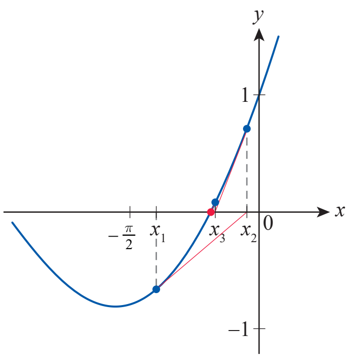


Figure 10
Zoomed-In Graph of
$f(x) = e^x + \sin x$

The largest negative root of f lies somewhere to the right of $-\pi/2$, as the close-up in Figure 10 indicates. An initial guess of 0 would work well to begin Newton's method, but for illustrative purposes we use $x_1 = -1.25$. Applying Newton's method, we obtain the following formula for x_{n+1}.

$$x_{n+1} = x_n - \frac{e^{x_n} + \sin x_n}{e^{x_n} + \cos x_n}$$

The first few approximations are as follows.

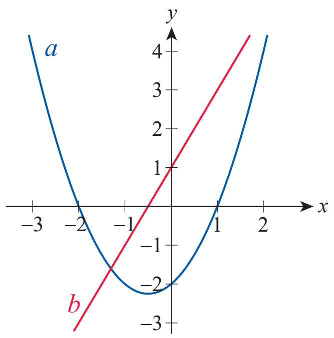
$$\begin{array}{ll} x_1 = -1.25 & x_2 \approx -0.149219 \\ x_3 \approx -0.534414 & x_4 \approx -0.587419 \\ x_5 \approx -0.588532 & x_6 \approx -0.588533 \end{array}$$

Since x_5 and x_6 agree to five decimal places, the largest negative root of f is approximately -0.58853.

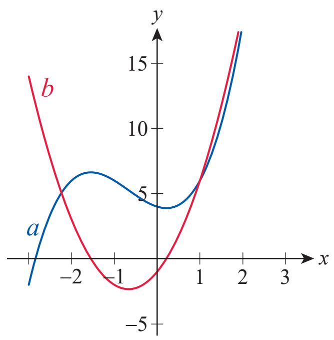
4.5 Exercises

1–4 The graphs of the first and second derivatives of a function f are given. Identify which one is which, and then sketch a possible graph of f. (Answers for the graph of f will vary.)

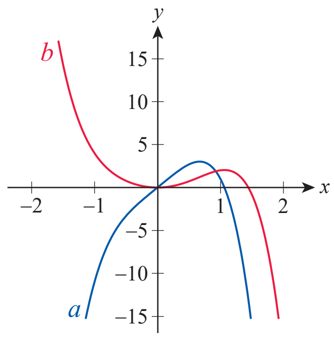
1.



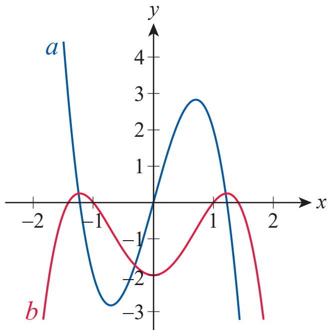
2.



3.



4.



5–48 Use the curve-sketching strategy to construct a graph of the function.

5. $f(x) = x^3 + 3x^2 - 9x$

6. $g(x) = -x^3 + 2x^2 - x + 4$

7. $h(x) = \frac{1}{4}x^4 + \frac{5}{3}x^3 + x^2 - 8x$

8. $F(x) = -\frac{3}{4}x^4 + x^3 + 9x^2 + 2$

9. $G(x) = (x^2 - 1)(x^2 - 2)$

10. $k(x) = x^5 - 2x^3 - 8x + 1$

11. $L(x) = x^5 - 3x^2$

12. $m(x) = 4x^3 - 5x^4$

13. $n(x) = \frac{-3}{x - 2}$

14. $H(x) = \frac{x^2 + 2}{x + 2}$

15. $R(x)=\frac{x}{x^2-4}$

16. $r(x)=\frac{2x^2+1}{x-3}$

17. $A(x)=|x-3|-2$

18. $f(x)=1.5-|x-2.2|$

19. $w(x)=x^{2/5}+\frac{2}{5}$

20. $u(x)=(x-2)\sqrt{x}$

21. $F(x)=2-(x-1)^{3/5}$

22. $G(x)=\sin^2 x-1$

23. $h(x)=e^{-x}+e^{2x}$

24.* $H(x)=-2\sqrt{x}\cdot 2^{-2x}$

25. $P(x)=x\ln x$

26. $H(x)=0.3\sqrt[3]{x}(x^2-1)$

27. $G(x)=x\sqrt{4-x^2}$

28. $L(x)=\frac{3}{x-2}$

29. $m(x)=\frac{x^2+7}{x-7}$

30. $K(x)=\frac{e^x}{x}$

31. $F(x)=e^x-e^{-2x}$

32. $v(x)=(x-1)^{3/5}$

33. $m(x)=-\frac{5}{(x-2)^2}$

34. $R(x)=\frac{x+2}{x-4}$

35. $G(x)=\frac{-x}{x^2-1}$

36. $t(x)=\frac{2x^2+2}{x-4}$

37. $H(x)=\frac{3}{4}\left(x-\frac{4}{3}\right)^{4/3}$

38. $w(x)=\frac{(x-1)^2}{2x^2-2}$

39. $k(x)=x-\sqrt[3]{x}$

40. $F(x)=x^{4/5}\left(x-\frac{4}{5}\right)$

41. $c(x)=x\sqrt[3]{1-x^2}$

42. $G(x)=-\sqrt{x}e^{-x}$

43. $Z(x)=2\sin x-\cos^2 x$

44. $K(x)=\sin x-\cos x$

45. $L(x)=x^{5/3}\ln|x|$

46. $G(x)=\sqrt{4x^2+3}$

47. $u(x)=7-\sqrt{9x^2+2x+1}$

48. $z(x)=e^{\cos x}$

49–54 First prove that $\lim_{x\to\pm\infty}(f(x)-g(x))=0$. This means that when $x\to\pm\infty$, the graph of $f(x)$ approaches that of $g(x)$. Use this observation as an aid in graphing $f(x)$. (In this case, we say that $f(x)$ is asymptotic to $g(x)$.)

49. $f(x)=\frac{x^3+5}{x+2},\quad g(x)=x^2-2x+4$

50. $f(x)=\frac{(x+1)^4+2}{3x+3},\quad g(x)=\frac{1}{3}(x+1)^3$

51. $f(x)=\sqrt{4x^2+5},\quad g(x)=|2x|$

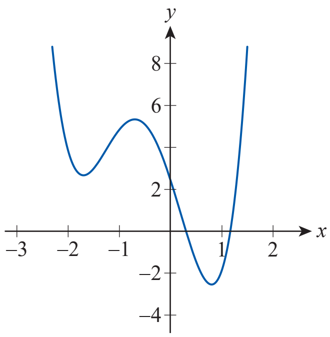
52. $f(x)=\sqrt{x^2-4x+5},\quad g(x)=|x-2|$

53. $f(x)=\sqrt[3]{x}+\frac{1}{x^2},\quad g(x)=\sqrt[3]{x}$

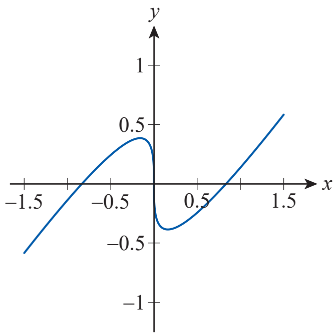
54. $f(x)=\sin x+\frac{1}{x},\quad g(x)=\sin x$

55–56 Sketch on paper a few of the tangent lines that are used to approximate the largest root of the indicated function by Newton's method, using the starting values of –1, 0, and 1, respectively. Does the method always work? Explain.

55.



56.



57–60 Use Newton's method to approximate the given number to five decimal places.

57. $\sqrt[4]{50}$

58. $\sqrt[10]{10}$

59. $\ln 5$

60. $\ln 100$

61–70 Use Newton's method to approximate the zero(s) of the given function to five decimal places. Restrict the domain to the given interval where indicated.

61. $f(x)=x^3-x+2$

62. $f(x)=2x^3+x^2-5x+1$

63. $f(x)=x^4-6.1x^3+4.7x^2-12.2x+5.4$

64. $f(x)=0.25x^4-2x^2+x+0.69$

65. $f(x)=x^5+x+1$

66. $f(x)=2x^5-5x^4+2x^3-4x^2+1$

67. $f(x)=4.2x-\sqrt{x+3}$

68. $f(x)=\sqrt{2+x^2}-1.1x$

69. $f(x)=2x^2-\cos(x-1);\quad \left(0,\frac{\pi}{2}\right)$

70. $f(x)=\sin(2x+1)-\frac{x}{2};\quad (0,1)$

71–76 Use Newton's method to solve the equation on the given interval. Approximate the root to six decimal places.

71. $\sin x = x^2$ on $\left(0, \frac{\pi}{2}\right)$

72. $2 - x^3 = e^x$ on $\mathbb{R}$

73. $x^4 = \arctan x$ on $(0, \infty)$

74. $\ln x = 2 - \sqrt{x}$ on $(0, \infty)$

75. $\cos x = \tan x$ on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

76. $\log_{1/2} x = \sin x$ on $(0, \infty)$

77–80 Recall from Exercise 62 of Section 4.2 that $c \in \mathbb{R}$ is said to be a fixed point of $f(x)$ if $f(c) = c$. Use Newton's method to approximate to four decimal places the fixed point(s) of the function on the given interval.

77. $f(x) = e^{-x}$ on $(0, \infty)$

78. $f(x) = \cos x$ on $\mathbb{R}$

79. $f(x) = 2\cot x$ on $(0, 2\pi)$

80. $f(x) = \log_{1/2} x$ on $(0, \infty)$

81–82 Use Newton's method to find the critical point(s) of the function correct to five decimal places.

81. $f(x) = x^5 - x^3 - 5x$

82. $f(x) = x^2 \sin x, \quad 0 < x < \pi$

83–87 Perform the first few iterations of Newton's method for the given function with the indicated first guess, and explain why the method doesn't work.

83. $f(x) = \sin x - \cos x; \quad x_1 = -\frac{\pi}{4}$

84. $f(x) = x^3 - 6x^2 + 12x - 6; \quad x_1 = 3$

85. $f(x) = \begin{cases} -\sqrt{-x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}; \quad x_1 = a \ (a \ne 0)$

86. $f(x) = \sqrt[3]{x}; \quad x_1 = a \ (a \ne 0)$

87. $f(x) = -x^3 + 9x^2 - 19x + 19; \quad x_1 = 3$

88. The following rule for approximating the square root of a has been known since ancient times.

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

Use Newton's method to derive this rule. (**Hint:** Start with the equation $x^2 - a = 0$.)

89. Generalizing Exercise 88, use Newton's method to derive a rule for approximating $\sqrt[k]{a}$, $k \ge 3$.

90. Using the approach you have taken in the previous two exercises, derive the following formula approximating $1/a$.

$$x_{n+1} = x_n(2 - ax_n)$$

91–96 Use the formulas you derived in Exercises 88–90 to approximate the given number to five decimal places.

91. $\sqrt{2}$

92. $\sqrt{50}$

93. $\sqrt[3]{10}$

94. $\sqrt[5]{30}$

95. $\frac{1}{7}$

96. $\frac{1}{19}$

4.5 Technology Exercises

97. Use a graphing utility to approximate π by generating the first 10 iterations of Newton's method for solving the equation $\sin x = 0$ with an appropriate starting value.

98. Repeat Exercise 97 for the equation $(x-5)^{50} = 0$ with the starting value of $x_1 = 6$. What do you find? Graph $f(x) = (x-5)^{50}$, and see if the graph gives insight into why things went wrong.

99–100 Perform the first two iterations of Newton's method with each of the given starting values in an attempt to find the positive root of $f(x)$; then use a graphing utility to come up with better approximations. What do you find? Graph $f(x)$, and see if the graph gives insight into why things went wrong.

99. $f(x) = x^3 - 2x - 1; \quad x_1 = 0.9, \quad x_1 = 0.8, \quad x_1 = -0.4$

100. $f(x) = x^4 - 6x^3 + 9.5x^2 - 1.5x - 4.9375;$
$x_1 = 3, \quad x_1 = 2.9, \quad x_1 = 0$