$$30{,}000 + \frac{50{,}000(x-3)}{\sqrt{1+(3-x)^2}} = 0$$

$$\frac{(x-3)}{\sqrt{1+(3-x)^2}} = -\frac{30{,}000}{50{,}000}$$

$$\frac{(x-3)^2}{1+(3-x)^2} = \frac{9}{25}$$ Square both sides.

$$25(x-3)^2 = 9 + 9(x-3)^2$$ Note that $(3-x)^2 = (x-3)^2$.

$$16(x-3)^2 = 9$$

$$x - 3 = \pm\frac{3}{4}$$ Divide by 16 and take the square root.

$$x = 3 \pm \frac{3}{4}$$

The point $3+\frac{3}{4}$ does not actually solve the original equation, so we only have to evaluate C at the two endpoints 0 and 3 and at the critical point $3-\frac{3}{4}=\frac{9}{4}$.

$$C(0) = 50{,}000\sqrt{10} \approx \$158{,}114$$

$$C\left(\frac{9}{4}\right) = \$130{,}000$$

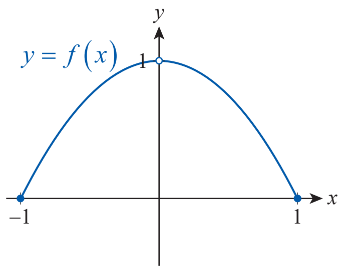
$$C(3) = \$140{,}000$$

From this comparison, we see that the minimal cost of installation can be achieved by laying the cable with an underground run of $\frac{9}{4} = 2\frac{1}{4}$ kilometers and a diagonal run underwater of $\sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{\frac{25}{16}} = 1\frac{1}{4}$ kilometers.

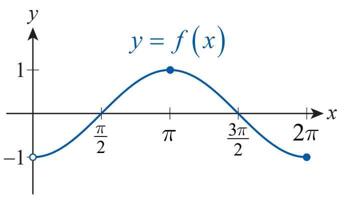
4.1 Exercises

1–4 Use the graph as an aid to identify the absolute extrema, if they exist, for the given function on the specified domain.

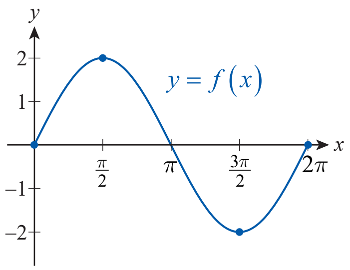
1. $f(x) = -x^2 + 1;\quad D = [-1, 0) \cup (0, 1]$



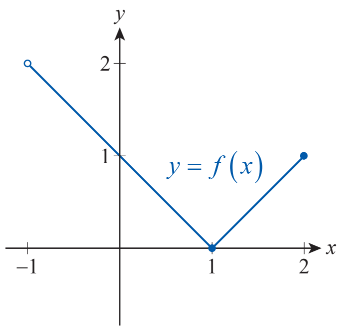
2. $f(x) = -\cos x;\quad D = (0, 2\pi]$



3. $f(x) = 2\sin x; \quad D = [0, 2\pi]$

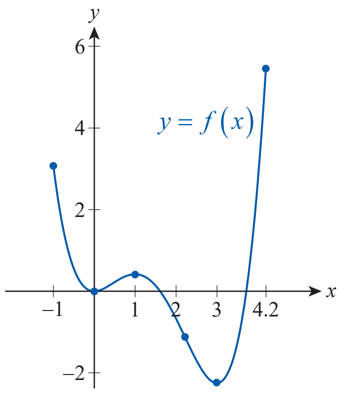


4. $f(x) = |x-1|; \quad D = (-1, 2]$



5–8 Use the graph to decide whether each highlighted point is a critical point, and then find and classify all relative and absolute extrema for the function over the given interval.

5.



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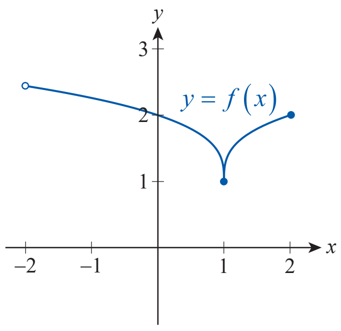

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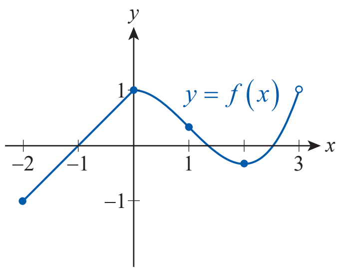
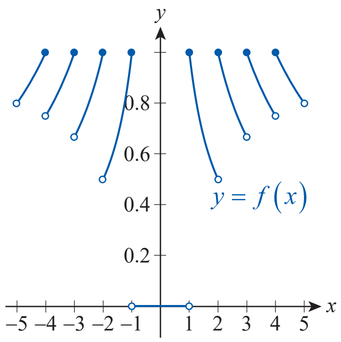


8.



9–20 Use graph paper to sketch the graph of the given function on the specified domain, and then use the graph to visually identify and classify any absolute extrema.

9. $f(x) = 2x+1; \quad D = [0, 3]$

10. $g(x) = -x-1; \quad D = (-1, 2]$

11. $h(x) = \frac{1}{2}x - 3; \quad D = \mathbb{R}$

12. $u(x) = -x^2; \quad D = (-1, 1)$

13. $v(x) = (x+1)(x-3); \quad D = [-2, 4]$

14. $k(x) = (x-4)^4; \quad D = \mathbb{R}$

15. $K(x) = x^7; \quad D = \mathbb{R}$

16. $m(x) = e^{-x+2}; \quad D = [2, \infty)$

17. $n(x) = \cos \pi x; \quad D = \left(0, \frac{3}{2}\right]$

18. $F(x) = \frac{1}{(x+1)^2}; \quad D = \mathbb{R}$

19. $G(x) = \frac{1}{x^2+1}; \quad D = \mathbb{R}$

20. $H(t) = \arcsin t; \quad D = [-1, 1]$

21–37 Sketch by hand the graph of a function f on the specified domain, with the specified properties. (Answers will vary.)

21. Defined on $[2,4]$, absolute maximum at 2, absolute minimum at 4

22. Defined on $[-1,2]$, absolute maximum at 0, absolute minimum at 1

23. Defined on $[-5,5]$, absolute maximum at 1, absolute minimum at 5

24. Defined on $\mathbb{R}$, absolute minimum at 2, no absolute maximum

25. Defined on $[-3,2]$, absolute maximum at –2, absolute minimum at 0, relative maximum at 1

26. Defined on $[0,6]$, absolute maximum at 2, relative minimum at 4, no absolute minimum

27. Defined on $[-1,1]$, absolute maximum occurs twice, no minimum

28. Defined on $(1.5,7)$, continuous, has relative maximum and minimum, but no absolute maximum or minimum

29. Defined on $(1.5,7)$, continuous, has both absolute maximum and minimum

30. Defined on $[-2,4]$, two relative maxima, but no absolute maximum

31. Defined on $(0,\infty)$, continuous, no relative or absolute extrema

32. Defined on $(-1,3]$, continuous, no absolute minimum, one relative minimum, absolute maximum occurs twice

33. Defined on $\mathbb{R}$, both the absolute maximum and absolute minimum occur infinitely often

34. Defined on $(0,\infty)$, infinitely many relative maxima and minima, no absolute maximum or minimum

35. Differentiable on $\mathbb{R}$, has one critical point, but no extrema

36. Defined on $(0,10)$, not differentiable at 5, but absolute maximum occurs at 5

37. Defined on $(0,10)$, discontinuous at 5, but absolute maximum occurs at 5

38–55 Find all critical points, if they exist, for the given function.

38. $f(x)=x^2-7x+1.5$

39. $g(x)=2x^3+3x^2-12x+1.5$

40. $h(x)=x^3+1.5x^2+3x-2.5$

41. $u(x)=-\frac{3}{2}x+2$

42. $v(x)=x^4-\frac{16}{3}x^3+2x^2+24x-1$

43. $k(x)=|2x-3|$

44. $K(x)=|3x^2+3x-18|$

45. $m(x)=\frac{2-x}{x^2-x+2}$

46. $n(x)=\frac{|x^2-2|}{2x^2+4}$

47. $F(t)=\sqrt{3+3t^2}$

48. $G(x)=x^{3/2}-3\sqrt{x}$

49. $T(s)=2\sqrt[3]{s}(s-2)$

50. $r(v)=\frac{v-1}{\sqrt{v}}$

51. $s(\alpha)=\cos\alpha+\cos^2\alpha$

52. $u(z)=\cot z+2z$

53. $t(x)=\sqrt{x}\ln x$

54. $U(t)=e^t\sin t$

55. $a(t)=\cos(\arctan t)$

56–77 Find all absolute extrema of the function on the given closed interval.

56. $f(x)=4x-x^2$ on $[0,6]$

57. $g(x)=3x^2-30x+7$ on $[0,8]$

58. $h(x)=x^3+1.5x^2-6x+3.5$ on $[-4,3]$

59. $u(x)=3x^4-8x^3+6x^2-24x-9$ on $[0,3]$

60. $v(x)=\frac{x^4}{4}-2x^2+4$ on $[-2,2]$

61. $k(x)=\frac{x^4}{2}+2x^3-x^2-6x+\frac{1}{2}$ on $[-3,1]$

62. $f(x)=|x+3|\cdot|x-3|$ on $[-4,4]$

63. $m(x)=|x+3|+|x-3|$ on $[-4,4]$

64. $n(x)=\frac{3x}{2x^2+2}$ on $[-4,4]$

65. $g(x)=\frac{x^2+5}{x+2}$ on $[-1.5,1.5]$

66. $F(x)=\frac{1}{1+x^2}$ on $[-10,10]$

67. $G(t)=\frac{1}{\sqrt{t}}+\sqrt{t}$ on $\left[\frac{1}{4},4\right]$

68. $k(s)=(s^2-1)\sqrt{s}$ on $[0,2]$

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69. $r(z) = \sin(\arccos z)$ on $[-1,1]$

70. $G(x) = \arctan x$ on $[-1,1]$

71. $w(x) = x\sqrt{8-x^2}$ on $\left[-2\sqrt{2}, 2\sqrt{2}\right]$

72. $T(s) = s^2e^{-s}$ on $[0,10]$

73. $r(x) = (\cos x)e^x$ on $\left[0, \frac{3\pi}{2}\right]$

74. $L(x) = x\ln x$ on $\left[\frac{1}{e^2}, e\right]$

75. $t(x) = \ln((e-1)\sin \pi x + 1)$ on $[0,1]$

76. $U(x) = \sqrt[3]{x}(x-3)$ on $[-1,3]$

77. $V(x) = 5 + (5+x)x^{5/7}$ on $[-5,1]$

78–89 Find and classify the absolute extrema, if they exist, of the function over the given domain.

78. $f(x) = 3x - 2;\quad D = (0,2]$

79. $g(x) = x^2 - 4;\quad D = (-2,2)$

80. $h(x) = 2x^3 - 5x;\quad D = \mathbb{R}$

81. $K(z) = \sqrt{4-z^2};\quad D = (-2,2)$

82. $r(z) = -\frac{2}{z};\quad D = [2,\infty)$

83. $n(x) = \frac{1}{(x+3)^2};\quad D = (-3,\infty)$

84. $t(x) = 10^{x/2};\quad D = \mathbb{R}$

85. $L(x) = \ln(x+1);\quad D = [0,\infty)$

86. $F(x) = \sec x;\quad D = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

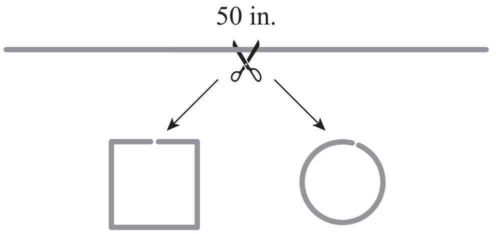
87. $t(z) = 2\cos \pi z + 2;\quad D = \mathbb{R}$

88. $u(x) = x - [\![x]\!];\quad D = [1,3)$

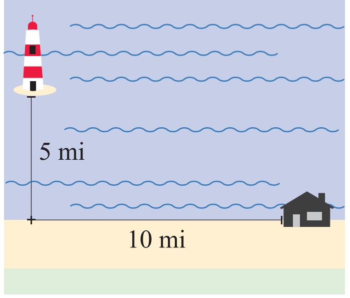
89. $v(x) = \arctan x;\quad D = \mathbb{R}$

90. Find two numbers whose sum is 50 and whose product is as large as possible. (**Hint:** Denote the numbers by x and $50 - x$, and maximize the product.)

91. A 50-inch piece of wire is cut into two pieces, which are then bent into a square and a circle, respectively. Where should the wire be cut in order to minimize the sum of the areas of these two shapes? (**Hint:** Start with the notation of Exercise 90, and use appropriate formulas from geometry.)



92. A lighthouse is 5 miles off a straight shoreline. Ten miles down the coast is a restaurant where the lighthouse keeper is planning to meet his friends. If he can row at 2.5 mph and walk at 4 mph, where should he land in order to make the fastest possible time to the restaurant?



93. Referring to Exercise 26 of Section 3.7, find the number of calculators that have to be produced in order to maximize profit.

94. The power output of a 12-volt car battery when a resistor is connected to it, is given by the formula $P = 12I - (R+r)I^2$, where I is the current (in amperes), and r stands for the (typically very small) so-called internal resistance of the battery. Suppose we are starting a car with a starter motor of resistance $R = 0.16$ ohms, and that the internal resistance of the battery is $r = 0.016$ ohms. Find the current that corresponds to the battery's maximum power output.

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Concept Check

95–105 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

95. If f attains both its absolute minimum and absolute maximum values on a closed interval, then f is a continuous function.

96. A continuous function on a closed interval can attain its absolute extrema only at critical points.

97. If $f(x)$ is a differentiable function and k is a constant, then $f(x)$ and $f(x)+k$ have the same critical points.

98. If $f(x)$ is a differentiable function and k is a nonzero constant, then $f(x)$ and $kf(x)$ have the same critical points.

99. If $f(x)$ is a differentiable function and k is a nonzero constant, then $f(x)$ and $f(x+k)$ have the same critical points.

100. If $f(x)$ is a differentiable function and k is a nonzero constant, then $f(x)$ and $f(kx)$ have the same critical points.

101. If $f(x)$ has a maximum at c, then so does $f(-x)$ at $-c$.

102. If $f(x)$ has a maximum at c, then $-f(x)$ has a minimum at c.

103. A function $f(x)$ can have more than one absolute maximum value.

104. If $f(x)$ is continuous on a closed interval I, then it attains its minimum value on I.

105. If $f(x)$ has no maximum on a closed interval I, then $f(x)$ must be discontinuous on I.

4.1 Technology Exercises

106–127. Use a graphing utility to verify the answers you obtained for Exercises 56–77.

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