3.6 Exercises

1–15 Use the Derivative Rule for Inverse Functions to determine $(f^{-1})'(a)$ for the indicated value of *a*. (In these and subsequent exercises, the domain of *f* is assumed to have been restricted so that the inverse exists and is differentiable, whenever appropriate.)

1. $f(x)=x^3;\quad a=8$

2. $f(x)=2x-1;\quad a=5$

3. $f(x)=\sqrt{x};\quad a=3$

4. $f(x)=\sqrt[3]{x+2};\quad a=-1$

5. $f(x)=x^2+5;\quad a=9$

6. $f(x)=x^{3/2};\quad a=27$

7. $f(x)=\dfrac{2x}{x-1};\quad a=4$

8. $f(x)=\dfrac{5}{(x-1)^3};\quad a=5$

9. $f(x)=\dfrac{3}{x^2+2};\quad a=1$

10. $f(x)=e^{2x};\quad a=5$

11. $f(x)=10^x;\quad a=10$

12. $f(x)=2^{\sqrt{x}};\quad a=8$

13. $f(x)=\sin x;\quad a=\dfrac{\sqrt{3}}{2}$

14. $f(x)=2\tan^{-1}x;\quad a=\dfrac{\pi}{2}$

15. $f(x)=\sin(x^2);\quad a=\sin 0.01$

16–30 Determine the value of $(g^{-1})'(b)$ at the given point (assume that the domain of *g* is appropriately restricted so that g^{-1} exists). (**Note:** Do *not* attempt to find a formula for g^{-1}.)

16. $g(x)=x^5+2x+1;\quad b=g(1)$

17. $g(x)=x^6-11x^4+x;\quad b=g(-1)$

18. $g(x)=x^{100}+x^{50}+1;\quad b=g(-1)$

19. $g(x)=\sqrt{x^4+x^2};\quad b=g(-2)$

20. $g(x)=(3x^8+x^3+1)^{3/2};\quad b=g(1)$

21. $g(x)=(2x^9-3\sqrt{x})^{2/5};\quad b=g(1)$

22. $g(x)=x^5+x+2;\quad b=2$

23. $g(x)=x^{17}+2x^{11}-2x+3;\quad b=4$

24. $g(x)=\dfrac{x^3+8}{\sqrt{x+1}};\quad b=g(2)$

25. $g(x)=\dfrac{x+1}{x^3};\quad b=\dfrac{3}{8}$

26. $g(x)=e^{x^4-x+2};\quad b=g(-2)$

27. $g(x)=x\sin x;\quad b=\dfrac{\pi}{2}$

28. $g(x)=10^{\cos(x^3+x)};\quad b=g(1)$

29. $g(x)=\tan\sqrt{x};\quad b=g(1)$

30. $g(x)=x^3e^{x^2+1};\quad b=e^2$

31–48 Determine the derivative of the given function.

31. $f(x)=\ln(x^3)$

32. $g(x)=(\ln x)^3$

33. $h(x)=\ln(x^2+3)$

34. $F(x)=\ln\left(x\sqrt{x^2+4}\right)$

35. $G(x)=x\ln\sqrt{x^2+4}$

36. $k(x)=\ln\dfrac{2x}{x^2+1}$

37. $L(x)=\dfrac{\ln 2x}{x^2+1}$

38. $f(x)=\ln\sqrt{\dfrac{x+3}{2x+5}}$

39. $g(x)=\ln\sqrt[3]{\dfrac{x+3}{x-3}}$

40. $H(x)=\ln(\ln x)$

41. $F(t)=\ln\left(\sqrt{t^2+4}+2t\right)$

42. $L(s)=\ln\dfrac{\sqrt{s^2+2}}{s^4+s^2+1}$

43. $T(x)=\ln|\cos x|$

44. $C(t)=\ln(\sin^2 t+1)$

45. $v(x)=\cos 2x(\ln(\cos 2x))$

46. $F(t)=\dfrac{\log_5 t}{t^2}$

47. $w(x)=x\log x$

48. $t(x)=\log_{3/2}\left((5x^2+4)^{3/2}\right)$

49–66 Use logarithmic differentiation to find y'.

49. $y=(x+1)(x+2)(x+3)(x+4)$

50. $y=\dfrac{(x+1)(x+2)}{(x+3)(x+4)}$

51. $y=\sqrt[3]{(2x-1)(x-5)(3x+1)}$

52. $y=\dfrac{(x^2-1)^{2/3}(5x^3+3)}{(x^2+x+2)(x^4-10)^{3/4}}$

53. $y=\dfrac{\sqrt[3]{x^3-5x^2+7}\,(x+2)}{x^{2/3}\sqrt{3x^2+4}}$

54. $y=\sqrt[3]{\dfrac{x^3-2x^2+1}{(x^2-1)(x^3+5)}}$

55. $y=\dfrac{x^2\sqrt[5]{x^3+3}}{\sqrt[4]{x^4+4}}$ **56.** $y=x^{x^2}$

57. $y=(\sin x)^{1/x}$ **58.** $y=(2x^2+1)^{\tan x}$

59. $y=(\cos x)^{\sqrt{x}}$ **60.** $y=(\sqrt[3]{x})^{\sqrt[3]{x}}$

61. $y=(\ln x)^x$ **62.** $y=\dfrac{(\ln x)^x(x^3-1)}{e^x+2}$

63. $y=x^{x^x}$ **64.** $y=(\sin x)^{\cos x}$

65. $y=(\ln x)^{\sin x}$ **66.** $y=(e^x)^x$

67. Mimic the procedure seen in the text to find a formula for the derivative of $y=\cos^{-1}x$.

68. Find a formula for the derivative of $y=\sec^{-1}x$ (see Exercise 67).

69. Find a formula for the derivative of $y=\cot^{-1}x$ (see Exercise 67).

70–93 Determine dy/dx. (Recall that $\arcsin x$ is just a different notation for $\sin^{-1}x$, and the same holds for the other inverse trigonometric functions.)

70. $y=\cos^{-1}(x^2)$ **71.** $y=\tan^{-1}(2x+1)$

72. $y=x\arcsin x$ **73.** $y=\ln(\arctan x)$

74. $y=(\operatorname{arccot} x)^2$ **75.** $y=\arccos\sqrt{x}$

76. $y=\tan^{-1}x+\dfrac{x}{1+x^2}$

77. $y=\arccos x-x\sqrt{1-x^2}$

78. $y=\dfrac{\operatorname{arccot} x}{x}$ **79.** $y=\arctan(e^x)$

80. $y=\operatorname{arccot}(\ln 3x)$ **81.** $y=\dfrac{1-\arctan x}{1+\arctan x}$

82. $y=\arccos x\cdot\operatorname{arccot} x$ **83.** $y=(\arcsin(x^3))^2$

84. $y=\sec^{-1}(e^{x^2})$ **85.** $y=\sec^{-1}(x^2+1)$

86. $y=\csc^{-1}(e^{-x})$ **87.** $y=\sec^{-1}\sqrt{x^2+1}$

88. $y=\sin(\arccos 3x)$ **89.** $y=(\arctan x)^x$

90. $y=(\arcsin x)^{\ln x}$

91. $y=\cos(\operatorname{arccsc}(x^2+1))$

92. $y=\tan\left(\operatorname{arcsec}\sqrt{1+e^{2x}}\right)$

93. $y=\cos\left(\operatorname{arccot}\dfrac{x-1}{\sqrt{2x-1}}\right)$

94–99 Find the equation of the line tangent to the graph of $y=f(x)$ at the indicated x-value. (If needed, round your answer to three decimal places.)

94. $f(x)=\log_2(x^2+1);\quad x=1$

95. $f(x)=\dfrac{(2+x)2^{\ln x}}{x^2e^x};\quad x=1$

96. $f(x)=\arcsin(\ln 3x);\quad x=\dfrac{1}{3}$

97. $f(x)=x\arccos\dfrac{x}{4}-\ln\dfrac{1}{x^2+1};\quad x=2$

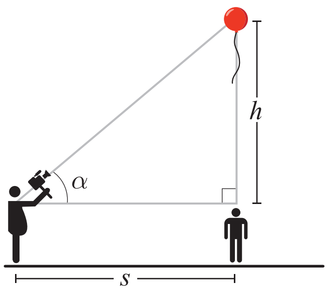
98. $f(x)=(\sin x)^x;\quad x=\dfrac{\pi}{2}$

99. $f(x)=x^{\ln(\arctan x)};\quad x=1$

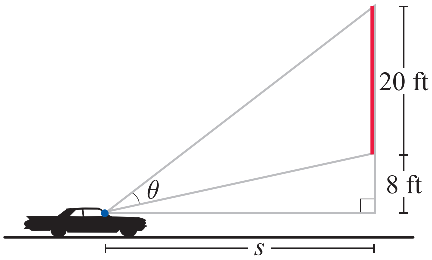
100. Differentiate $f(x)=\arcsin x+\arccos x$. What information about f can you glean from your answer?

101. Repeat Exercise 100 for the function $f(x)=\arcsin(1/x)-\operatorname{arccsc} x$.

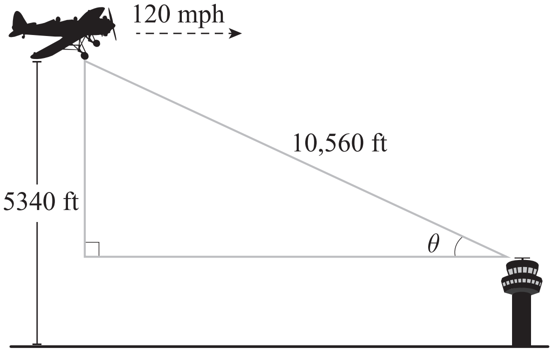
102. A father is filming his child releasing a helium-filled balloon. Assuming that the balloon rises vertically, let the distance between father and child be denoted by s and the height of the balloon, measured from the child, be denoted by h. Find a formula for the angle of elevation α of the camera as it is following the rise of the balloon. Then differentiate with respect to time to find $d\alpha/dt$.



103. The height of the screen of a drive-in movie theater is 20 ft and it is mounted 8 ft above the eye level of a driver who is parked s feet from the screen. Find a formula for the angle θ at which the screen is viewed by this driver. Then differentiate to find the rate of change of the viewing angle as a function of the distance s.



104. An air traffic controller observes a small plane flying horizontally toward the tower and determines from the instrument readings that the distance between the tower and the plane is 10,560 ft, the flying altitude is 5340 ft, and the speed of the plane is 120 mph.



a. Find the angle of elevation θ at which the controller first sees the plane, if the tower is 60 ft high.

b.* Find the angular rate of change $d\theta/dt$ when the plane is 1.25 miles from the controller.

105. Give an alternative definition to $\cot^{-1} x$ so as to make the function continuous and satisfy the identity $\cot^{-1} x = (\pi/2) - \tan^{-1} x$. Graph the function. (**Hint:** Appropriately restrict the domain of $\cot x$. You might also think about the relationship between the graph of the function to be defined and that of $\tan^{-1} x$.)

Concept Check

106–109 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

106. The tangent lines to the graphs of $\ln x$ and $\ln 3x$ have the same slope for all x.

107. If $y = \log \pi$, then $y' = \dfrac{1}{\ln 10} \cdot \dfrac{1}{\pi}$.

108. The derivative of $\csc^{-1} x$ is negative everywhere.

109. The functions $f(x) = \ln x$ and $g(x) = \log_c x$ are constant multiples, hence so are their derivatives.