10.6 Exercises

1–24 Determine whether the alternating series converges and give a reason for your answer.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$

2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{\sqrt{n}}$

3. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n+1}{3n-2}$

4. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{2n-1}$

5. $\sum_{n=2}^{\infty}(-1)^{n}\frac{3}{n\ln n}$

6. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^{2n}-1}{4^n}$

7. $\sum_{n=0}^{\infty}(-1)^{n}\frac{n^2+1}{n^2+2}$

8. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2 2^n}$

9. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n}{\ln 2n}$

10. $\sum_{n=1}^{\infty}(-1)^{n}\frac{3^n}{n^3}$

11. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln(n+1)}{n}$

12. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\sqrt{n}}{n+2}$

13. $\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{n+1}{5\sqrt{n}}\right)^n$

14. $\sum_{n=1}^{\infty}(-1)^{n+1}\sin\frac{1}{n}$

15. $\sum_{n=0}^{\infty}(-1)^{n}\frac{\sqrt{n}+3}{\sqrt{n+3}}$

16. $\sum_{n=0}^{\infty}(-1)^{n}\frac{2^n}{n!}$

17. $\sum_{n=0}^{\infty}(-1)^{n}\frac{1}{\sqrt[3]{n^2+1}}$

18. $\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{4}{5}\right)^n$

19. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\tan^{-1}n}$

20. $\sum_{n=1}^{\infty}(-1)^{n}\tan^{-1}\left(\frac{1}{n}\right)$

21. $\sum_{n=1}^{\infty}(-1)^{n}e^{-n}\sin n$

22. $\sum_{n=1}^{\infty}\left(\frac{-1}{1.1}\right)^n$

23. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{1+\frac{1}{n}}$

24. $\sum_{n=2}^{\infty}\frac{(-1)^n}{(\ln n)^3}$

25–30 Approximate the sum of the alternating series, accurate to at least the indicated number of decimal places.

25. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^3}$; accurate to 2 decimal places

26. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n3^n}$; accurate to 4 decimal places

27. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n+1)!}$; accurate to 5 decimal places

28. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2^n n!}$; accurate to 6 decimal places

29. $\sum_{n=2}^{\infty}\frac{(-1)^{n}\ln n}{e^n}$; accurate to 3 decimal places

30. $\sum_{n=2}^{\infty}\frac{(-1)^{n}\ln n}{2n^4}$; accurate to 4 decimal places

31–58 Determine whether the given series converges absolutely, converges conditionally, or diverges. Use any convergence test discussed so far in this chapter.

31. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{n^2+1}$

32. $\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$

33. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n^{1/n}}$

34. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{(2n)!}$

35. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\sqrt[3]{n}}{\sqrt{n}}$

36. $\sum_{n=1}^{\infty}(-1)^{n+1}\cos\frac{1}{n}$

37. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2+4}$

38. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{5^n}{n!}$

39. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+4}$

40. $\sum_{n=2}^{\infty}(-1)^{n}\frac{1}{n\sqrt{\ln n}}$

41. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\ln n}{n^{1.1}}$

42. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{5n-1}$

43. $\sum_{n=1}^{\infty}\frac{(-2)^n}{n^2}$

44. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n(\sqrt{n}+2)}$

45. $\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$

46. $\sum_{n=1}^{\infty}(-1)^{n+1}n\sin\frac{1}{n}$

47. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^4}{4^n}$

48. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\tan^{-1}n}{n^2}$

49. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$

50. $\sum_{n=1}^{\infty}\left(\ln\frac{2}{n}\right)^n$

51. $\sum_{n=2}^{\infty}\left(-\frac{\ln n}{n}\right)^n$

52. $\sum_{n=1}^{\infty}\left(\frac{1-2n}{5n}\right)^n$

53. $\sum_{n=2}^{\infty}(-1)^{n+1}\left(\frac{2^n}{n^2}\right)^n$

54. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^3}{e^n}$

55. $\sum_{n=2}^{\infty}\frac{(-\pi)^n}{\left(3+\frac{1}{n}\right)^n}$

56. $\sum_{n=1}^{\infty}(-1)^n\log\left(5+\frac{1}{n^2}\right)$

57. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{2}\right)^n$

58. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}3^n}{2^n n!}$

59. Kate claims that the series

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{9}+\cdots+\frac{1}{n}-\frac{1}{n^2}+\cdots$$

converges by Leibniz's Test, because it is an alternating series of the form $\sum(-1)^{n+1}a_n$ and $a_n \to 0$. Is she right? Why or why not?

60. Prove that the series consisting of the positive terms of the alternating harmonic series diverges to infinity.

61. Repeat Exercise 60 for the negative terms of the alternating harmonic series.

62.* Prove the first part of Riemann's Rearrangement Theorem; that is, the statement that the terms of a conditionally convergent alternating series can be rearranged so that the sum of the series is any preselected real number. (**Hint:** Let s be the desired, preselected sum, and start adding up the positive terms of the series until their sum first becomes greater than s. Then add enough negative terms until the resulting sum becomes less than n, and continue.)

63.* Prove the second and third parts of Riemann's Theorem; namely, that the terms of the series in Exercise 62 can be rearranged to diverge to ∞ or $-\infty$, or to diverge in an oscillating manner.

64.* Prove the following so-called Polynomial Test for infinite series: If $p(x)$ and $q(x)$ are polynomials with degrees r and s, respectively, then the series $\sum\frac{p(n)}{q(n)}$ is convergent if and only if $s > r + 1$ (we are assuming $q(n) \neq 0$ for any value of the summation index n).

65. Prove that if the series $\sum a_n$ is absolutely convergent, then $\sum a_n^2$ is convergent. Then give an example to show that the statement is not true if $\sum a_n$ is conditionally convergent.

10.6 Technology Exercises

66–68 Use a graphing utility to solve the problem.

66. Find an approximation of e with an error no greater than 10^{-8}, knowing that $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}=\frac{1}{e}$. How many terms did you use?

67. Find an approximation of π with an error no greater than 10^{-3}, knowing that $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=\frac{\pi}{4}$. How many terms did you use?

68. Find an approximation of $\ln 2$ with an error no greater than 10^{-3}, knowing that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=\ln 2$. How many terms did you use?