3.4 Exercises

1–9 Identify $f(x)$ and $u = g(x)$ such that $F(x) = f(u) = f(g(x))$. Also find $h(x)$ wherever $F(x) = f(g(h(x)))$. (Answers will vary.)

1. $F(x) = (3x - 2.5)^6$
2. $F(x) = 2(x^3 - 5x^2 + \pi)^{-4}$
3. $F(x) = 2\sqrt[3]{x^2 - 9}$
4. $F(x) = \dfrac{-3}{5 + \sqrt{x^3 + x}}$
5. $F(x) = \sin \dfrac{1}{x^2 + 1}$
6. $F(x) = 3\cos\left(\dfrac{\tan x}{2}\right)$
7. $F(x) = \csc(3e^x)$
8. $F(x) = \sec\left(e^{2+\sqrt{x}}\right)$
9. $F(x) = \dfrac{3}{\sqrt{\ln(x^2 + 1)}}$

10–60 Find the derivative of the given function.

10. $f(x) = (2x^2 + x)^7$
11. $g(x) = 3(x^5 - \pi x^2 + 7.5)^{11}$
12. $h(x) = \dfrac{1}{2}(x^8 + 5x^3 - ex)^{100}$
13. $F(x) = -3(5 + 2\sqrt{x})^{-5}$
14. $G(x) = (2x^2 - 3x + 1)^{2/3}$
15. $k(x) = -5(x^5 - 2x^3 + 10.5x)^{-2/5}$
16. $f(x) = \sqrt{2 - 4x}$
17. $g(x) = \sqrt{x^2 - 5x + 2}$
18. $h(x) = (4x + 5)^{21}(3x - 7)^{13}$
19. $q(x) = 2(x^3 - 5x)^{2/3}(x + 3)^{5/4}$
20. $r(t) = \dfrac{1}{3t + 1}$
21. $k(z) = \dfrac{1}{1 + 5z - 2z^2}$
22. $F(x) = \left(\dfrac{2x - 3}{1 - 7x}\right)^{10}$
23. $S(v) = \left(\dfrac{2v + 1}{v^2 - 5}\right)^{-3}$
24. $G(y) = \left(\dfrac{3y^2 - 1}{2 + 4y}\right)^{7/5}$
25. $T(s) = \left(\dfrac{s^2 - 1}{s^2 + 1}\right)^{-2/3}$
26. $G(x) = \dfrac{(5 - \pi x^2)^2}{(1 + 2x)^3}$
27. $H(x) = \dfrac{\sqrt{x^2 - 2}}{(x^2 + 2)^2}$
28. $R(x) = \sqrt{\dfrac{1}{x^2 - 1}}$
29. $B(t) = \sqrt[3]{\dfrac{t}{2t^2 + 1}}$
30. $K(s) = \sqrt{\dfrac{2s - 5}{3s + 1}}$
31. $t(x) = \sin(\cos x)$
32. $Q(x) = 2\tan(\sin x)$
33. $P(x) = x\tan^2 x$
34. $w(x) = \cot(x^2)$
35. $U(z) = 5\sec^2 z$
36. $R(x) = x\sqrt{\sin x}$
37. $C(x) = \sin^2(\tan x)$
38. $U(v) = \csc\left(\dfrac{v}{\cos v}\right)$
39. $V(x) = e^{\cos x}$
40. $R(\theta) = e^{\theta \tan\theta}$
41. $w(x) = \sin\sqrt{2x + 1} + e^{\tan\sqrt{2x+1}}$
42. $t(x) = 10^{\sqrt{x}}$
43. $f(x) = \pi 2^{\sin(\pi x)}$
44. $u(x) = 2^{x^2} - 4^{\sqrt{x}}$
45. $t(s) = \tan(2^s)$
46. $u(x) = \cot^2(2^{\sin x})$
47. $E(x) = 5^{5^x}$
48. $K(x) = \sqrt[3]{3^x} + 3^{\sqrt[3]{x}}$
49. $N(x) = \cos^2\left(e^{\cos(x^2)}\right)$
50. $u(t) = \tan^3(t^3 + 3^t)$
51. $C(x) = \cos^2(x^2)$
52. $F(x) = 5^{x^5}$
53. $t(s) = \sqrt{\cos(10^s)}$
54. $G(t) = \sec^{-3}(5^t)$
55. $H(s) = \sin(2^s)\tan(2^s)$
56. $w(s) = \sin(\tan(2^s))$
57. $T(z) = \sin(e^z) + e^{\sin z}$

58.* $q(x) = \sin(\cos(\tan(\cot x)))$

59.* $U(\theta) = \theta + \tan(\theta + \tan(\theta + \tan\theta))$

60.* $v(x) = \left(1 + \left(2 + (3 + 4x)^5\right)^6\right)^7$

61–68 Find an equation for the tangent line to the graph of the given function at the specified point.

61. $f(x) = \sqrt{2x^2 + 1}$; $(2, 3)$
62. $g(x) = (x^2 + 3x + 4)^{2/3}$; $(1, 4)$
63. $q(x) = \cos(\tan x)$; $(0, 1)$
64. $S(x) = \sin(x^2) + \sin^2 x$; $(0, 0)$
65. $M(x) = \dfrac{e^{\cos x}}{x}$; $\left(\pi, \dfrac{1}{e\pi}\right)$
66. $a(x) = 10^{\sqrt{x}}$; $(1, 10)$
67. $h(x) = \dfrac{3x + 1}{\sqrt{x^2 + 3}}$; $(1, 2)$
68. $u(x) = \pi^{\pi^{\sin x}}$; $(0, \pi)$

69–76 Find all x-values where the line tangent to the given curve is horizontal.

69. $f(x)=(x^2-8x+15)^{100}$

70. $g(x)=\dfrac{2x+3}{x^2-2}$

71. $h(x)=\sqrt{x^2+1}$

72. $T(x)=\tan^{10}x$

73. $w(x)=\sec(x^2+2)$

74. $t(x)=\cos(\cos x)$

75. $k(x)=e^{x/(x^2+1)}$

76. $q(x)=\pi^{\cos^2 x}$

77–84 Determine the second derivative of the function.

77. $p(x)=(x^2+5)^{20}$

78. $r(t)=\sqrt{t^2+5}$

79. $g(x)=5\cos^2 x$

80. $c(x)=e^{\tan x}$

81. $F(t)=t\sin(t^2)$

82. $d(x)=5^{5^x}$

83. $G(x)=\sin^2 x+\cos^2 x$

84. $U(s)=\sec\sqrt{s}$

85. Suppose that $f(1)=1$, $f'(1)=-2$, $g(1)=1$, and $g'(1)=5$. If $F(x)=(f\circ g)(x)$ and $G(x)=(g\circ f)(x)$, find $F'(1)+G'(1)$.

86. Let $P(x)=x(x+1)(x+2)\cdots(x+10)$. If $F(x)=(P\circ P)(x)$, find the value of $F'(0)$.

87. Find a formula for the n^{th} derivative of $f(x)=\cos(kx)$, $k\in\mathbb{R}$. (**Hint:** Use the Chain Rule and recognize a pattern.)

88. Repeat Exercise 87 for the function $g(x)=2^{kx}$.

89. Use the Chain Rule to prove that the function $f(x)=\sin(1/x^2)$ is differentiable for $x\neq 0$.

90. Use the Chain Rule to construct a second proof of the Quotient Rule. (**Hint:** Rewrite $f(x)/g(x)$ as $f(x)\cdot[g(x)]^{-1}$.)

91. Use the Chain Rule to prove that the derivative of an even function is odd and vice versa.

92. Find all points where the line tangent to the graph of $y=\sqrt[3]{\cos x}$ is horizontal, as well as those where it is vertical.

93.* A spherical balloon is being inflated so that its radius is increasing at a rate of $dr/dt=0.1$ in./s. Find the rate at which the volume of the balloon is increasing when its radius is $r=4$ in. (**Hint:** Notice that $V(t)=V(r(t))$ and use the Chain Rule.)

94.* Pouring sand is forming a conical shape so that the radius of the bottom of the cone is always twice its height throughout the process. If the height of the cone is increasing at a rate of $dh/dt=0.5$ mm/s, find the rate at which the volume of the cone is increasing when its height is $h=50$ mm. (See the hint given in Exercise 93.)

95. The position function of a vibrating loudspeaker cone is given by $x(t)=10^{-3}\cos 1500t$, where distance is measured in meters, time in seconds. As indicated by the position function, the cone is at one of its extreme positions at $t=0$. Use the above information to find **a.** the maximum velocity of the cone and **b.** the maximum acceleration of the cone.

96. The position function for damped harmonic motion of an object of mass m is

$$x(t)=Ae^{-\frac{k}{2m}t}\cos(\omega t),$$

where A is the amplitude and k and ω are constants specific to the motion. Find the velocity and acceleration functions for this motion.

97. Unless conditions are extreme, most gases obey the so-called *Ideal Gas Law*, which says $PV=nRT$, where P stands for pressure measured in pascals (Pa), V for volume, n for the number of moles (mol) of gas in the container, T denotes temperature measured in kelvins (K), and R is the *universal gas constant*, which is the same for all gases. Suppose 5 moles of gas are being slowly compressed by a piston in a container so that $dV/dt=-2\cdot 10^{-8}\ \text{m}^3/\text{s}$. Assuming that temperature is being kept constant at $T=293$ K throughout the process, find the rate of change of pressure with respect to time when $V=10^{-3}\ \text{m}^3$. (Use $R\approx 8.315\ \text{J}/(\text{mol}\cdot\text{K})$.)

3.4 Technology Exercises

98–99 The Maclaurin polynomial of order 2 of the function $f(x)$ is used to approximate $f(x)$ near $x=0$. It is defined as

$$P_2(x)=f(0)+f'(0)x+\frac{1}{2}f''(0)x^2.$$

Find the Maclaurin polynomial of order 2 for $f(x)$. Then use a graphing utility to graph f along with its Maclaurin polynomial. (We will learn more about Maclaurin polynomials in Section 10.8.)

98. $f(x)=\cos(\sin x)$

99. $f(x)=\dfrac{1}{x^2+1}$