1.1 Exercises

1–22 Find the domain and range of the given relation.

1. $R = \{(-3,1), (-3,5), (-3,-1), (0,0), (1,2)\}$

2. $S = \{(3,-1), (2.6,6), (\pi,0.5), (e,100)\}$

3. $T = \{(4,5.98), (-2,-8), (-2,0), (3,\cos 3)\}$

4. $U = \{(4,4), (4,\pi), (\pi,4), (4,0)\}$

5. $F = \{(\text{Tanisha}, \text{swimming}), (\text{Don}, \text{biking}), (\text{Peter}, \text{skating}), (\text{David}, \text{skateboarding})\}$

6. $L = \{(\text{Lin}, \text{Chinese}), (\text{Chuck}, \text{English}), (\text{Sarah}, \text{German}), (\text{Daniel}, \text{Hungarian})\}$

7. $A = \{(x,y) \mid x \in \mathbb{Z},\ y = 2x + 3\}$

8. $B = \left\{(x,y) \mid x \in \mathbb{R},\ y = \frac{x}{2}\right\}$

9. $C = \{(x, -2x + 7) \mid x \in \mathbb{Z}\}$

10. $D = \{(2x, 5y) \mid x \in \mathbb{N},\ y = x + 1\}$

11. $3x = y + 5$

12. $\sqrt{2}x - 1.2y = 3$

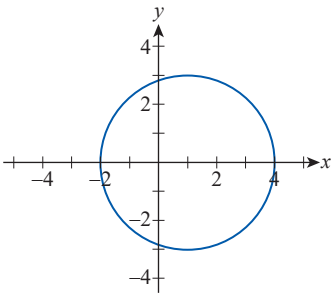
13. $x = 5$

14. $y = \pi$

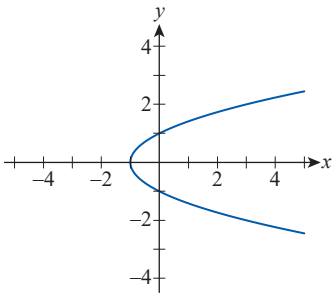
15. $x = 3y^2 - 1$

16. $y = |x| - 2$

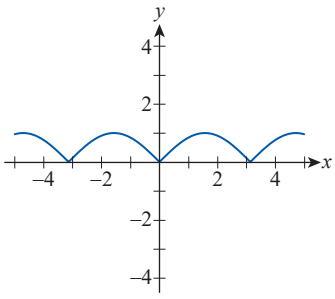
17.



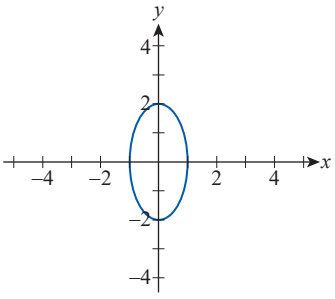
18.



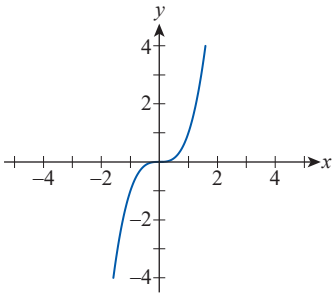
19.



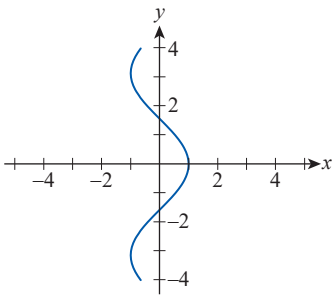
20.



21.



22.



23–26 Find the domain and range of the given relation. Choose an appropriate domain on which the given relation makes sense. (Answers will vary.)

23. $\{(x,y)\,|\,\text{student } x \text{ is registered for course } y\}$

24. $\{(x,n)\,|\,x \text{ wears size } n \text{ shoes}\}$

25. $\{(x,y)\,|\,y \text{ is the father of } x\}$

26. $\{(P,n)\,|\,\text{person } P \text{ weighs } n \text{ pounds}\}$

27–30 List the ordered pairs in the given relation R on the set $A=\{1,2,3,4,5\}$.

27. $(a,b)\in R$ if and only if $a=b$

28. $(a,b)\in R$ if and only if $a<b$

29. $(a,b)\in R$ if and only if $a \mid b$

30. $(a,b)\in R$ if and only if $a+b=5$

31–34 Determine whether the given relation is a function. If the relation is not a function, explain why.

31. $A=\{(1,3),(-2,4),(0,4)\}$

32. $B=\{(0,0),(0,1),(2,3),(4,5),(6,7)\}$

33. $C=\{(-1,2),(\pi,3),(-1,0),(1,2)\}$

34. $D=\{(1,2),(2,1),(3,4),(4,3)\}$

35–40. Determine whether the relations given in Exercises 17–22 are functions. For those that are not, explain why.

41–52 Determine whether the given equation is a function. If the equation is not a function, explain why.

41. $y=3x-4$

42. $x=3y-4$

43. $x^2+y^2=9$

44. $x+y^2=9$

45. $x^2+y=9$

46. $y=\sqrt[3]{x}$

47. $x=x^3-y$

48. $xy=4$

49. $x=\pi$

50. $y=\dfrac{3x}{x^2+1}$

51. $F=5r^2\pi$

52. $V=\dfrac{4}{3}r^3\pi$

53–58 Express y explicitly as a function of x from the given relation.

53. $\dfrac{x+3y}{2}=5$

54. $\dfrac{x-3y}{5}=\dfrac{2y+7x}{3}$

55. $3x^2-y=5-x+2y$

56. $x+7-3y=(x-2)^2+y$

57. $yx^2-y=3x+1$

58. $x+1=yx^2$

59–66 Find the value of the given function for **a.** $f(-2)$, **b.** $f(x+1)$, **c.** $f(x+h)$, and **d.** $\dfrac{f(x+h)-f(x)}{h}$.

59. $f(x)=\dfrac{1}{3}x+2$

60. $f(x)=\dfrac{5x-3}{2}$

61. $f(x)=x^2-3$

62. $f(x)=3x^2-5x+\dfrac{1}{2}$

63. $f(x)=\sqrt{x}$

64. $f(x)=\dfrac{1}{\sqrt{x+2}}$

65. $f(x)=\dfrac{1}{x+1}$

66. $f(x)=(x-1)^3+5$

67–72 Identify the domain, codomain, and range of the given function.

67. $f\colon \mathbb{N}\to\mathbb{N},\quad f(x)=x+1$

68. $g\colon \mathbb{N}\to\mathbb{Z},\quad g(x)=3x-2$

69. $h\colon \mathbb{Z}\to\mathbb{Z},\quad h(x)=x^2$

70. $F\colon \mathbb{R}\to\mathbb{R},\quad F(x)=2x^4+1$

71. $G\colon [0,\infty)\to\mathbb{R},\quad G(x)=\sqrt{x}$

72. $H\colon \mathbb{Q}^+\to\mathbb{Q},\quad H(x)=\dfrac{1}{x}$ (Note that $\mathbb{Q}^+$ stands for the set of positive rational numbers.)

73–82 Find the implied domain of the given function.

73. $f(x)=\dfrac{x+1}{x^2-x-6}$

74. $g(x)=\sqrt{3x+2}$

75. $h(x)=\dfrac{2}{\sqrt{x^2-4x+3}}$

76. $F(t)=\dfrac{1}{\sqrt{4-t^2}}$

77. $G(s)=\sqrt{2-s}+\sqrt{s}$

78. $D(h)=\dfrac{\frac{1}{\sqrt{1+h}}-1}{h}$

79. $R(x)=\dfrac{1}{|2x+3|}$

80. $H(z)=z^{3/2}-2$

81. $F(\theta)=\dfrac{2}{1-\cos\theta}$

82. $\varphi(x)=\dfrac{5}{\sin x-\frac{\sqrt{2}}{2}}$

83–88 Turn the formula into a function by finding the argument(s) of the function. Identify any functions of two variables.

83. $C = 2\pi r$

84. $V = \frac{4}{3}r^3\pi$

85. $C = \frac{5}{9}(F - 32)$

86. $A = 6a^2$

87. $V = \frac{1}{3}b^2h$

88. $E = \frac{1}{2}mv^2$

89–94 Use the vertical line test to decide whether y is a function of x.

89. $y^3 + 1 = x$

90. $2x^2 + 2y^2 = 18$

91. $y^2 + 1 = x$

92. $x = (y - 2)^2$

93. $x = y^3 - 2y$

94. $yx^2 = 1$

95–101 Find all open intervals of monotonicity (intervals where the function is increasing or decreasing) for the given function.

95. $f(x) = (x - 1)^2$

96. $g(x) = 4x - x^2$

97. $h(x) = x^3 - 12x$

98. $k(x) = \frac{x^2}{x^2 + 1}$

99. $F(x) = |x - 1|$

100. $G(x) = 2x + |3x - 1|$

101. $H(x) = |x + 1| + |x - 2|$

102–110 Discuss the symmetry of the given equation. Give reasons. (**Hint:** See Example 9.)

102. $y = x^2 - 1$

103. $x = y^2 - 1$

104. $x^4 + y^4 = 5$

105. $|x| + |y| = 2$

106. $x - |y| = 2$

107. $xy = 2$

108. $y = \frac{2x^3 - x}{x^4 + x^2}$

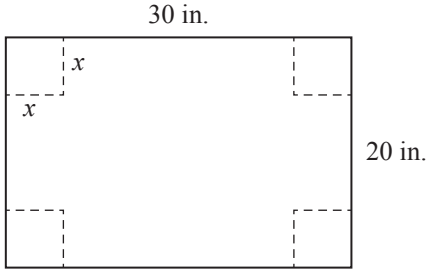
109. $y^2 + 6x = x^3$

110. $y = (x - 1)^2$

111. Express the perimeter of a square as a function of its area.

112. Express the area of an equilateral triangle as a function of its perimeter.

113. An open-top box is constructed from a 20 in. by 30 in. piece of cardboard by cutting out a square of side length x from each of the four corners and folding up the sides, as shown in the figure below. Express the volume of the box as a function of x.



114. Express the surface area A of a cube as a function of its volume V.

115. The height of a circular cone is equal to the diameter of its base. Express its volume V as a function of the radius r of the base.

116. Express the volume of a sphere as a function of its surface area.

117. Knowing that water boils at 212 °F, which corresponds to 100 °C, and the fact that freezing occurs at 32 °F, which is 0 °C, obtain the linear function $C(F)$ that expresses the Celsius temperature C as a function of the Fahrenheit reading F.

118.* The organizers of an educational leadership seminar series have found that the seminar attracts 100 participants when the registration fee is set to \$150. They estimate that for each increase of \$10 in the registration fee, they will end up with 5 fewer registered participants. Express the revenue R as a function of the registration fee F.