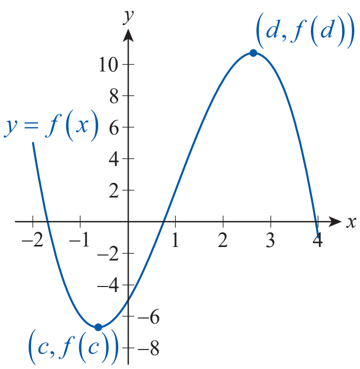


Figure 5

Solution

We begin by observing that f' is positive on the interval (c, d), negative on the intervals $(-\infty, c)$ and (d, ∞), and that $f'(c) = f'(d) = 0$. This tells us that f is increasing on (c, d), decreasing on $(-\infty, c)$ and (d, ∞), and that f has a relative minimum at $x = c$ and a relative maximum at $x = d$. Further, f' has a critical point at 1 and appears to be differentiable there (the graph of f' is nicely smooth), so $f''(1) = 0$; moreover, since f' is increasing to the left of 1 and decreasing to the right of 1, it must be the case that f changes concavity as it passes through the point $(1, 2)$.

Putting all these observations together, the graph of f must be something along the lines of the one in Figure 5, though the actual values of $f(c)$ and $f(d)$ can be nothing more than a rough guess.

The values of f' from the original graph tell us approximately how fast the graph of f rises or falls near a given point—for instance, since $f'(1) = 8$, the "slope" of f at $(1, 2)$ should be 8.

4.7 Exercises

1–8 Verify by differentiating that $F(x)$ is an antiderivative of $f(x)$.

1. $f(x) = \dfrac{1}{\sqrt{x}} + \dfrac{1}{x^2}, \quad F(x) = 2\sqrt{x} - \dfrac{1}{x}$

2. $f(x) = 2(x-1)(x+5), \quad F(x) = \dfrac{2}{3}x^3 + 4x^2 - 10x$

3. $f(x) = -x(x+2)(x-4), \quad F(x) = -\dfrac{1}{4}x^4 + \dfrac{2}{3}x^3 + 4x^2 + \dfrac{5}{3}$

4. $f(x) = 6\cos 3x, \quad F(x) = 2\sin 3x$

5. $f(x) = 5\sec^2(5x+1), \quad F(x) = \tan(5x+1) + 5$

6. $f(x) = \dfrac{x^2+1}{\sqrt{x}}, \quad F(x) = \dfrac{\sqrt{x}}{5}(10 + 2x^2)$

7. $f(x) = \dfrac{2x}{x^2+7}, \quad F(x) = \ln(x^2+7)$

8. $f(x) = \pi^{2x}, \quad F(x) = \dfrac{\pi^{2x}}{2\ln\pi}$

9–20 Find an antiderivative of the function.

9. $f(x) = 1$

10. $g(x) = 2x + 2$

11. $h(x) = 4x^3 - x$

12. $u(x) = x^5 + x^3 + \pi$

13. $v(x) = \sec^2 x + 3x$

14. $k(x) = \dfrac{2}{x}$

15. $f(x) = 5e^x$

16. $m(x) = \dfrac{1}{2\sqrt{x}}$

17. $u(t) = -\dfrac{4}{t^3}$

18. $v(s) = \dfrac{1}{6s^{2/3}}$

19. $w(z) = \dfrac{1}{\sqrt{1-z^2}}$

20. $g(s) = \dfrac{1}{1+s^2} + 1 + s^2$

21–32 Find the general antiderivative of $f(x)$; then find the particular antiderivative $F(x)$ that satisfies $F(1) = 1$.

21. $f(x) = 2x - 3$

22. $f(x) = 3x^2 + \dfrac{1}{2}$

23. $f(x) = \dfrac{1}{\sqrt{x}}$

24. $f(x) = 1$

25. $f(x) = 0$

26. $f(x) = -\dfrac{1}{x}$

27. $f(x) = x^3 - \dfrac{1}{x^2}$

28. $f(x) = \dfrac{-1}{3x^{2/3}}$

29. $f(x) = (\ln 10)10^x$

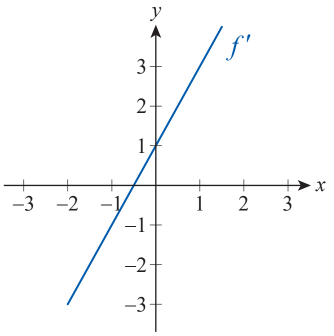
30. $f(x) = \sin x$

31. $f(x) = \frac{\pi}{4}\sec^2\left(\frac{\pi}{4}x\right) + 1$

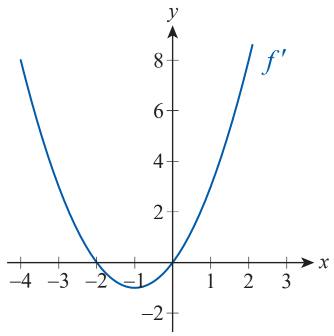
32. $f(x) = \frac{2x}{x^2 + 4}$

33–36 Given the graph of f' and the knowledge that f passes through the point $(3,1)$, sketch a possible graph for f.

33.



34.



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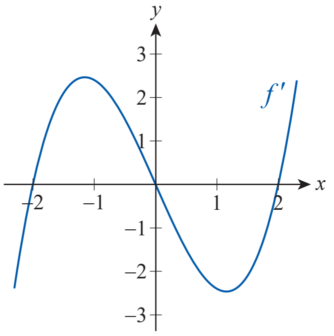
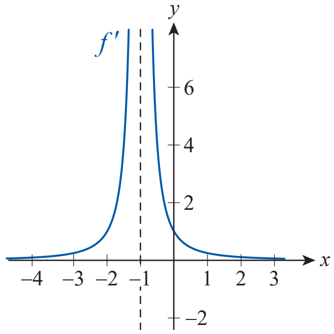


36.



37–60 Find the general antiderivative of the given function, and check your answer by differentiation. (If necessary, rewrite the function before antidifferentiation.)

37. $f(x) = 6x^2 - 4x + 1.5$

38. $g(x) = 5x^3 - \pi x$

39. $h(x) = 3x^5 - 10x^4 + x^2 + 7$

40. $u(x) = -7x^4 + \frac{1}{2}x^3 + 6x^2 - 8x + \frac{5}{2}$

41. $v(x) = 3(x+6)(2x+1)$

42. $k(x) = -x(x+3)(7x-5)$

43. $h(x) = x^3\sqrt{x}$

44. $m(x) = \frac{3}{\sqrt{x}} + 2x\sqrt[3]{x}$

45. $n(x) = \frac{x^3 + 7x}{x^2}$

46. $f(t) = \frac{t^2 - t}{\sqrt{t} + 1}$

47. $a(y) = \left(\sqrt[3]{y^4} - 1\right)^2$

48. $w(z) = \frac{2}{z} + \frac{2}{\sqrt{z}}$

49. $g(t) = e^{3t} - 3\sec t \tan t$

50. $s(t) = 2 \cdot 10^{1.5t}$

51. $t(\theta) = \theta + \cos\theta$

52. $c(\theta) = \theta^2 + \csc^2\theta$

53. $v(x) = (\csc x - \cot x)\csc x$

54. $t(x) = -\sec^2 x\left(\cos^2 x + \sin^2 x\right)$

55. $w(x) = \frac{\cos x}{\cos^2 x - 1}$

56. $u(x) = \frac{2\tan 2x}{2\cos^2 x - 1}$

57. $a(x) = \frac{5}{1 + 9x^2}$

58. $b(x) = \frac{1}{\sqrt{1 - 4x^2}}$

59. $c(x) = \frac{4}{|5x|\sqrt{25x^2 - 1}}$

60. $d(x) = \frac{-3}{\sqrt{4 - 36x^2}}$

61–76 Find $f(x)$ that satisfies the specified conditions. (When no initial conditions are specified, find the general antiderivative.)

61. $f''(x) = \pi, \quad f'(1) = 0, \quad f(1) = 0$

62. $f''(x) = 1 - 4x, \quad f'(-1) = 1, \quad f(-1) = -4$

63. $f'''(x) = 0, \quad f''(2) = 2, \quad f'(2) = 2, \quad f(2) = 2$

64. $f'''(x) = x + 1, \quad f''(0) = 1, \quad f'(0) = 2, \quad f(0) = 3$

65. $f''(x) = \sqrt[3]{x}, \quad f'(1) = 0, \quad f(1) = \frac{1}{7}$

66. $f''(x) = x + \frac{1}{\sqrt{x}}, \quad f'(4) = 6, \quad f(4) = 0$

67. $f'''(x) = \sqrt{x} + 1, \quad f''(0) = 1, \quad f'(0) = -1, \quad f(0) = 7$

68. $f''(x) = \sqrt[3]{x}(x-3), \quad f'(0) = 0, \quad f(0) = 0$

69. $f'(x) = \dfrac{4}{1+4x^2}, \quad f\left(\dfrac{1}{2}\right) = \pi$

70. $f'(x) = \dfrac{-1}{\sqrt{1-3x^2}}, \quad f\left(\dfrac{\sqrt{3}}{3}\right) = 0$

71. $f'''(x) = -\cos 2x, \quad f''(0) = 1, \quad f'(0) = 1, \quad f(0) = -1$

72. $f'''(x) = \cos x - \sin x$

73. $f''(x) = 2^{5x}$

74. $f'''(x) = e^x + e, \quad f''(0) = 1, \quad f'(0) = 2, \quad f(0) = 3$

75. $f''(x) = \cos x - e^{2x}, \quad f'(0) = -2, \quad f(0) = 1$

76. $f'''(x) = \sin 10x + 10x + 10, \quad f''(0) = 0,$
$f'(0) = 3.5, \quad f(0) = -0.5$

77–85 Use $-32\ \text{ft/s}^2$ for the acceleration caused by gravity ($-9.81\ \text{m/s}^2$ in the metric system). Ignore air resistance. (**Hint:** See Example 4.)

77. A soccer ball is kicked upward from a height of 3 feet with an initial velocity of 48 feet per second. How high will it go?

78. A student drops a pen from a classroom window on the fourth floor of the mathematics building. If the window is 48 ft above ground level, how long is the pen in the air and with what speed does it hit the ground?

79. A hiker throws a pebble into a canyon that is 350 meters deep, with a downward initial velocity of $10\ \text{m/s}$. For how many seconds is the pebble in the air and what is the speed of impact?

80. A baseball is thrown upward from a height of 1.5 meters with an initial velocity of 30 meters per second. How high will it go, and for how long is it going to rise?

81.* With what initial velocity do we need to throw a tennis ball vertically upward in order for it to reach the top of a 60 ft campus flagpole?

82. An air rifle shoots a pellet at 1200 feet per second. What is the horizontal range of the rifle, that is, how far from where the pellet is shot will it hit the ground, if we shoot horizontally from a height of 5 feet?

83. A golf ball is hit horizontally at 40 meters per second from the top of a slight hill that is 1.5 meters high. If the terrain around the hill is nearly flat, approximately how far will the golf ball fly?

84. Prove that the position function of an object thrown vertically from an initial height of h_0 feet with an initial velocity of v_0 feet per second is $h(t) = -16t^2 + v_0 t + h_0$.

85. Repeat Exercise 84 using the metric system (meters and seconds) to arrive at the formula $h(t) = -4.905t^2 + v_0 t + h_0$.

86. The acceleration due to gravity on the lunar surface is approximately $-5.25\ \text{ft/s}^2$. How high would the soccer ball of Exercise 77 fly on the moon?

87. Find out what would happen in the situation described in Exercise 83 under lunar conditions. (See Exercise 86 for the acceleration due to gravity on the moon.)

88. The rate of growth of a rabbit population in a certain state park, where food supply is limited and predators are present, is proportional to $e^{-0.1t}$, where t is time measured in months. If the initial population size is 300 rabbits, which grows to 400 in three months, find the population size in a year. (**Hint:** Let $P(t)$ stand for population size, and use $\dfrac{d}{dt}P(t) = ke^{-0.1t}$.)

89. The rate of growth of a population of a certain virus in a medical experiment is proportional to $\sqrt[3]{t}$, where t is time measured in days. If the initial population size is 1000, which grows to 1500 in a day, find the population size in five days. (See and appropriately modify the hint given in Exercise 88.)

90.* A modern Formula One car is able to come to a complete stop from $200\ \text{km/h}$ (124.3 mph) using a braking distance of only about 65 meters. Assuming constant deceleration (which is not fully realistic), what multiple of g is this? (**Hint:** $1\ \text{m/s} = 3.6\ \text{km/h}$)

91. The Bugatti Veyron, the fastest production grand tourer from 2010 to 2017, can go from 0 to $100\ \text{km/h}$ in 2.5 seconds. Find its position function when accelerating from a standstill and the distance covered during the first 1.5 seconds. What is the car's acceleration time from 0 to 60 mph? (Use the simplifying assumption that acceleration is constant. Also see the hint provided in Exercise 90.)

92. Jerry the mouse is running toward his hole at a steady speed of $11\ \text{ft/s}$. Still 20 feet from his destination, he is discovered by Tom the cat, who is 2 feet behind Jerry at that moment. If Tom can reach his top speed of $40\ \text{ft/s}$ in 3 seconds, will he be able to catch Jerry? (Suppose the locations of Tom, Jerry, and the mousehole remain collinear throughout the pursuit.)

93.* Assume that an airplane needs to reach a liftoff speed of 180 mph and that it can achieve the same on a runway that is 0.8 miles long. Assuming constant acceleration during takeoff, what would this acceleration be?

94. The acceleration function of a particle moving along the x-axis is $a(t)=3\sqrt{t}-\frac{1}{\sqrt{t}}$ units/s^2. If it starts at the origin with an initial velocity of 2 units per second, find the position function of the particle. Where will it be in 5 seconds?

95. Repeat Exercise 94 for the acceleration function $a(t)=(2-t)\sqrt{t}$, if the particle starts from rest at the point $(3,0)$. Where will it be in 5 seconds, and when will its instantaneous velocity be zero?

96. It follows from our discussions in Section 3.6 as well as the present section that an antiderivative of $-1/\sqrt{1-x^2}$ can be written as $-\sin^{-1}x$. Use the graphs of inverse trigonometric functions provided in Section 3.6 to argue that $\cos^{-1}x$ is also an antiderivative of $-1/\sqrt{1-x^2}$. (It follows that the general antiderivative of $-1/\sqrt{1-(kx)^2}$, $|kx|<1$ is $(1/k)\cos^{-1}(kx)+C$. See also Exercise 67 of Section 3.6.)

Concept Check

97–103 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

97. If $f(x)$ has an antiderivative on an interval I, then it has infinitely many antiderivatives on the same interval.

98. All polynomials have antiderivatives on the entire real line $\mathbb{R}$.

99. It is possible for a function to have an unique antiderivative on an interval I.

100. Whenever F_1 and F_2 are both antiderivatives of f on an open interval, then F_1-F_2 is a constant function.

101. If a function has an antiderivative on the interval $(-a,a)$ for some $a>0$, then it has exactly one antiderivative whose graph goes through the origin.

102. Every antiderivative of a polynomial function of degree n has degree $n+1$.

103. If $F(x)$ is an antiderivative of $f(x)$, and $G(x)$ is an antiderivative of $g(x)$ on an interval I, then $F(x)\cdot G(x)$ is an antiderivative of $f(x)\cdot g(x)$ on the same interval.